THE SSH MODEL IN THE MOMENTUM REPRESENTATION

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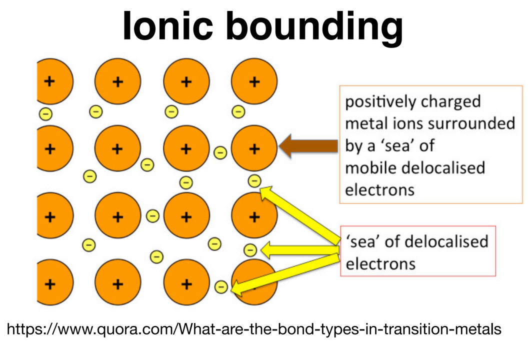
ABSTRACT

This work is based on the Su-Schrieffer-Heeger model, which describes a system of non-interacting polarized fermions, i.e. without spin, moving in a one-dimensional superlattice. We analyze the Hamiltonian of the system in second quantization, in which the optical lattice has discretized the space, and take into account that the basis that diagonalizes the kinetic energy is the one of momentum. In the first case, let us consider a finite chain; we show that the discrete Sine transform type-I respects the finite boundary con- ditions of the system, hence, it is the proper transform to be used. This transformation arises from linear combinations of plane waves and allows us to express our Hamiltonian in the momentum basis in such a way that will allow us to extend the study of the system to an arbitrary number of sites. In the second case, when periodic boundary conditions are considered, the usual Fourier transform can be used; this case will be shortly discussed in this poster as well.

INTRODUCTION

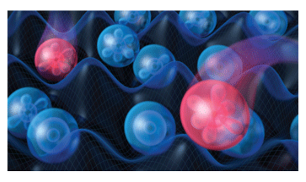
Ionic bounding

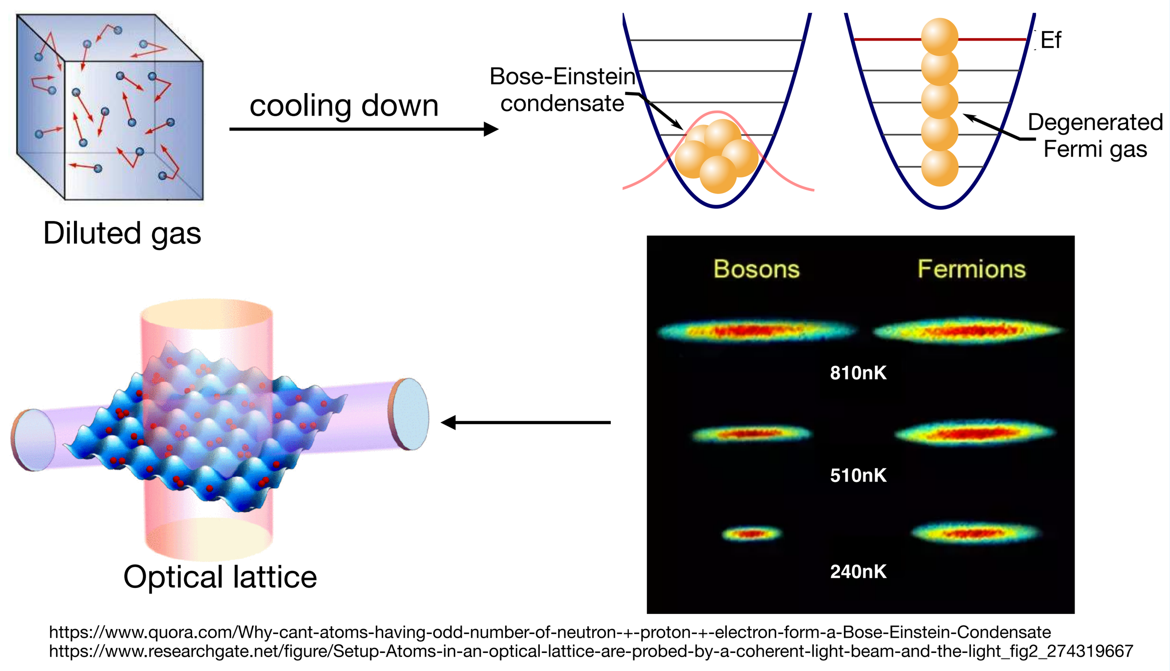


https://www.quora.com/What-are-the-bond-types-in-transition-metals

Transfer of electrons from a metal to a non-metal in order to obtain a full valence shell for both atoms.

