Universidad de los Andes
MATE-4330

Functional Analysis

Problem Sheet 14

Compact operators (II).

Hand in: May 7, 2010

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1. Let $X = C[0,1]$ and $k \in C[0,1]^2$. Show that the following operator is compact:

$$T : X \to X, \qquad (Tx)(t) = \int_0^t k(s,t)x(s)\,\mathrm{d}s$$

2. Let X and T as in the previous exercise. Show that $\sigma(T) \setminus \{0\} = \emptyset$. Show that for every $\lambda \in \mathbb{C} \setminus \{0\}$ and every $y \in X$ there exists exactly one $x \in X$ such that $(T - \lambda)x = y$.

3. Let H_1, H_2 be Hilbert spaces and $T \in L(H_1, H_2)$. Then the following is equivalent:
   (a) T is compact.
   (b) T^* is compact.
   (c) T^*T is compact.

4. Let H_1, H_2 be Hilbert spaces and $T \in L(H_1, H_2)$ a compact operator. Let $(P_n)_{n\in\mathbb{N}}$ be a monotonically increasing sequence of projections with $P \xrightarrow{s} \mathrm{id}$. Then $\|K - KP_n\| \to 0$.