MATE-1215: Calculo integral – Ecuaciones diferencial

Tarea 2

Problem 1. Let c be a positive constant. Sketch the directional fields and find solutions of the following differential equations:

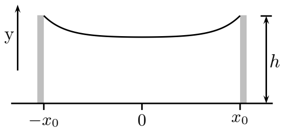
(a) $\dfrac{\mathrm{d}x}{\mathrm{d}t} = x \ln\left(\dfrac{c}{x}\right), \quad x > 0,$ (b) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = xy.$

Problem 2. Solve the following initial value problems. What are the maximal domains of the solutions?

(a) $y'(x) - x(y(x) + y(x)^2) = 0, \quad y(0) = 2,$
(b) $y'(x) = xy(x) + x\,, \quad y(2) = 3.$

Problem 3.

Galileo Galilei conjectured that a chain hanging freely from two posts of the same height h at distance $2x_0$ describes a parabola.



Physical considerations show that the curve described by the chain satisfies

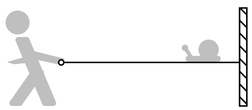
$$y''(x) = \gamma\sqrt{1 + (y'(x))^2}, \quad x \in [-x_0, x_0],$$

where $\gamma \neq 0$ is a constant depending only on x_0 and the length of the chain. Prove or disprove Galilei's conjecture. How are γ, the length of the chain and x_0 related?

Hint. Solve the differential equation $z' = \gamma\sqrt{1+z^2}$ for suitable initial value.

Problem 4. A snail sits on the fixed end of an extendable rubber band of length L. At time $t = 0$ it starts to move with constant velocity v away from the fixed end. At the same time, Achilles takes the lose end of the rubber band and starts running with velocity w away from the fixed end so that the band extends. Will the snail ever reach Achilles? If so, when will this happen?

The snail and Achilles are assumed to be immortal. Moreover, the rubber band is assumed to be infinitely extendable and the snail is a point.



Problem 5. Let C be the curve with parametrization $x = f(t)$ and $y = g(t)$ where $f(t) = \sin t$, $g(t) = t + 2\cos t$. Find the tangent on C at $(1, \pi/2)$. At what points are the tangents to C parallel to the coordinate axes? Sketch the curve C.

Problem 6. Sketch the cardioid κ given by

$$x(t) = (1 - \cos t)\cos t, \quad y(t) = (1 - \cos t)\sin t, \qquad t \in [0, 2\pi].$$