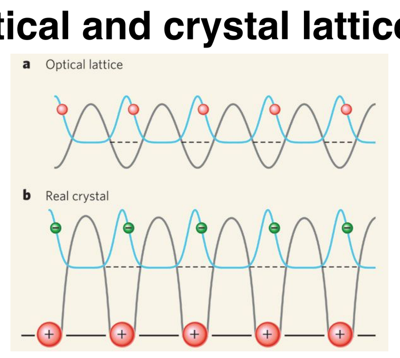
Ultracold atoms: atoms that are at temperatures close to 0 Kelvin (absolute zero), typically below temperatures of some hundreds of nanokelvins (μK).





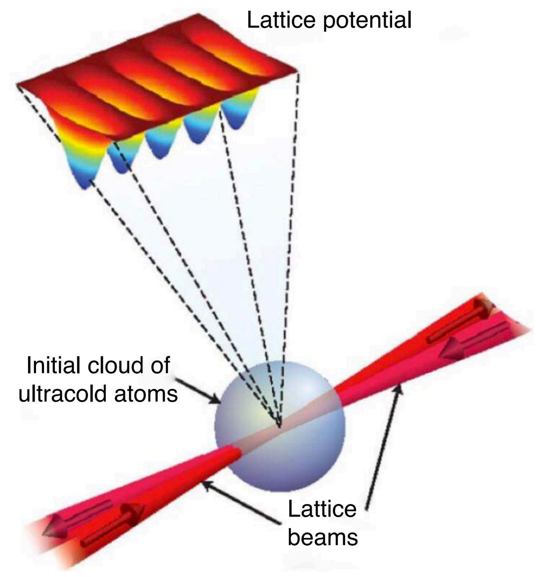
https://www.quora.com/Why-cant-atoms-having-odd-number-of-neutron-+-proton-+-electron-form-a-Bose-Einstein-Condensate
https://www.researchgate.net/figure/Setup-Atoms-in-an-optical-lattice-are-probed-by-a-coherent-light-beam-and-the-light_fig2_274319667

Optical and crystal lattice



https://www.nature.com/articles/453736a

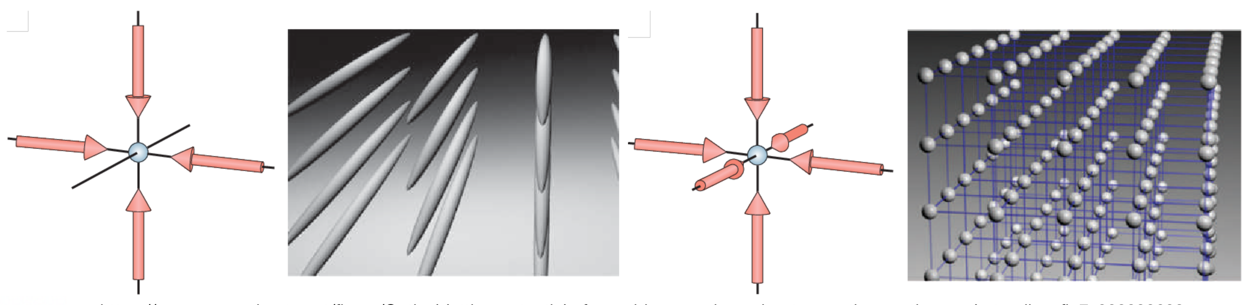
a. An optical lattice looks like a crystal but it is made of light and **b.** a crystal lattice is an arrangement of atoms, ions or molecules in a crystalline material.



