Universidad de los Andes
MATE-2201

Analysis 1

Problem Sheet 12

Integration; power series; Taylor expansion.

Hand in: April 23, 2009

---

1. (a) Find $\lim_{n\to\infty} \sqrt[n]{n!}$.

   (b) Find $\lim_{n\to\infty} \frac{1}{n}\sqrt[n]{n!}$.

2. (a) Use power series to find

$$\sum_{n=1}^{\infty} \frac{k}{3^{k-1}}, \qquad \sum_{n=1}^{\infty} \frac{k}{(k+1)!}.$$

   (b) Find the power series representation of arctan at 0 and show that

$$\frac{\pi}{4} = \sum_{k=0}^{\infty} \frac{(-)^k}{2k+1} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} \pm \cdots$$

3. Find the Taylor series of

   (a) $f : \mathbb{R} \to \mathbb{R}, \quad f(x) = (x^2 - 3x + 1)(x - 2)$ at $p = 2$,

   (b) $g : \mathbb{R} \to \mathbb{R}, \quad g(x) = \sin(x)$ at $p = \frac{\pi}{2}$

   and determine the radius of convergence.

4. Show that the following function is arbitrarily often differentiable and find its Taylor series at 0. What is its radius of convergence? Where is the Taylor series equal to φ?

$$\varphi : \mathbb{R} \to \mathbb{R}, \qquad \varphi(x) = \begin{cases} \exp(-x^{-2}), & x \neq 0, \\ 0, & x = 0. \end{cases}$$