Universidad de los Andes Departamento de Matemáticas

MATE-1215: Calculo integral – Ecuaciones diferencial

Tarea 3

Problem 1. Let a and b be a positive constants. Find the area of the region that lies inside the curves given in polar coordintes by

$$C_1 : r = a \sin\vartheta, \qquad C_2 : r = b \sin\vartheta, \qquad \vartheta \in \mathbb{R}.$$

Find the length of the curve which bounds this region.

Problem 2. Proof or find a counter example:

(a) Every convergent sequence of real numbers is bounded.

(b) Every unbounded sequence of real numbers is divergent.

(c) Let $(a_n)_{n\in\mathbb{N}}$, $(b_n)_{n\in\mathbb{N}}$ be sequences of real numbers. If $(a_n)_{n\in\mathbb{N}}$ converges and $(b_n)_{n\in\mathbb{N}}$ diverges to ∞, then $\lim_{n\to\infty}\big(a_n + b_n\big) = \infty$.

(d) Show that a sequence $(a_n)_{n\in\mathbb{N}}$ of real numbers converges if and only if the sequences $(a_{2n})_{n\in\mathbb{N}}$ and $(a_{2n+1})_{n\in\mathbb{N}}$ converge and have the same limit.

Problem 3. Do the following sequences converge? If they converge, find the limit.

(a) The Fibonacci sequence,

(b) $\left(\dfrac{2^n}{n!}\right)_{n\in\mathbb{N}}$,

(c) $\left(\sqrt{n^2+3n} - n\right)_{n\in\mathbb{N}}$,

(d) $\left(\dfrac{1 + 2n\sin(n) + 3n^3}{\pi - 3n - 2n^3}\right)_{n\in\mathbb{N}}$.

Problem 4. Does the sequence $(a_n)_{n\in\mathbb{N}}$ defined by

$$a_0 := 1, \ \ a_{n+1} := a_n + \frac{1}{a_n}, \quad n \in \mathbb{N},$$

converge? If it converges, find the limit.