Localization and Artificial Gauge Fields in Quantum Optical Lattices

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Bose-Hubbard

Modeling cold neutral atoms in a lattice:

$$\hat{H} = -t \sum_{\langle i, j \rangle} \left(\hat{a}_i^t \hat{a}_j + h.c. \right) + \frac{U}{2} \sum_i \hat{n}_i \left(\hat{n}_i - 1 \right)$$

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Which is obtained from the many-body system:

$$\hat{H} = \int d^3x \, \hat{\psi}(x)^t \left[\frac{\hat{p}^2}{2m} + V_0(x) + V_T(x) \right] \hat{\psi}(x) + \frac{1}{2} \frac{4\pi a_s \hbar}{m} \int d^3x \, \hat{\psi}(x)^t \hat{\psi}(x)^t \hat{\psi}(x) \hat{\psi}(x)$$

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Keys:

Fermions: interaction → 0 always

Bosons: interaction > 0 → energy costs and gap

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Harper's equation

Electrons in 2D lattice with magnetic field → Potential: $\bar{A}=(0,Hx,0)$

Bloch energy function (Tight-Binding)

$$W(\bar{k})=2t(\cos(k_x a_x)+\cos(k_y a_y))$$

Peierls substitution.

$$\hbar\hat{k}\rightarrow\hat{p}-\frac{e}{c}\hat{A}$$

[2] Douglas R. Hofstadter Phys. Rev. B 14, 2239, (1976)

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Ansatz: $\psi(ma, na)=e^{i\nu n}g(m)$

New effective equation and conditions:

$$\begin{pmatrix} g(m+1) \\ g(m) \end{pmatrix}=\begin{pmatrix} \varepsilon-\cos(2\pi m\alpha-\nu) & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} g(m) \\ g(m-1) \end{pmatrix}$$

$$\alpha=Hea^2/ch$$
$$\varepsilon=E/t$$

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Conditions : (i) $\begin{pmatrix} g(m+1) \\ g(m) \end{pmatrix}=Q\begin{pmatrix} g(1) \\ g(0) \end{pmatrix}$ → Q bounded mat.

(ii)Q periodic in ‘m’: → $\alpha=p/q$ (iii) Det(Q) = 1

(iv) ν is irrelevant → ν = π / 2q

→ $|Tr(Q(\varepsilon,\alpha,\nu=\pi/2q))|\leq 4$

[2] Douglas R. Hofstadter Phys. Rev. B 14, 2239, (1976)

BH model adaptation

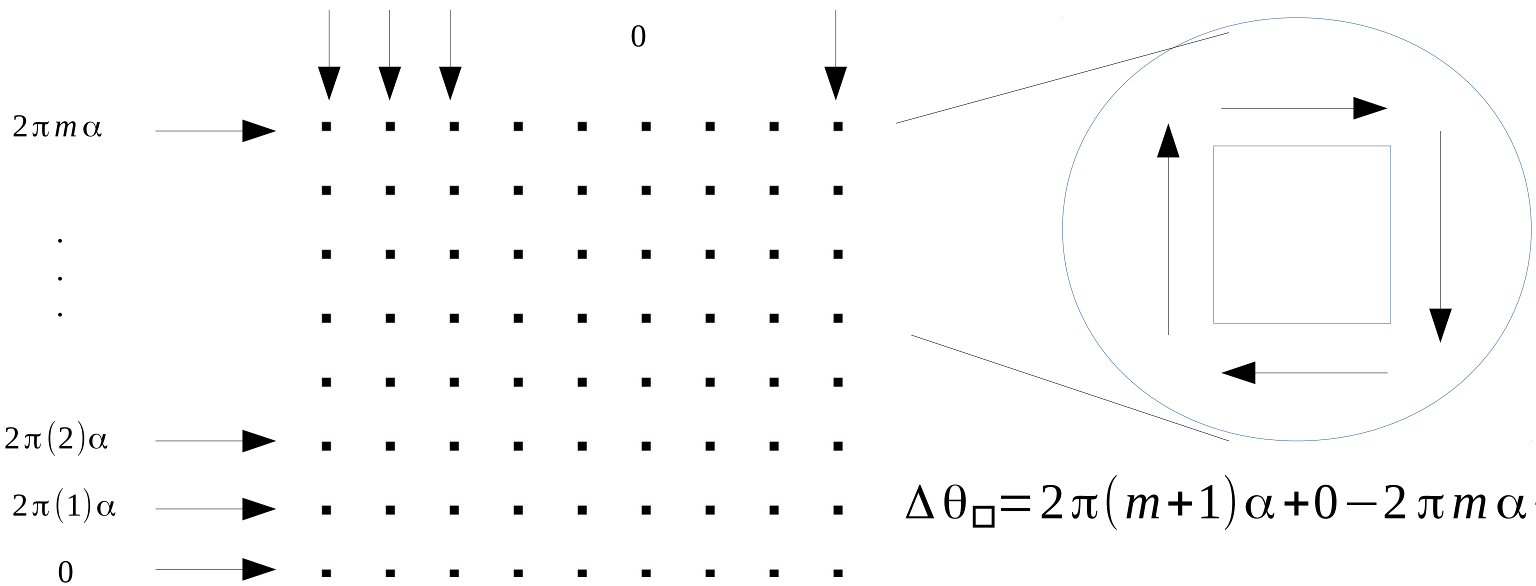
Rothe, H.J. Lattice Gauge Theories, An introduction, 2005

HB ↔ Harper's equation

$$\hat{H} = -t \sum_{\langle i,j \rangle} \left(e^{-i\theta_{ij}} \hat{a}_i^{t} \hat{a}_j + h.c \right) + \frac{U}{2} \sum_i \hat{n} (\hat{n} - 1)$$

Magnetic phase depends on the direction of jumps:

$$\theta_{ij} = \begin{Bmatrix} 2\pi m \alpha \\ 0 \end{Bmatrix}$$



$$\Delta\theta_{\square} = 2\pi(m+1)\alpha + 0 - 2\pi m \alpha - 0 = 2\pi\alpha$$

*Btw, Wilson's loop W(□) is invariant under U(1) transformations. {θij} → W(□)

BH model adaptation

By the way...

