Universidad de los Andes, MATE-4241
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Operator Theory

Problem Sheet 7

Hand in: 23th of September 2010

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1. Let H be a complex Hilbert space and S, T selfadjoint linear operators on H.

   (a) Let $z \in \varrho(T)$ and $\lambda \in \mathbb{C} \setminus \{z\}$. Show that $\lambda \in \sigma_{\text{ess}}(T)$ if and only if there exists a sequence $(x_n)_{n\in\mathbb{N}} \subseteq \mathcal{D}(T)$ such that

   $$x_n \not\to 0, \quad x_n \xrightarrow{w} 0 \quad \text{and} \quad \big((T-z)^{-1} - (\lambda - z)^{-1}\big)x_n \to 0 \qquad \text{for } n \to \infty.$$

   (b) Assume that there exists a $z \in \varrho(S) \cap \varrho(T)$ such that $(S-z)^{-1} - (T-z)^{-1}$ is compact. Show that then $\sigma_{\text{ess}}(S) = \sigma_{\text{ess}}(T)$.

2. Let H be a complex Hilbert space and $S(H \to H)$ be a closable linear operator. Show that the deficiency numbers are constant in connected components of its domain of regularity $\Gamma(S)$.