MATE-1215: Calculo integral – Ecuaciones diferencial

Tarea 1[1]**.**

Problem 1. Evaluate the following integrals:

(a) $\displaystyle\int \frac{1}{x}\cos(\ln x)\sin(\ln x)\,\mathrm{d}x,$

(b) $\displaystyle\int \frac{1}{1-\cos x}\,\mathrm{d}x,$

(c) $\displaystyle\int \mathrm{e}^x\sqrt{1+\mathrm{e}^{2x}}\,\mathrm{d}x,$

(d) $\displaystyle\int \frac{(\tan x)^3+(\tan x)^2}{(\cos x)^3\sin x}\,\mathrm{d}x,$

Problem 2. (a) Determine for what values of x the integrand is defined and evaluate the integrals:

$$\int \frac{x^2}{\sqrt{4x-x^2}}\,\mathrm{d}x, \qquad \int \frac{x+4}{x^3-1}\,\mathrm{d}x.$$

(b) Sketch the curves given by

$$y = 5\ln x, \quad y = x\ln x, \qquad x>0,$$

and find the area enclosed by them.

Problem 3. (a) Let $f:[0,\infty)\to\mathbb{R}$, $f(x):=\sqrt{x}$. Explain why $\displaystyle\int_0^\infty f(x)\,\mathrm{d}x$ is an improper integral. Does it converge or diverge? Compute its value if possible.

(b) Does the improper integral $\displaystyle\int_0^\pi \tan x\;\mathrm{d}x$ exist? Justify your answer and compute the value of the integral if it exists.

(c) Does the integral $\displaystyle\int_0^\infty \frac{1}{x^2+2x+1}\,\mathrm{d}x$ exist? If it exists, compute its value, otherwise explain why it does not exist.

Problem 4. (a) Let C be the curve discribed by $y=\cosh x$, $0\le x\le\pi$. Compute the arc length of C.

(b) The curve $y=\sqrt{2x-x^2}$, $0\le x\le 1$ is rotated about the x-axis. Find the area of the surface.

[1] Hand in until 25 August 2009