Universidad de los Andes
MATE-2201

Analysis 1

Problem Sheet 2

Induction; binomial coefficients, ordered sets.

Hand in: 3 February 2009

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1. Show the following formulae:

   (a) $\sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2, \quad n \in \mathbb{N},$

   (b) $\sum_{k=1}^{2n} (-1)^{k+1} \frac{1}{k} = \sum_{k=1}^{n} \frac{1}{n+k}\ , \quad n \in \mathbb{N}.$

2. Show that for each $n \in \mathbb{N}_0$ the sum $4^{2n+1} + 3^{n+2}$ is divisible by 13.

3. For $n \in \mathbb{N}_0$ and $m \in \mathbb{N}$ let

$$a(m,n) := \#\{(x_1, \ldots, x_m) \in \mathbb{N}_0^m \ : \ \sum_{j=1}^{m} x_j \leq n\},$$

$$b(m,n) := \#\{(x_1, \ldots, x_m) \in \mathbb{N}_0^m \ : \ \sum_{j=1}^{m} x_j = n\}.$$

   (a) Show that $a(m,n) = b(m+1,n)$, $m \in \mathbb{N},\ n \in \mathbb{N}_0$.

   (b) Show that $a(m,n) = \binom{n+m}{m}$, $m \in \mathbb{N},\ n \in \mathbb{N}_0$.

   Hint: Show $a(m,n-1) + a(m,n-1) = a(m,n)$ and use induction on $n+m$.

4. Let $(K, +, \cdot, >)$ be an ordered field and $a,\ x,\ x',\ y,\ y' \in K$. Show the following statements (Cor. 3.9). Justify every step.

   (iii) $x < y \implies x + a < y + a,$

   (iv) $x < y \ \wedge \ x' < y' \implies x + x' < y + y',$

   (v) $x < y \ \wedge \ a > 0 \implies a \cdot x < a \cdot y,$
   $x < y \ \wedge \ a < 0 \implies a \cdot x > a \cdot y,$

   (vi) $0 \leq x < y \ \wedge \ 0 \leq x' < y' \implies 0 \leq x' \cdot x < y' \cdot y,$

   (ix) $x > 0 \implies x^{-1} > 0,$

   (x) $0 < x < y \implies 0 < y^{-1} < x^{-1},$

   (xi) $x > 0 \wedge y < 0 \implies xy < 0.$