Universidad de los Andes — Departamento de Matemáticas

MATE-1207: Calculo vectorial

Tarea 3

2 pts. **Problem 1.** Evaluate $\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \, \sqrt{1+\cos^2 x}\, \mathrm{d}x\, \mathrm{d}y$.

2 pts. **Problem 2.** Sketch and find the volume of

$$E = \{(x,y,z) \,:\, 0 \le z \le \pi/2,\ x^2+y^2 \le (1-\cos z)^2\}.$$

2 pts. **Problem 3.** Find the mass of the solid

$$\sqrt{y} \le x \le \pi^{1/5}, \quad 0 \le y \le \pi^{2/5}, \quad 0 \le z \le \sin x^5$$

if the mass density is $\rho(x,y,z) = y$.

2 pts. **Problem 4.** Let S_1 be the sphere with radius 2 centred at the origin and S_2 be the sphere with radius 2 and centre $(0,0,2)$. Find the volume of $(S_1 \setminus S_2) \cap \{z \ge 0\}$.

2 pts. **Problem 5.** Let $D_1 = \{(x,y,z) \,:\, x^2+y^2 \le 4\}$ and $D_2 = \{(x,y,z) \,:\, 4x^2+4y^2+z^2 \le 64\}$. Sketch the intersection $D_1 \cap D_2$ and find its volume.

2 pts. **Problem 6.** Evaluate $\int_1^2 \int_0^{\sqrt{2y-y^2}} 1 \,\mathrm{d}x\,\mathrm{d}y$.

2 pts. **Problem 7.** Evaluate $\iiint_E xyz\,\mathrm{d}V$ where E lies between the spheres with radius $R=2$ and $R=4$ and centre in the origin and above the cone with angle $\Phi = \pi/3$ and centre at the origin. Sketch E.

2 pts. **Problem 8.** Use the transformations $x = (u-2v)/5$ and $y = (2u+v)/5$ to find

$$\iint_R \sqrt{6x^2+9y^2+4xy}\;\mathrm{d}A$$

where $R = \{(x,y) \,:\, 6x^2+9y^2+4xy \le 4\}$.

4 pts. **Problem 9.** Considere en el espacio tridimensional los cilindros

$$y^2+z^2 = 1 \quad \text{y} \quad x^2+z^2 = 1.$$

(a) Use Maple para hacer el dibujo.

(b) Plantee la integral con todos sus límites para haller el volumen del sólido acotado por la de intersección de los dos cilindros en el primer octante usando

- coordenadas cartesianas,
- coordenadas cilíndricas.

(c) Evalúe (a mano) el volumen del sólido en el primer octante.

(d) Haga una estimativa del valor del volumen del sólido en el primer octante usando sumas de Riemann (ver Lab3) para particiones `RS(2,2), RS(2,4), RS(10,20), RS(100,100).`

Extra problem for those who believed that $\frac{\partial^2 f}{\partial x \partial y}(x,y) = \frac{\partial^2 f}{\partial y \partial x}(x,y)$ by *definition*:

Problem 1. Let

$$f : \mathbb{R}^2 \to \mathbb{R}, \quad f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y) \neq 0, \\ 0, & (x,y) = 0. \end{cases}$$

Then f is continuously differentiable, in $(0,0)$ all second order partial derivatives exist but

$$\frac{\partial^2 f}{\partial x \partial y}(0,0) \neq \frac{\partial^2 f}{\partial y \partial x}(0,0).$$

There exist also functions where the order of integration of iterated integrals cannot be changed without changing the value of the integral:

Problem 2. Let $I = [0,1]$ and define for all $n \in \mathbb{N}$

$$g_n(t) = \begin{cases} n(n+1), & (n+1)^{-1} < t < n^{-1}, \\ 0, & \text{else.} \end{cases}$$

Let $f : I \times I \to \mathbb{R}, \quad f(x,y) = \sum_{n=1}^{\infty} \Big(g_n(x) - g_{n+1}(x) \Big) g_n(y)$. Then

$$\int_0^1 \int_0^1 f(x,y) \,\mathrm{d}x \,\mathrm{d}y \neq \int_0^1 \int_0^1 f(x,y) \,\mathrm{d}y \,\mathrm{d}x.$$