Universidad de los Andes
MATE-2201

Analysis 1

Problem Sheet 12

Differentiation; integration; power series. Hand in: November 5, 2009

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1. (a) Find $\lim_{n\to\infty} \sqrt[n]{n!}$.

   (b) Find $\lim_{n\to\infty} \frac{1}{n} \sqrt[n]{n!}$.

2. For $n \in \mathbb{N}$ define

$$f_n : (0,\infty) \to \mathbb{R}, \quad f_n(x) = 2n(\sqrt[n]{2x} - 1).$$

   (a) Find the pointwise limit of $(f_n)_{n\in\mathbb{N}}$.

   (b) Show that $(f_n)_{n\in\mathbb{N}}$ converges uniformly on every compact interval in $(0,\infty)$.

   (c) Does $(f_n)_{n\in\mathbb{N}}$ converge uniformly in $(0,\infty)$?

   Hint. Write f_n as an integral.

3. (a) Use power series to find

$$\sum_{n=1}^{\infty} \frac{n}{3^{n-1}}, \qquad \sum_{n=1}^{\infty} \frac{n}{(n+1)!}.$$

   (b) Find the power series representation of arctan at 0 and show that

$$\frac{\pi}{4} = \sum_{n=0}^{\infty} \frac{(-)^n}{2n+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \pm \cdots$$

4. Recall that $(C([0,1]), \|\cdot\|_\infty)$ is a Banach space. Show that

$$T : C([0,1]) \to \mathbb{C}, \qquad f \mapsto \int_0^1 f \,\mathrm{d}x$$

   is a bounded linear map and find $\|T\|$. Show that T is continuous. Is it differentiable? If so, find its derivative.