$$\alpha = Hea^2/hc$$

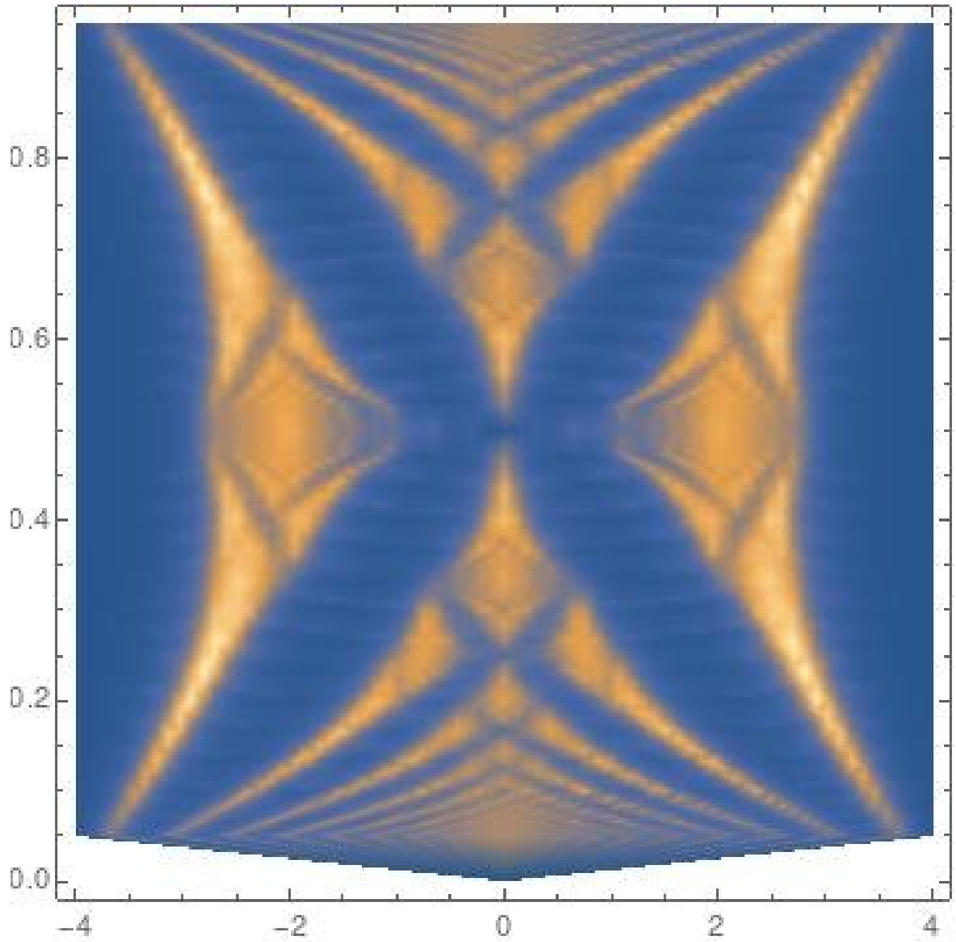
$$H = \alpha \frac{hc}{e\,a^2}$$

If we were using electrons then it would be imperative to apply an ultra intense magnetic field to get $\alpha \approx 1$ (not practical)

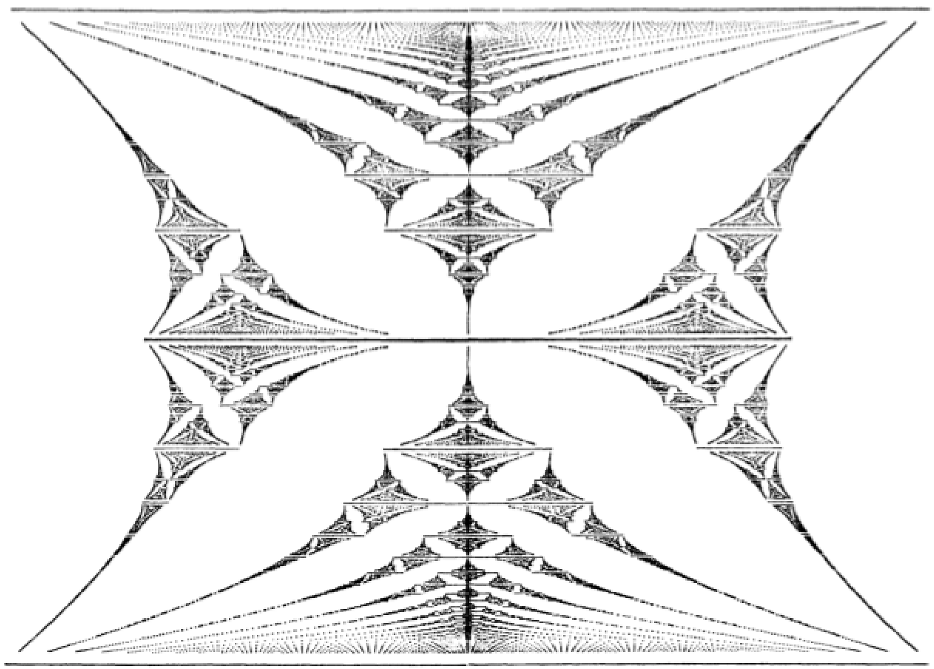
In contrast, while simulating magnetic fields we can achieve an ultra intense magnetic field by reducing lattice spacing $H \sim 1/a^2$

Segundo detalle:
Adición de fases en redes ópticas:
Light-induced gauge fields for ultracold atoms
N. Goldman, G. Juzeliūnas, P. Öhberg and I B Spielman, IOPscience (2014)

Hofstadter's Butterflies (exact diagonalization)



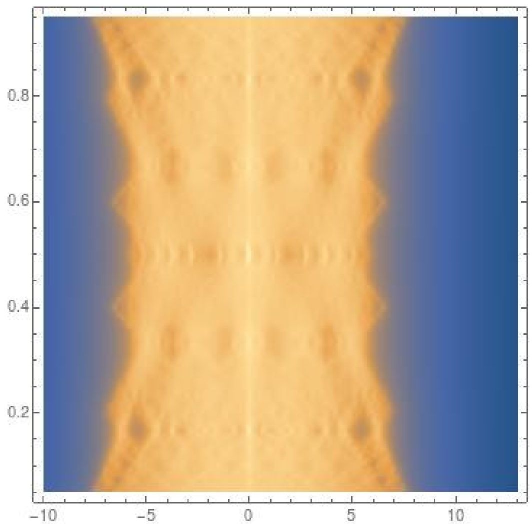
40x40 lattice

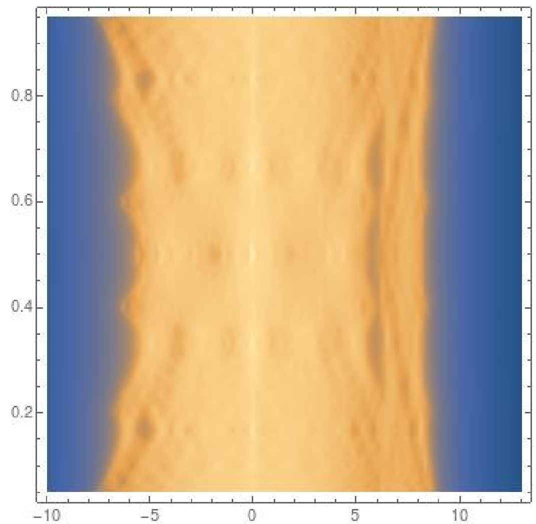


Energy vs magnetic field (phase α) for one Bloch electron in a 2D lattice in presence of a uniform magnetic field [2].