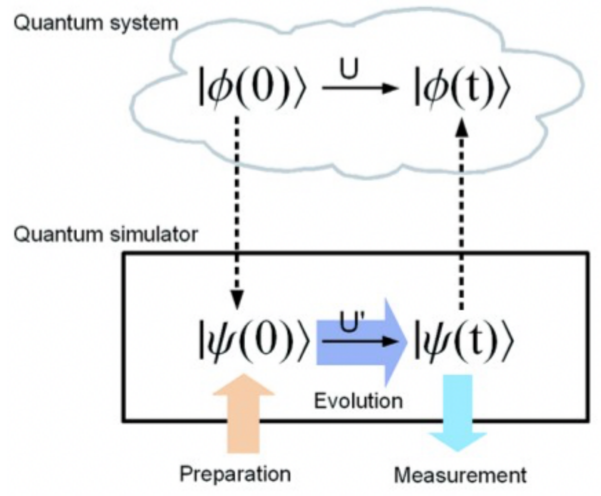
Optical lattice:

- Made by the interference of counter-propagating laser beams.
- Makes a perfect periodic potential.
- Ultra low temperatures, therefore atoms are congregated in sites of potential minima.
- The resulting arrangement resembles a crystal lattice.
- Highly controllable system, therefore it can be used for quantum simulation.
- Tunable interactions.

Optical lattices can be built for different dimensions.



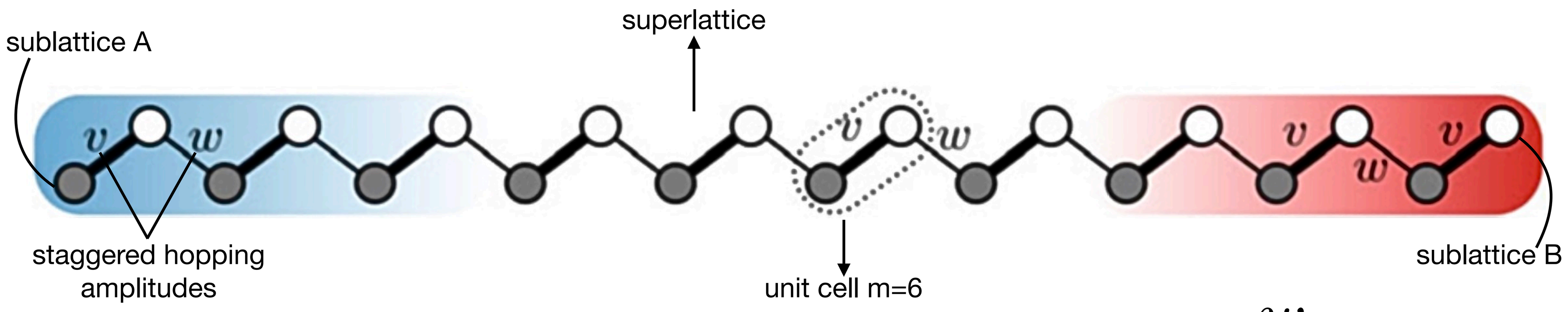
https://www.researchgate.net/figure/Optical-lattice-potentials-formed-by-superimposing-two-or-three-orthogonal-standing_fig7_200030688



Quantum simulators: they permit the study of quantum systems that are difficult to study in the laboratory and intractable on classical computers. Quantum simulators are controllable quantum systems that can be used to simulate other quantum systems.

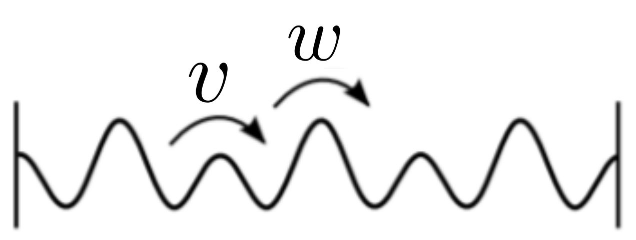
http://inspirehep.net/record/1251940/plots

THE SU-SCHRIEFFER-HEEGER MODEL



L. O. János K. Asbóth and A. Palyi, *A Short Course On Topological Insulators* (Springer International Publishing Switzerland, 2016).

The SSH model [1] describes polarized fermions hopping on a one-dimensional lattice with staggered hopping amplitudes.

