

Segundo Parcial (Examen supletorio) 200820, 7 Octubre 2008

CÓDIGO: _____ NOMBRE: _____

Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Sum

Solve the following problems. Give all necessary steps to find the solutions. Hand in this sheet and all sheets of paper you used. Put your name on every sheet of paper you hand in.¹

Good luck!

4 pts.

Problem 1. Determine the maximal domain of continuity of

$$f(x, y) = \begin{cases} \frac{(x-1)y}{(x-1)^2 + (x-1)y + y^2}, & (x, y) \in \mathbb{R}^2 \setminus \{(1, 0)\}, \\ 0, & (x, y) = (1, 0). \end{cases}$$

4 pts.

Problem 2. A function $x = f(y, z)$ with $f(\pi, 1) = \pi$ is defined implicitly by the equation $\tan \frac{y}{z} - 2y \cos x - \frac{y^2 z}{x} = \pi$. Find all partial derivatives of first order of f at $(\pi, 1)$.**Problem 3.** Let $T : \mathbb{R}^2 \setminus \{(0, 0)\} \rightarrow \mathbb{R}$, $T(x, y) = xe^{x^2+y^2}$ be a temperature field.

6 pts.

(a) Compute all partial derivatives of first and second order of T .

2 pts.

(b) Find the rate of change of T at $(1, 2)$ in direction to the point $(3, 1)$?

2 pts.

(c) Compute the largest possible rate of change of T at $(1, 2)$. In what direction is no change of T ?

2 pts.

(d) Find the tangent plane to the graph of T at $(1, 2, T(1, 2))$.

2 pts.

(e) Compute the tangent line to the level curve given by $f(x, y) = e^2$ at $(1, 1)$.

4 pts.

(f) Evaluate $T(0, 1)$ and use linear approximation to find $T(0.2, 0.9)$.

2 pts.

(g) The position of a particle moving in the temperature field is $\vec{p}(t) = (2-t, \cos(t\pi))$. What change of temperature does the particle experience at time $t = 2$?

4 pts.

Problem 4. Let $f(x, y) = \exp(4x^2 + y^2)$. Describe in words and sketch the graph, some level sets and gradient vectors of f .**Problem 5.** Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$, $f(x, y) = 3xy$.

9 pts.

(a) Find all local and global extrema and all saddle points.

9 pts.

(b) Let

$$g : \mathbb{R}^2 \rightarrow \mathbb{R}^2, \quad g(x, y) = x^2 + y^2 - 2.$$

Find all local and global extrema of f under the restriction $g(x, y) = 0$.

¹Resuelva las siguientes preguntas (sin desarrollo sus respuestas no valen!). Escriba ordenadamente y devuelva esta hoja con todas las hojas que haya utilizado. Escriba su nombre en cada hoja que haya utilizado. Respete el juramento uniandino: cualquier caso de fraude será reportado. Buena suerte!