Universidad de los Andes
MATE-2201

Analysis 1

Problem Sheet 8

Uniform continuity, power series.

Hand in: March 19, 2009

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1. Let $I = (a, b)$ a nonempty real interval and $f : I \to \mathbb{R}$ continuous.
   (a) f is injective if and only if f is strictly monotonic.
   (b) If f is injective, then its inverse $f^{-1} : f(I) \to \mathbb{R}$ is continuous.

2. Show that $f : [0, \infty) \to \mathbb{R},\ x \mapsto \sqrt{x}$, is uniformly continuous but not Lipschitz continuous.

3. (a) $f_n : \mathbb{R} \to \mathbb{R}, \quad f_n(x) = \begin{cases} n^2 x, & 0 \le x \le \frac{1}{n}, \\ 2n - n^2 x, & \frac{1}{n} < x \le \frac{2}{n}, \\ 0, & x > \frac{2}{n}. \end{cases}$

   Show that $(f_n)_{n\in\mathbb{N}}$ is pointwise convergent, but not uniformly convergent.

   (b) Is the sequence of functions $(f_n)_{n\in\mathbb{N}}$ with

$$f_n(x) : [0, 1] \to \mathbb{R}, \qquad f_n(x) = x^n$$

   pointwise convergent? Is it uniformly convergent?

4. **Properties of the exponential function**

   Show

   (a) $\exp(\bar{z}) = \overline{\exp(z)}, \quad z \in \mathbb{C}$,
   (b) $\exp(z + w) = \exp(z)\exp(w), \quad z, w \in \mathbb{C}$,
   (c) $\exp(n) = \mathrm{e}^n, \quad n \in \mathbb{Z}$,
   (d) $\exp(z) \neq 0, \quad z \in \mathbb{C}$,
   (e) $|\exp(\mathrm{i}x)| = 1 \iff x \in \mathbb{R}$.