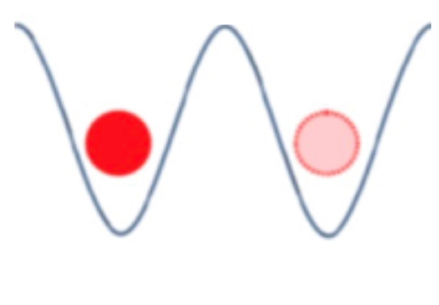


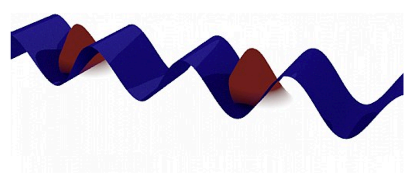
Non interacting fermions $\longrightarrow$ single-particle Hamiltonian

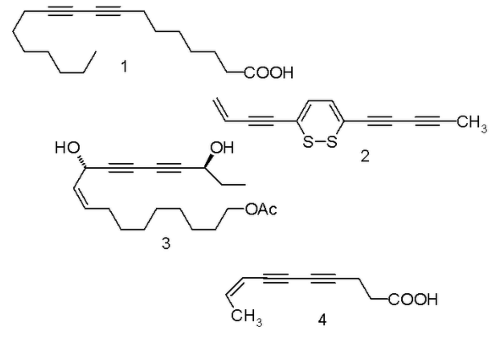
$$\hat{H} = -v\sum_{m=1}^{N}\left(|m,B\rangle\langle A,m| + h.c\right) - w\sum_{m=1}^{N-1}\left(|m+1,A\rangle\langle B,m| + h.c\right)$$

where $|m,\alpha\rangle = \hat{a}^\dagger_{m,\alpha}|\emptyset\rangle$ with $m \in 1,2,...,N$ and $\alpha \in \{A,B\}$. $v,w \geq 0$

One particle per cell $\longrightarrow$ half filling $\longrightarrow$ simplest insulators like polyacetylene







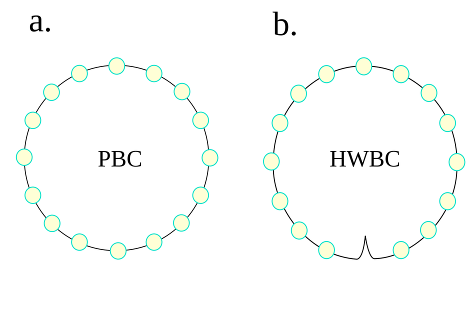
Su, W. P. and Schrieffer, J. R. and Heeger, A. J. *Soliton excitations in polyacetylene*, 1980.

Discrete Fourier transform:

Rewriting the Hamiltonian in the momentum space $\longrightarrow$ To study it in the thermodynamic limit and to extend the problem to an arbitrary number of sites

$\downarrow$

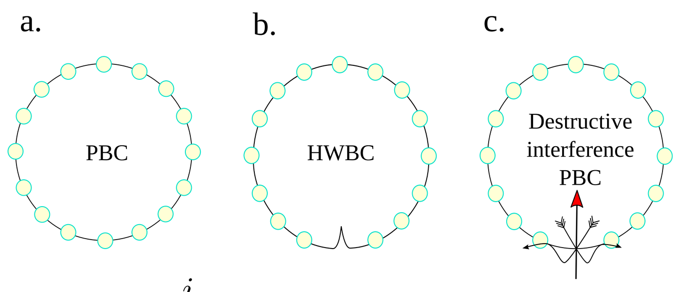
Unitary Fourier transform $\longrightarrow$ Expansion in plane waves



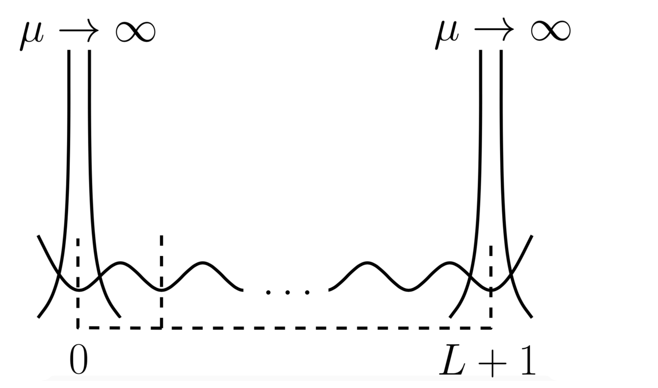
Transform used to describe systems with periodic boundary conditions.

