Universidad de los Andes
MATE-2201

Analysis 1

Problem Sheet 6

Series.

Hand in: September 17, 2009

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1. Find the 5-adic and 7-adic fraction of $\frac{1}{5}$. Prove your assertion.

2. Do the following series converge? Prove your answer.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$, (b) $\displaystyle\sum_{n=1}^{\infty} \Big(\frac{n}{n+1}\Big)^{n^2}$,

   (c) $\displaystyle\sum_{n=2}^{\infty} b_n$, where $b_{2m} := \frac{1}{(2m)^2}$, $b_{2m+1} = -\frac{1}{2m}$,

   (d) $\displaystyle\sum_{n=1}^{\infty} \Big(a + \frac{1}{n}\Big)^n$ where $a \in \mathbb{R}$.

3. (a) For $n \in \mathbb{N}$ let $a_n := b_n := \frac{(-1)^n}{\sqrt{n+1}}$ and $c_n := \sum_{k=0}^{n} a_k b_{n-k}$. Show that $\sum_{n=0}^{\infty} a_n$ converges, but $\sum_{n=0}^{\infty} c_n$ diverges.

   (b) Let $(a_n)_{n\in\mathbb{N}} \subseteq \mathbb{R}$ a monotonically decreasing sequence such that $\sum_{n=1}^{\infty} a_n$ converges in $\mathbb{R}$. Show that

$$\lim_{n\to\infty} n\, a_n = 0.$$

4. Koch's snowflake curve: Given a polygon, the middle third of each side of the polygon is removed and an equilateral triangle is attached instead.
   If the initial polygon is an equilateral triangle, then the *snowflake curve* is the limit if the procedure described above is iterated inifintely often. Find the circumference and the area of the snowflake if each of the sides of the initial triangle has length $a > 0$. Prove your assertions.