Experimental evidence:
Stöckmann, H.-J., et al, Physical Review Letters. 80 (15): 3232–3235. (1998);
Geim, A. K., et al, Nature. 497 (7451): 594–597 (2013);
Kim, P., et al Nature. 497 (7451): 598–602 (2013);
Martinis, J., et al, Science. 358 (6367): 1175–1179 (2017)

Energy spectrum for 2 bosons with U>0

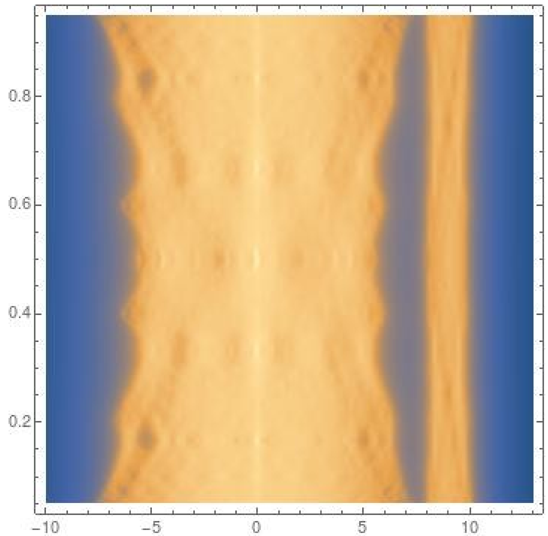


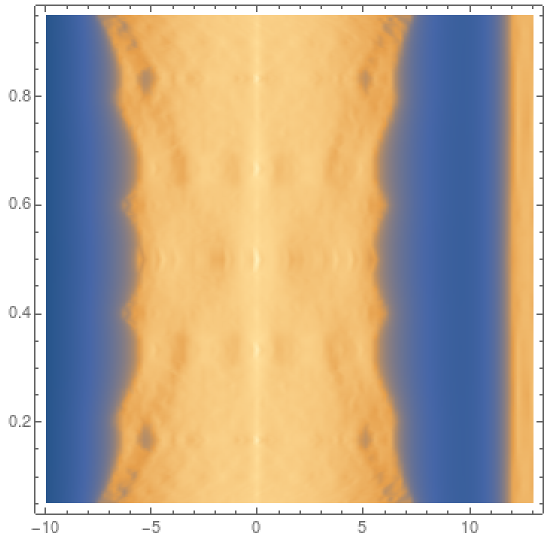


Caso a): 2 bosones en el sistema {1,0}U{1,1}U{2,0}

U=0

U=6





U=8

U=12

What happens for large U with more than 2 bosons?

Since the energy costs of states like (9,0,0,…,0) provided by the interaction term, well… we just cut off the these states, so the new basis is smaller and the calculations take less time.

We propose an ideal base (effective when U is large): {(min,min,min,…,min)}

If we had 9 particles in 9 places: (1,1,…,1),{S(2,0,1,…)}

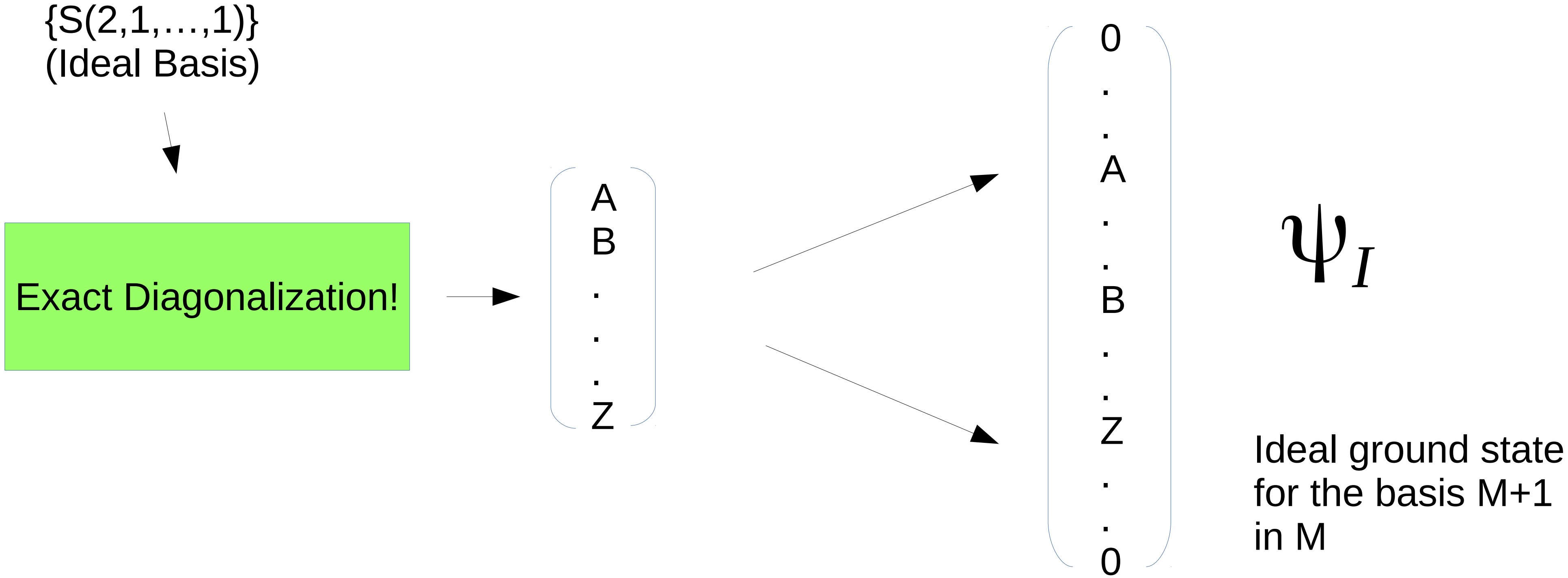
If we had 10 particles in 9 places {S(2,1,…)}

Now focus on the case M+1 particles in M places...

