Universidad de los Andes
MATE-2201

Analysis 1

Problem Sheet 1

Sets; induction; binomial coefficients.

Hand in: August 13, 2009

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1. For sets A, B and C show at least two of the following statements:

   (a) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$,

   (b) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

   (c) $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$,

   (d) $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.

2. (a) Find a bijection $\mathbb{N} \times \mathbb{N} \to \mathbb{N}$ (proof!).

   (b) Show that $\mathbb{Q}$ is countable.

3. Show the following formulae:

   (a) $\displaystyle\sum_{k=1}^{n} k^3 = \Big(\frac{n(n+1)}{2}\Big)^2, \quad n \in \mathbb{N},$

   (b) $\displaystyle\sum_{k=1}^{2n} (-1)^{k+1}\frac{1}{k} = \sum_{k=1}^{n} \frac{1}{n+k}\ , \quad n \in \mathbb{N}.$

4. For $n \in \mathbb{N}_0$ and $m \in \mathbb{N}$ let

$$a(m,n) := \#\{(x_1, \ldots, x_m) \in \mathbb{N}_0^m \ :\ \sum_{j=1}^{m} x_j \le n\},$$

$$b(m,n) := \#\{(x_1, \ldots, x_m) \in \mathbb{N}_0^m \ :\ \sum_{j=1}^{m} x_j = n\}.$$

   (a) Show that $a(m,n) = b(m+1,n)$ for all $m \in \mathbb{N}$ and $n \in \mathbb{N}_0$.

   (b) Show that $a(m,n) = \binom{n+m}{m}$ for all $m \in \mathbb{N}$ and $n \in \mathbb{N}_0$.

   Hint: Show $a(m,n-1) + a(m-1,n) = a(m,n)$ and use induction on $n+m$.