

## Tarea 2

4 pts.

**Problem 1.** Determine el dominio de continuidad de

$$f(x, y) = \begin{cases} \frac{xy}{x^2+xy+y^2} & \text{si } (x, y) \neq (0, 0), \\ 0 & \text{si } (x, y) = (0, 0). \end{cases}$$

4 pts.

**Problem 2.** Sea

$$f(x, y) = \frac{2x^2y}{x^4 + y^2}, \quad (x, y) \in \mathbb{R}^2 \setminus (0, 0).$$

(a) Halle el límite  $\lim_{(x,y) \rightarrow (0,0)} f(x, y)$ , si existe; si no existe, explique porque no existe.(b) Describa algunas curvas de nivel. ¿Para cuál  $k$  son las curvas de nivel vacías?

4 pts.

**Problem 3.** Verifica para los siguientes funciones que las curvas de nivel y las líneas de flujo del campo vectorial gradiente son perpendiculares haciendo en el mismo plano las curvas de nivel y el campo vectorial gradiente. También incluya un gráfico de la superficie (del grafo de la función):

(a)  $f(x, y) = x^4 + y^4 - 4xy + 1$ ,

(b)  $f(x, y) = y^2 - x^2$ .

4 pts.

**Problem 4.** Find all local and global extrema and saddle points of

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad f(x, y) = x^2 + y^2 - xy - 2x + y.$$

4 pts.

**Problem 5.** An ellipsoid with semiaxes  $a = 4$  cm,  $b = 5$  cm and  $c = 6$  cm shall be coated with a 0.1 cm thick layer of varnish. Use linear approximation to estimate the volume of varnish.

For problems 6 and 7 let  $T(x, y) = ye^{-x^2-y^2}$ ,  $(x, y) \in \mathbb{R}^2$ , be a temperature field in the  $xy$ -plane.

4 pts.

**Problem 6.** (a) Find the linear approximation of  $T$  at  $(x_0, y_0) = (0, 1)$ .(b) At what points  $(x, y, T(x, y))$  is the tangent plane to the graph of  $T$  perpendicular to the line  $L : \vec{r}(t) = (1, 2, \pi) + t(0, 0, 1)$ ?

4 pts.

**Problem 7.** (a) A particle moves on the plane. Its position at time  $t \in \mathbb{R}$  is given by  $p(t) = (t + \cos(t), \sin(t))$ . What is the rate of change of temperature that the particle experiences at time  $t = \pi$ ?(b) Suppose another particle is located at the point  $(1, 1)$ . In what direction must it move to experience the maximal possible change of temperature? What is this maximal value? What change of temperature does the particle feel if it moves parallelly to the vector  $\vec{v} = \langle 3, 4 \rangle$ ?(c) At what point  $(x, y)$  is the change of temperature in  $x$ -direction largest?