The fidelity

We define the fidelity as

$$F_G = |\langle \psi_I | G \rangle|^2$$



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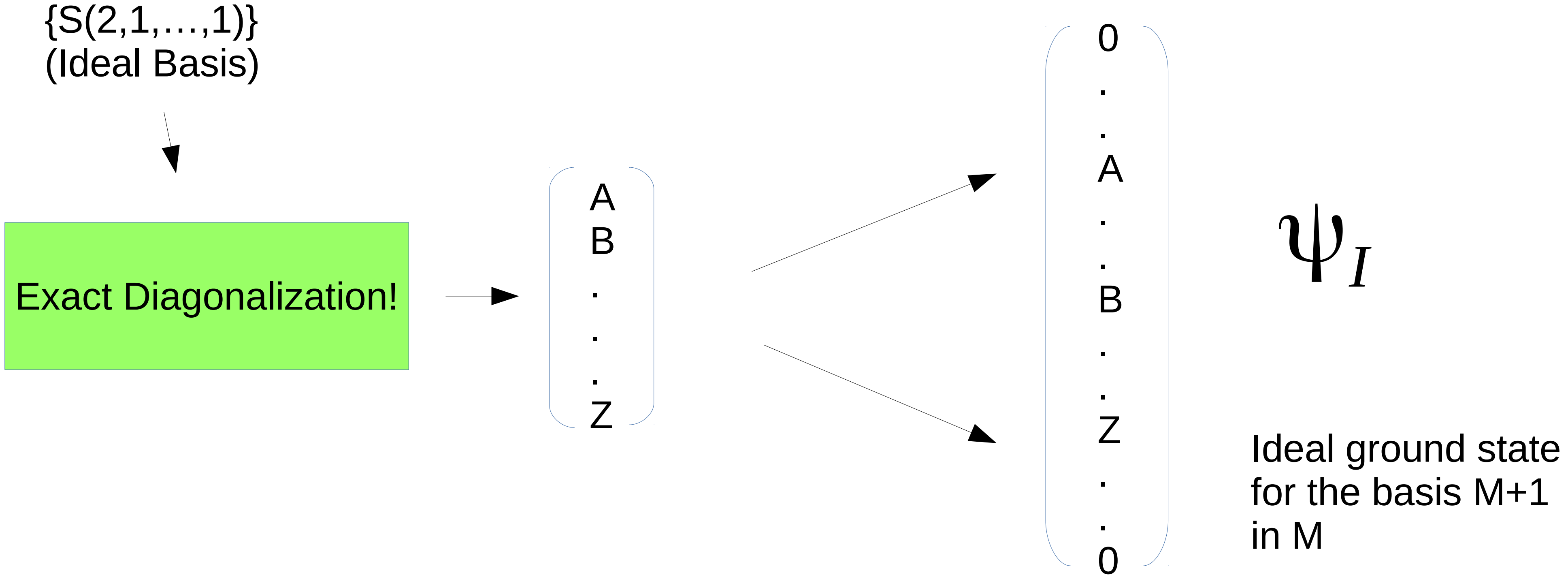
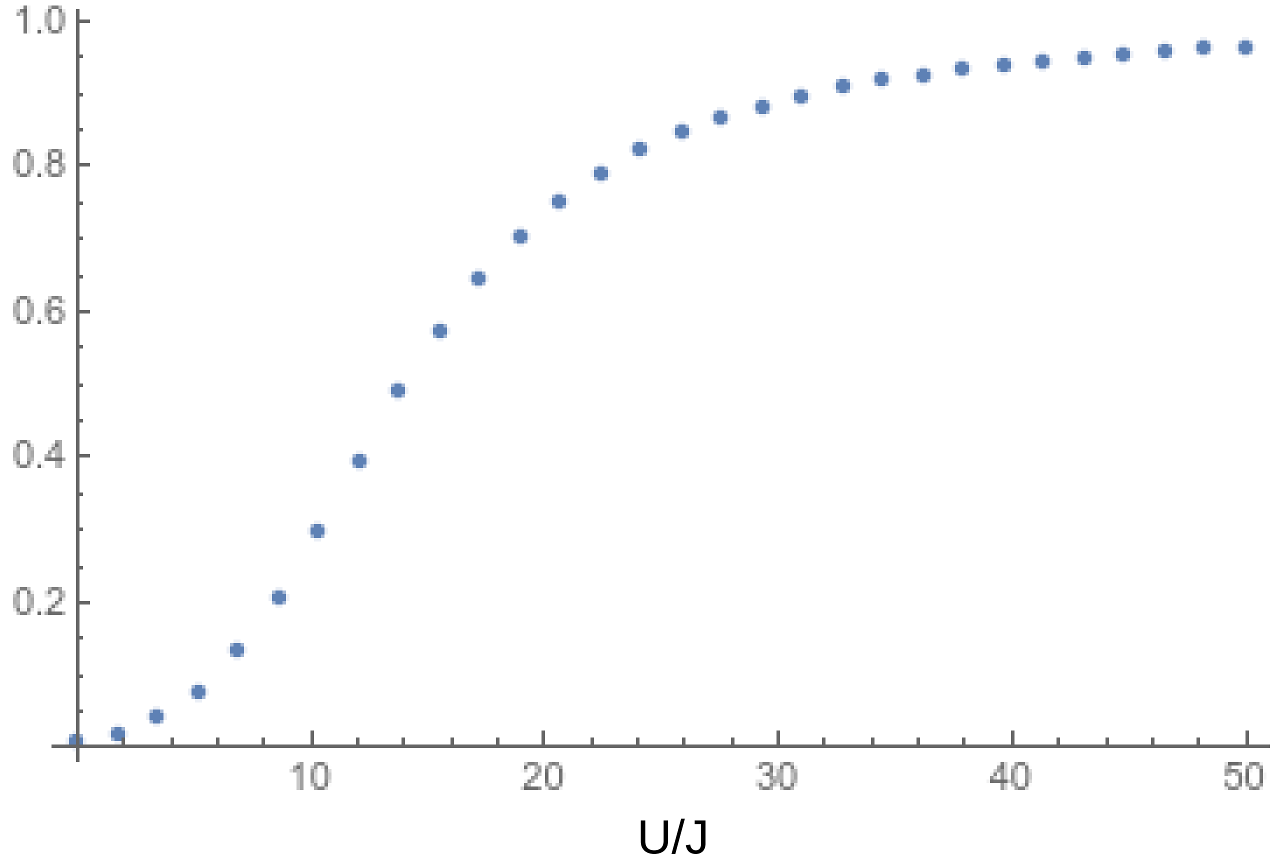


Let's see the results...

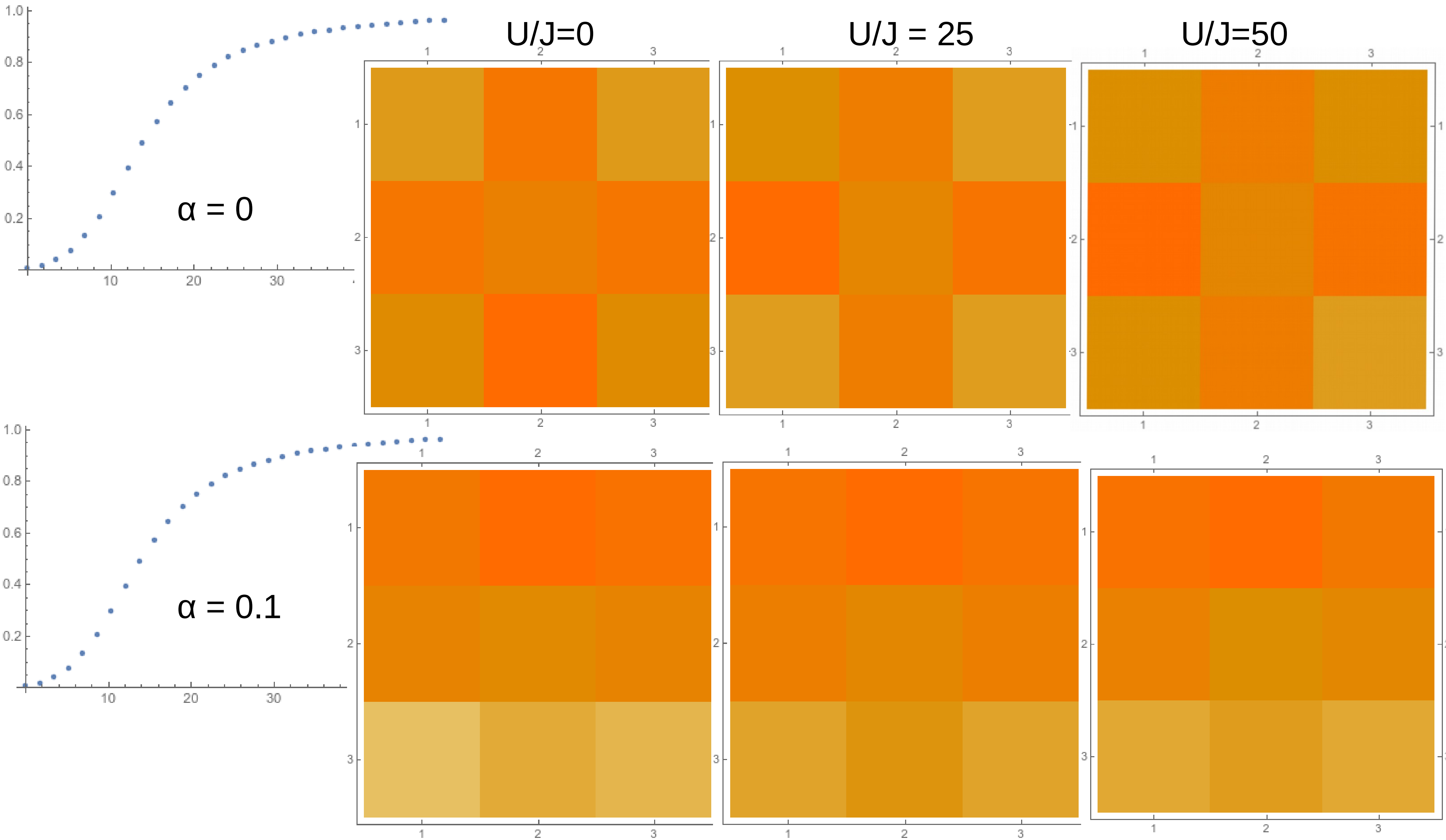
Fidelity without magnetic field

$$F_G = |\langle \psi_I | G \rangle|^2$$

3x3 lattice
for 10
particles
in 9 sites



Some results...