$$|+k\rangle = \frac{1}{\sqrt{L+1}}\sum_{j=0}^{L+1} e^{ikj}\hat{a}^\dagger|\emptyset\rangle \quad \text{and} \quad |-k\rangle = \frac{1}{\sqrt{L+1}}\sum_{j=0}^{L+1} e^{-ikj}\hat{a}^\dagger|\emptyset\rangle \quad \text{with} \quad k = \frac{2\pi m}{L+1}; m = 0,1,2,...,L+1$$

Discrete Sine transform type I [2]:



$$|\zeta^-\rangle = \frac{i}{\sqrt{2}}(|-k\rangle - |+k\rangle)$$



The values at the ends are zero

But our systems satisfies hard wall boundary conditions.

$$|\zeta^-\rangle \longrightarrow \hat{a}^\dagger_\zeta|\emptyset\rangle = \sqrt{\frac{2}{L+1}}\sum_{j=0}^{L+1}\sin(kj)\hat{a}^\dagger_j|\emptyset\rangle; \text{ where } k = \frac{\zeta\pi}{L+1}$$

Therefore the transformation of the staggered operators from the position representation to the momentum representation and vice versa are:

$$\hat{a}^{(\dagger)}_j = \sqrt{\frac{2}{L+1}}\sum_{\zeta=0}^{L+1}\sin\left(\frac{\pi\zeta j}{L+1}\right)\hat{a}^{(\dagger)}_\zeta;\ \zeta = 1,2,...,L$$

$$\hat{a}^{(\dagger)}_\zeta = \sqrt{\frac{2}{L+1}}\sum_{j=0}^{L+1}\sin\left(\frac{\pi\zeta j}{L+1}\right)\hat{a}^{(\dagger)}_j;\ j = 1,2,...,L$$

Using the transformation of the staggered operators, the Hamiltonian takes the following form:

$$\hat{H} = -v\sum_m(\hat{a}^\dagger_{mA}\hat{a}_{mB} + \hat{a}^\dagger_{mB}\hat{a}_{mA}) - w\sum_m(\hat{a}^\dagger_{m+1A}\hat{a}_{mB} + \hat{a}^\dagger_{mB}\hat{a}_{m+1A})$$

The expression of the delta is: $\delta_{\zeta\zeta'} = \frac{2}{L+1}\sum_m \sin\left(\frac{\pi m\zeta}{L+1}\right)\sin\left(\frac{\pi m\zeta'}{L+1}\right)$

We have:

$$\hat{H} = -v\sum_m\frac{2}{L+1}\left(\sum_{\zeta\zeta'}\sin\left(\frac{\pi m\zeta}{L+1}\right)\sin\left(\frac{\pi m\zeta'}{L+1}\right)\boxed{\hat{a}^\dagger_{\zeta A}\hat{a}_{\zeta' B}} + \sum_{\zeta\zeta'}\sin\left(\frac{\pi m\zeta}{L+1}\right)\sin\left(\frac{\pi m\zeta'}{L+1}\right)\boxed{\hat{a}^\dagger_{\zeta B}\hat{a}_{\zeta' A}}\right) +$$

$$-w\sum_m\frac{2}{L+1}\left(\sum_{\zeta\zeta'}\sin\left(\frac{\pi(m+1)\zeta}{L+1}\right)\sin\left(\frac{\pi m\zeta'}{L+1}\right)\boxed{\hat{a}^\dagger_{\zeta A}\hat{a}_{\zeta' B}} + \sum_{\zeta\zeta'}\sin\left(\frac{\pi(m-1)\zeta}{L+1}\right)\sin\left(\frac{\pi m\zeta'}{L+1}\right)\boxed{\hat{a}^\dagger_{\zeta B}\hat{a}_{\zeta' A}}\right)$$

$$\hat{H} = -\sum_\zeta \epsilon(\zeta)(\hat{a}^\dagger_{\zeta A}\hat{a}_{\zeta B} + \hat{a}^\dagger_{\zeta B}\hat{a}_{\zeta A}) - w\sum_{\zeta\zeta'}F(\zeta,\zeta')(\hat{a}^\dagger_{\zeta A}\hat{a}_{\zeta' B} - \hat{a}^\dagger_{\zeta B}\hat{a}_{\zeta' A})$$

where $\epsilon(\zeta) = v + w\cos\left(\frac{\pi\zeta}{L+1}\right)$ and $F(\zeta,\zeta') = \frac{2}{L+1}\frac{\sin\left(\frac{\pi\zeta'}{L+1}\right)\sin\left(\frac{\pi\zeta}{L+1}\right)}{\cos\left(\frac{\pi\zeta}{L+1}\right) - \cos\left(\frac{\pi\zeta'}{L+1}\right)}$

