Universidad de los Andes
MATE-2201

Functional Analysis

Problem Sheet 11

Adjoint operators; spectrum of linear operators.

Hand in: April 16, 2010

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1. Let H_1, H_2, H_3 be Hilbert spaces and $S(H_1 \to H_2)$ and $T(H_2 \to H_3)$ be densely defined linear operators.
   (a) If $T \in L(H_2, H_3)$, then TS is densely defined and $(TS)^* = S^*T^*$.
   (b) If S is injective and $S^{-1} \in L(H_2, H_1)$, then TS is densely defined and $(TS)^* = S^*T^*$.
   (c) If R is injective and $R^{-1} \in L(H_2, H_1)$, then R^* is injective and $(R^*)^{-1} = (R^{-1})^*$.

2. Let $X = C([0,1])$ and $a \in C([0,1])$. Show that

$$A : X \to X, \qquad (Ax)(t) = a(t)x(t)$$

   is a bounded linear operator. Find $\|A\|$, $\sigma(A)$, $\sigma_{\mathrm{p}}(A)$, $\sigma_{\mathrm{c}}(A)$ and $\sigma_{\mathrm{r}}(A)$.

3. (a) Let $H = C([0,1])$ and

$$T : H \to H, \qquad (Tx)(t) = \int_0^t x(s)\mathrm{d}s.$$

   Find $\sigma(T)$, $\sigma_{\mathrm{p}}(T)$, $\sigma_{\mathrm{c}}(T)$ and $\sigma_{\mathrm{r}}(T)$.

   (b) Let $H = \{f \in C([0,1]) : x(0) = 0\}$ and

$$S : H \to H, \qquad (Sx)(t) = \int_0^t x(s)\mathrm{d}s.$$

   Find $\sigma(S)$, $\sigma_{\mathrm{p}}(S)$, $\sigma_{\mathrm{c}}(S)$ and $\sigma_{\mathrm{r}}(S)$.

4. Let X be a Banach space, $T \in L(X)$ and $P \in \mathbb{C}[X]$ a polynomial. Then

$$\sigma(P(T)) = P(\sigma(T)).$$