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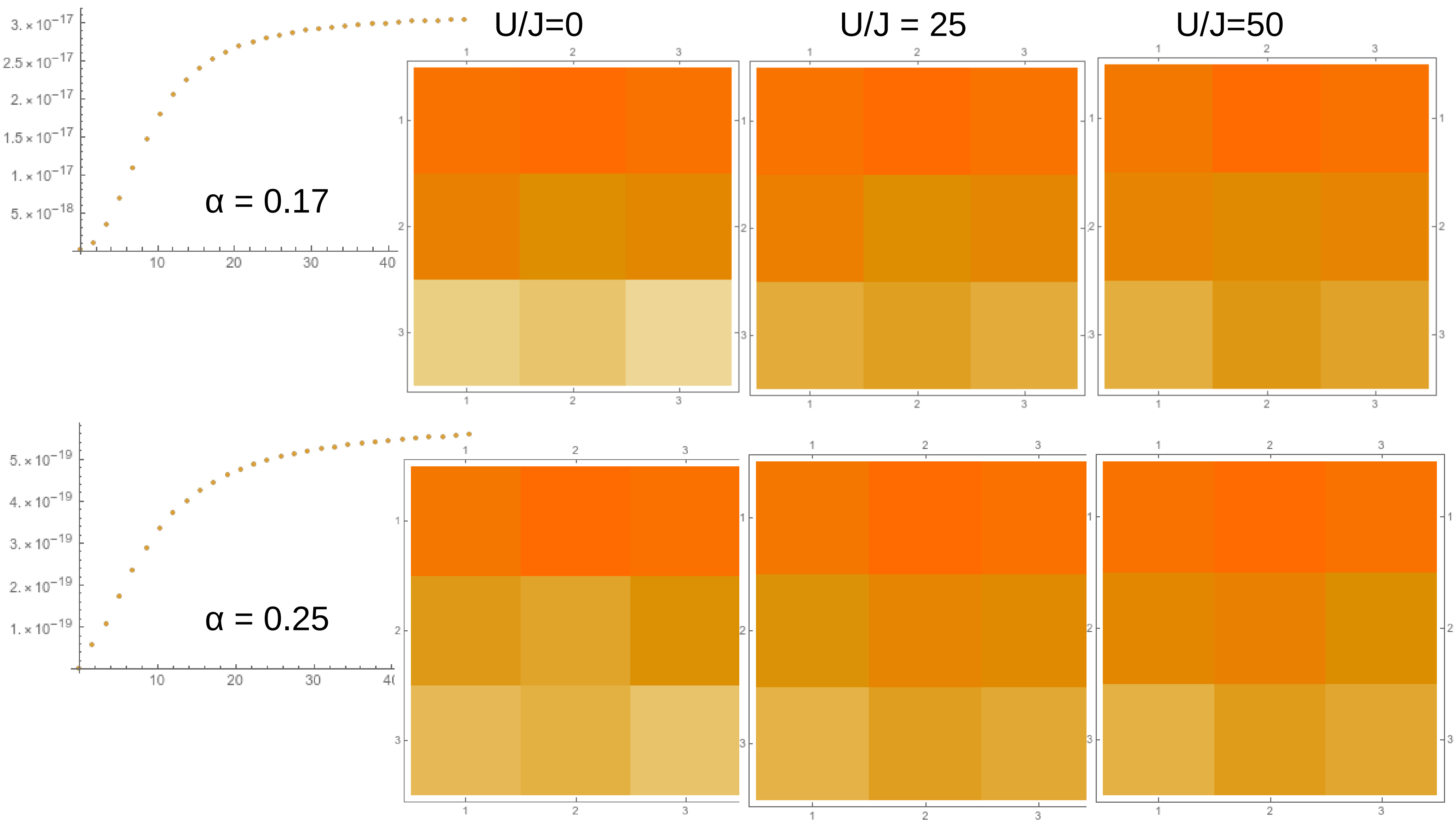

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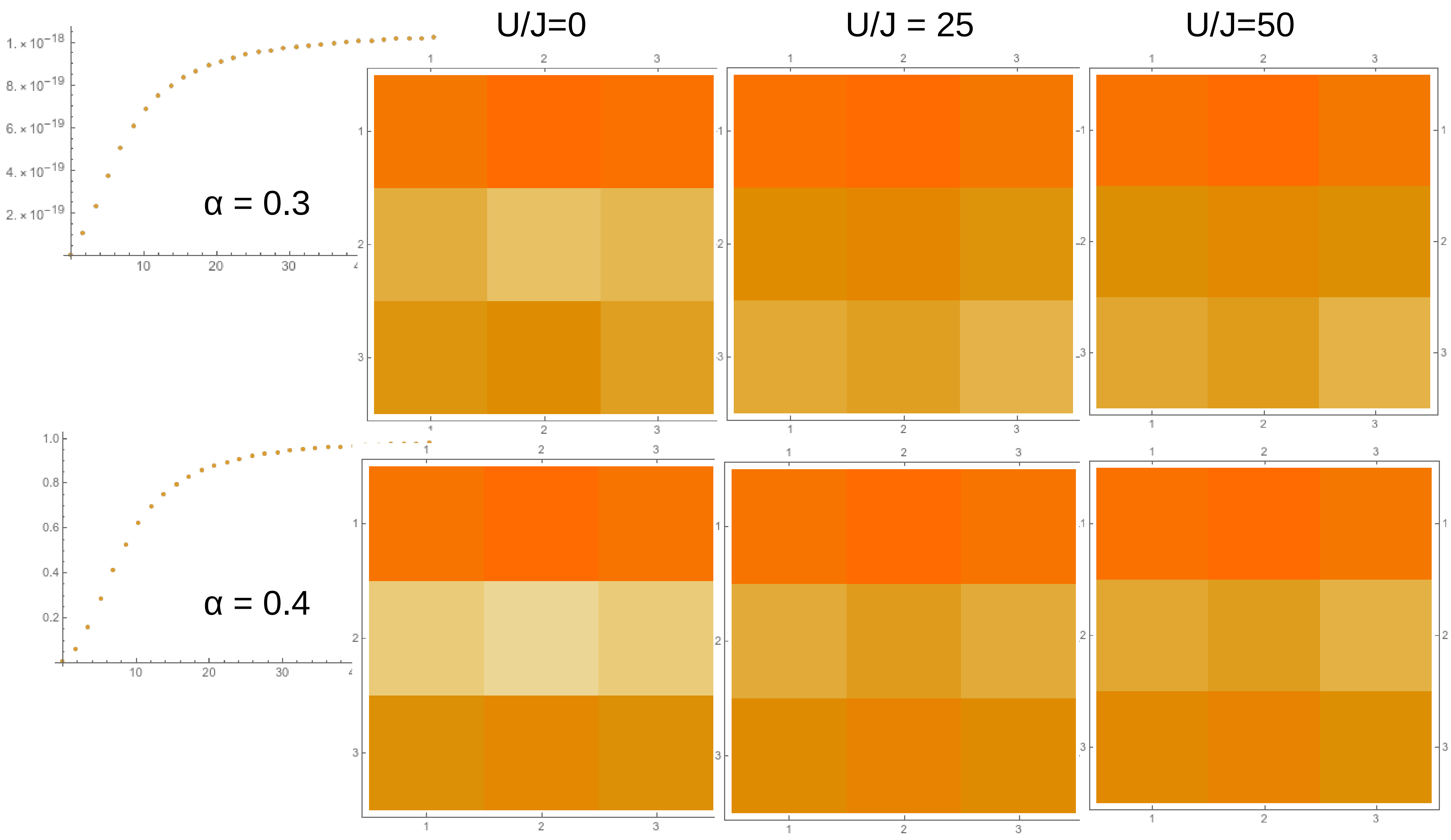