Now, considering periodic boundary conditions [3], *m=N+1* is identified with site *m=1* or m very large, and using the Fourier transform we have:

$$\tilde{f}(\zeta) = \frac{1}{\sqrt{2}}\sum_{-\infty}^{\infty} f(x)e^{-i\zeta x} \ \& \ f(x) = \frac{1}{\sqrt{2}}\sum_{-\infty}^{\infty}\tilde{f}(\zeta)e^{i\zeta x}$$

$$\hat{H} = -v\sum_{n=1}^{N}\frac{1}{2\pi}\left(\sum_{\zeta\zeta'}e^{-i\zeta n}e^{i\zeta' n}\hat{a}^\dagger_{\zeta B}\hat{a}_{\zeta' A} + \sum_{\zeta\zeta'}e^{-i\zeta n}e^{i\zeta' n}\hat{a}^\dagger_{\zeta A}\hat{a}_{\zeta' B}\right) +$$

$$-w\sum_{n=1}^{N-1}\frac{1}{2\pi}\left(\sum_{\zeta\zeta'}e^{-i\zeta(n+1)}e^{i\zeta' n}\hat{a}^\dagger_{\zeta A}\hat{a}_{\zeta' B} + \sum_{\zeta\zeta'}e^{-i\zeta n}e^{i\zeta'(n+1)}\hat{a}^\dagger_{\zeta B}\hat{a}_{\zeta' A}\right)$$

$$= -v\sum_\zeta(\hat{a}^\dagger_B\hat{a}_A + \hat{a}^\dagger_A\hat{a}_B)_\zeta - w\sum_\zeta(e^{-i\zeta}\hat{a}^\dagger_A\hat{a}_B + e^{i\zeta}\hat{a}^\dagger_B\hat{a}_A)_\zeta$$

$$= \sum_\zeta \hat{H}(\zeta) \Longrightarrow \hat{H}(\zeta) = d_0(\zeta)\hat{I} + \vec{d}(\zeta).\hat{\vec{\sigma}}$$

where $d_0 = d_z = 0 \longrightarrow \vec{d}(\zeta)$ is confined to a plane.

The matrix representation is given by:

$$\hat{H}(\zeta) := E_\zeta\begin{pmatrix} 0 & e^{i(\theta_\zeta - \frac{\zeta}{2})} \\ e^{-i(\theta_\zeta - \frac{\zeta}{2})} & 0\end{pmatrix}$$

were: $\theta_\zeta = \tan^{-1}\left(\frac{(v-w)\tan\left(\frac{\zeta}{2}\right)}{v+w}\right)$

The eigenvalues are:

$$\pm E_\zeta = \pm\sqrt{v^2 + w^2 + vw\cos(\zeta)}$$

And the eigenvectors are:

$$|\pm\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm e^{-i(\theta_\zeta - \frac{\zeta}{2})}\end{pmatrix}$$

were: $\theta_\zeta - \frac{\zeta}{2}$ is a geometric *Berry phase.*

PERSPECTIVES

- Study the Berry phase of the system and the differences between the topology of the system with PBC and the system with HWBC.
- Implement the Bogoliubov-de Gennes method to the fermionic unidimensional system described by de SSH model in the momentum space.

$$\hat{H} = \sum_k \hat{C}^\dagger \mathbb{H}_{BdG}\hat{C}$$

- Diagonalize the coupling matrix to visualize the edge modes with it's eigenenergy and wave function localized in the finite SSH fermionic chain.

REFERENCES

[1] W. P. Su, J. R. Schrieffer, and A. J. Heeger, Phys. Rev. B 22, 2099 (1980).

[2] S. A. Martucci, "Symmetric convolution and the discrete sine and cosine transforms," IEEE Trans. Signal Process. SP-42, 1038–1051 (1994).

[3] Simon, D., Osawa, S. and Sergienko, A.(2019). Topological Transitions and Bulk Wavefunctions in the SSH Model. Journal of Physics Condensed Matter 31(4).