

Analysis 1

Problem Sheet 11

l'Hospital; local and global extrema; Riemann integration.

Hand in: October 29, 2009

1. (a) Find all local and global extrema of

$$f : [0, \infty) \rightarrow \mathbb{R}, \quad f(x) = \frac{2 \sin x}{2 - \cos^2 x}.$$

- (b) Find the following limits if they exist:

$$(a) \lim_{x \rightarrow \infty} \left(x - \sqrt[3]{x^3 - x^2 + 1} \right), \quad (b) \lim_{a \rightarrow \infty} \left(1 + \frac{x}{a} \right)^a.$$

2. Let $(a_n)_{n \in \mathbb{N}} \subseteq \mathbb{R}$ and suppose that $a_n \geq 0$ for all $n \in \mathbb{N}$. Show:

$$\sum_{k=1}^{\infty} a_n \text{ converges} \implies \sum_{k=1}^{\infty} \frac{\sqrt{a_n}}{n} \text{ converges.}$$

3. Let $a \in \mathbb{R}_+$ and let $f : \mathbb{R} \rightarrow \mathbb{R}$, $f = \exp$. Use Riemann sums $s(f, P)$ and $S(f, P)$ to find

$$\int_0^a \exp(x) dx.$$

4. (a) Does the improper integral $\int_0^{\infty} \frac{\sin t}{t} dt$ exist?

- (b) Does $\int_0^1 D(t) dt$ exist, where D is the Dirichlet function

$$D : [0, 1] \rightarrow \mathbb{R}, \quad D(t) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \cap [0, 1], \\ 0 & \text{if } x \in [0, 1] \setminus \mathbb{Q}. \end{cases}$$