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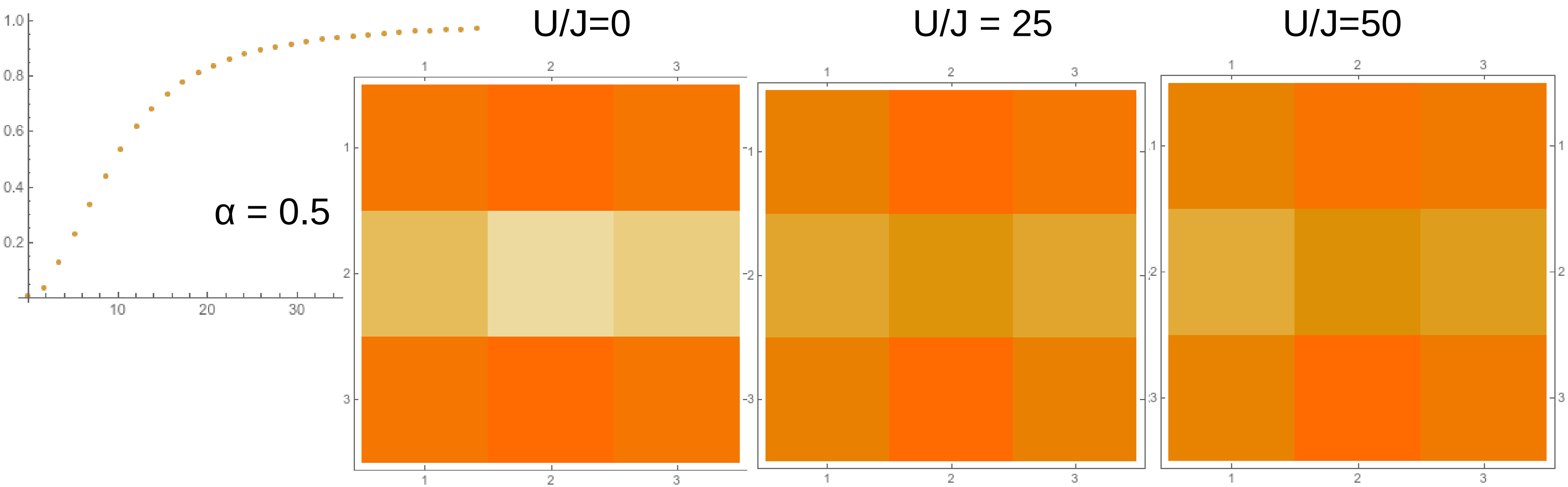
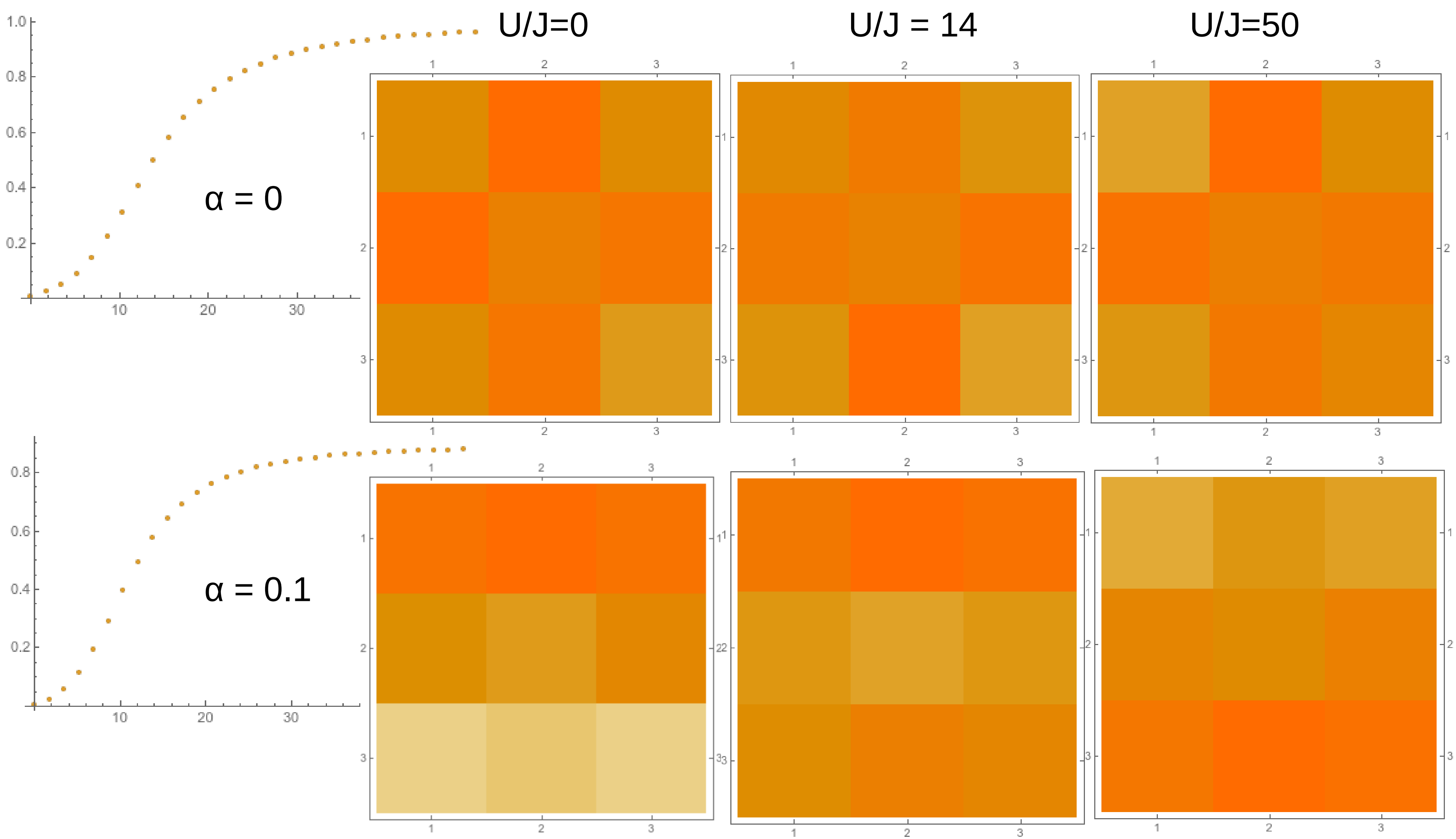
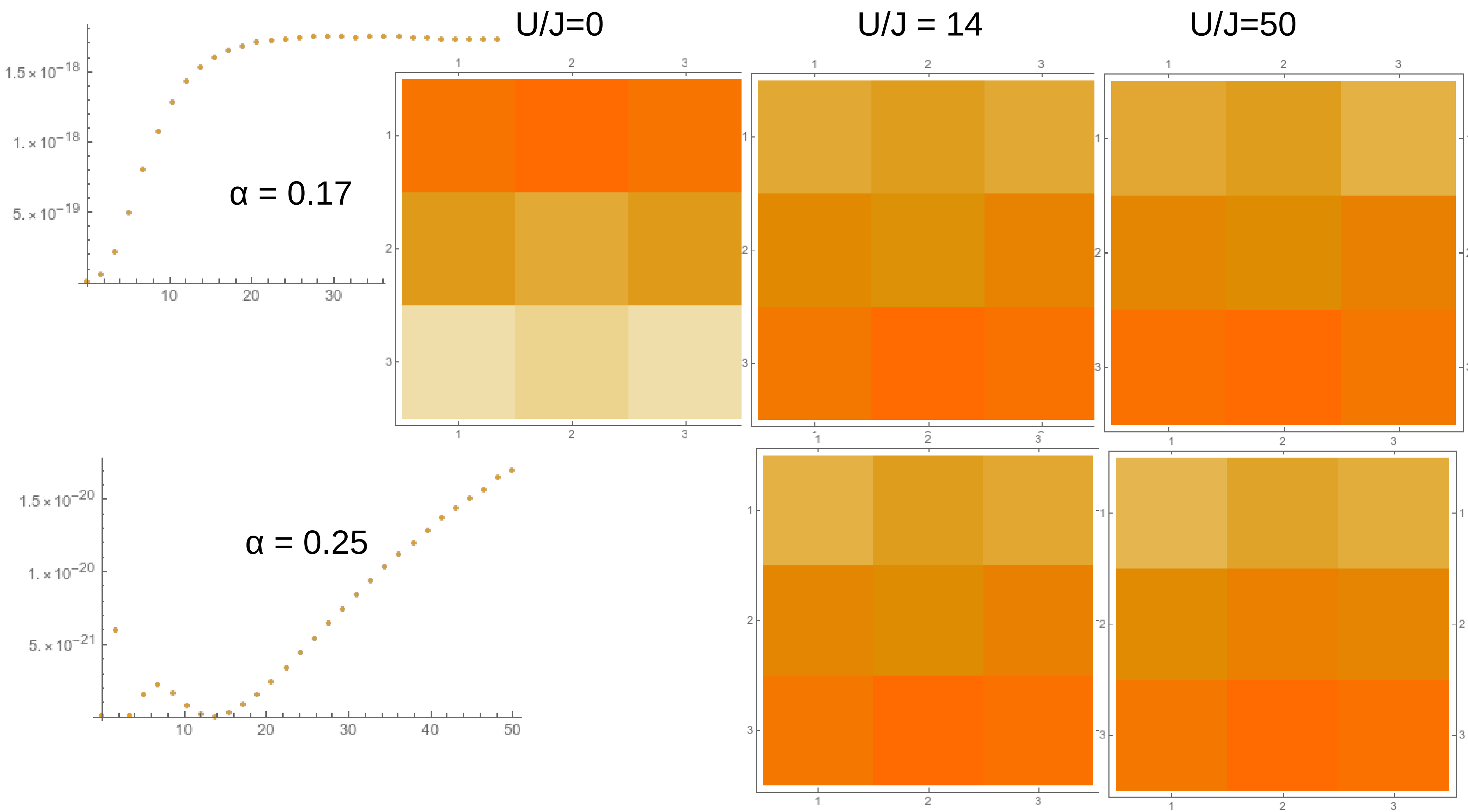


And what happen if we take M-1 particles in M particles?…

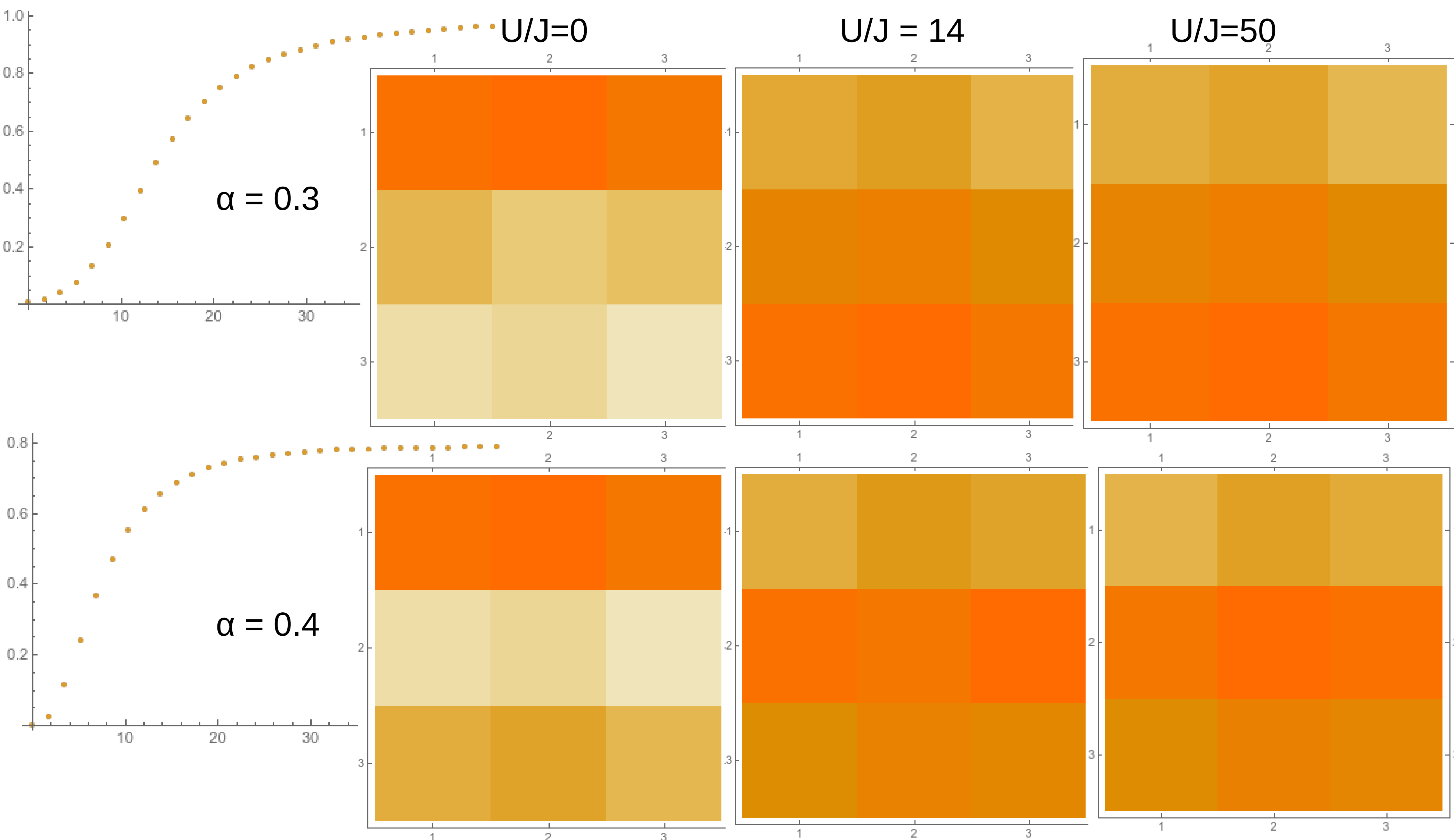
Ideal basis → {S(0,1,…,1)}

Some results...




U/J=0
U/J = 14
U/J=50
α = 0.17
α = 0.25
1.5 × 10−18
1. × 10−18
5. × 10−19
1.5 × 10−20
1. × 10−20
5. × 10−21

Some results...



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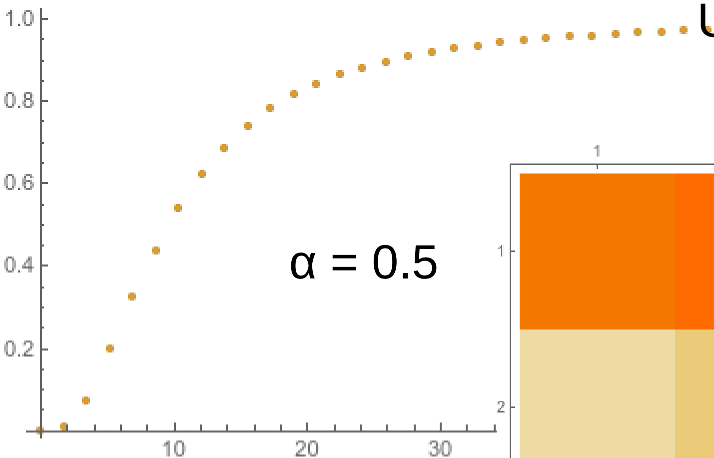
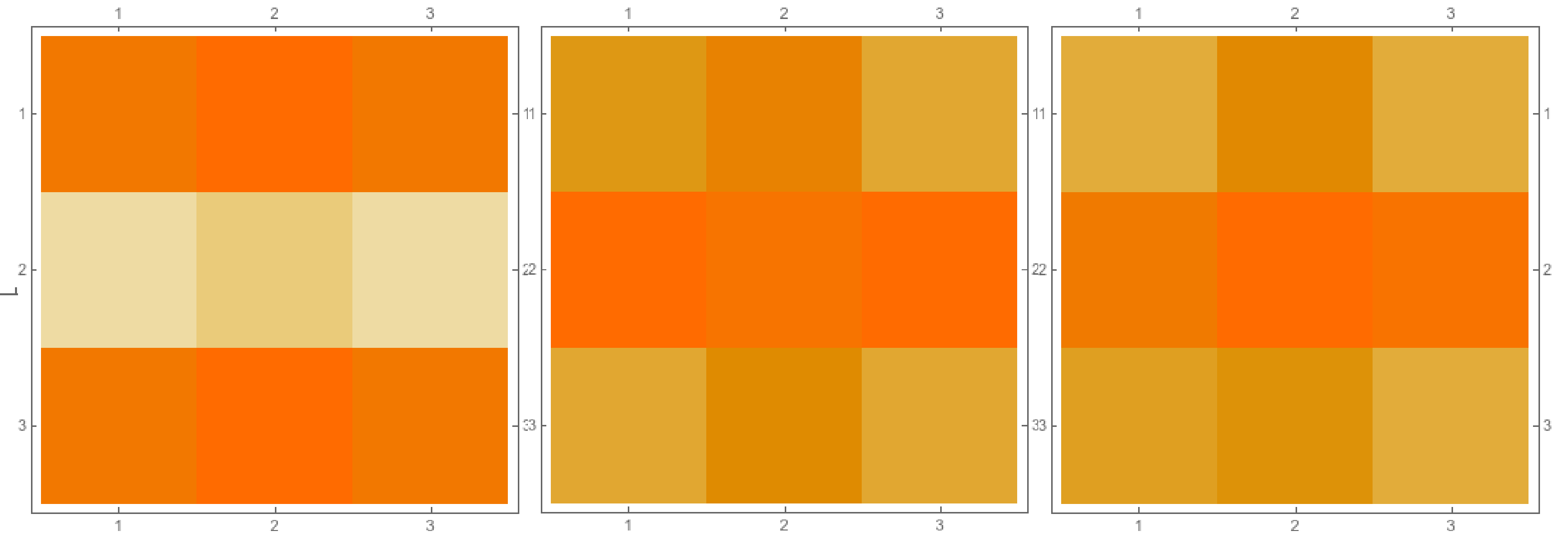


U/J=0

U/J = 14

U/J=50



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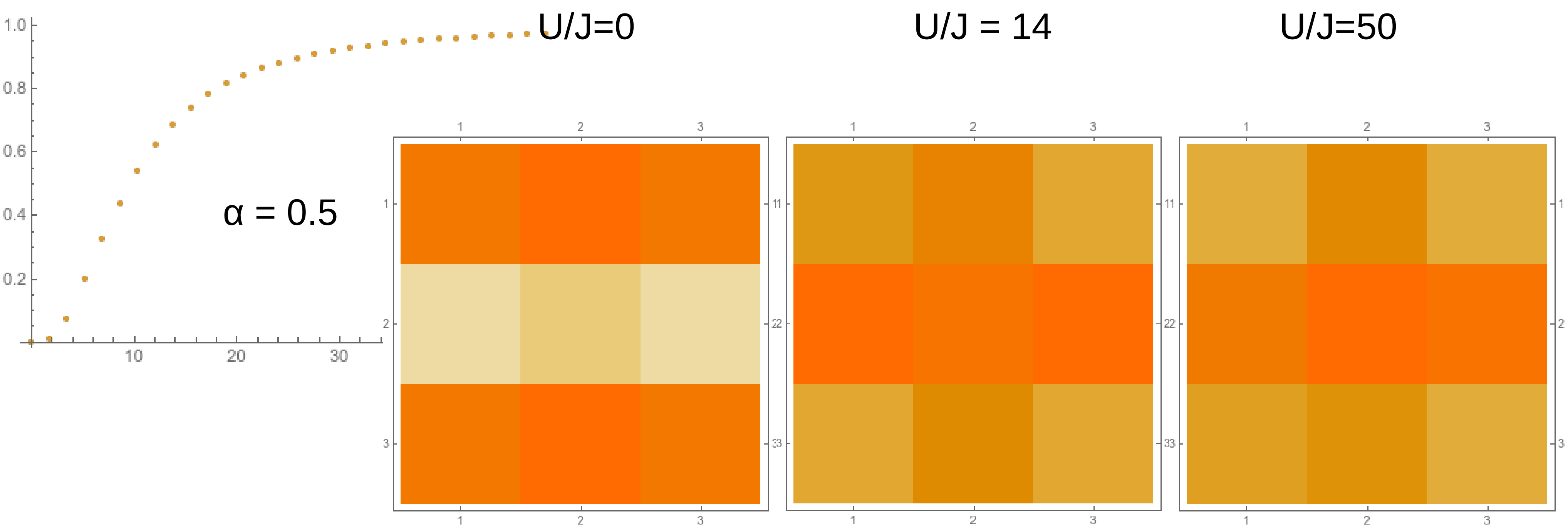

-Synthetic magnetic forces…
- Aharonov-Bohm cages...

Thank you!

The group

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