

**Novena Escuela de Física-Matemática
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Topological Order and Beyond

**Departamento de Matemáticas – Departamento de Física
Universidad de los Andes**

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Morning Lectures

Nicolas Regnault (École Normale Supérieure, Paris, Francia)

An introduction to topological phases and entanglement

Topological phases do not fall into the Landau's paradigm of symmetry breaking. The absence of local order parameter to characterize these phases has motivated the development of new ways to probe them. Among those new approaches, entanglement measurements and characterizations have allowed a deeper understanding of these systems.

Using concrete examples such as the Su-Schrieffer-Heeger or the AKLT models, we will illustrate several concepts of quantum information applied to topological matter. In particular, we will unveil how the edge excitations, a key signature of topological phases, are encoded within the bulk groundstate, giving flesh to the bulk-edge correspondence.



Raoul Santachiara (Université Paris-Sud, Francia)

Bootstrapping in two and higher dimension: the unexpected power of conformal symmetry and its relation with topological order

Conformal invariance is a fundamental symmetry in nature and has been studied in Physics since the introduction of the Maxwell equations.

Conformal transformations are the angle-preserving transformations. In general dimension, the conformal group is composed by the Poincaré group (translations and rotations) plus dilations and inversions. Among the most recent breakthrough results in theoretical physics, one has shown that, in any dimension, the constraints coming from conformal invariance together with general consistency assumptions can be sufficient to provide non-perturbative and beyond mean field theory results for strongly interacting systems. For instance, this approach has provided the critical exponents of the critical 3D Ising model.

In two dimension the group of conformal transformations becomes infinite, thus opening the way for a series of powerful analytical approaches. For instance the analytic behavior of the symmetry functions of the 2d conformal group, called the conformal blocks, have been at the basis of the first prediction of non-Abelian statistics in fractional quantum Hall systems.

In these lectures, I will introduce to the basic concepts concerning the implementation of conformal symmetry at quantum level and the functions that appear in this theory. I will in particular stress the relation between the topological states in 2D and the quantum conformal symmetry. New directions of research will be also discussed.



Norbert Schuch (Max-Planck-Institut fr Quantenoptik, Garching, Alemania)

Topological order: A simple perspective from stabilizer codes and tensor networks (Norbert Schuch)

Topological order is a form of order which cannot be characterized locally (such as for symmetry breaking) and in which the entanglement in the system organizes in a global way. In my lectures, I will discuss a number of important examples which exhibit topological order. In particular, Kitaev's Toric Code model will be presented in detail, including its ground space structure and the nature of its excitations. In the second part of the lecture, I will explain how Tensor Networks provide an entanglement-based perspective on systems with complex entanglement, which allows for a simple local characterization of systems with topological order and allows us to extend the description above to a larger class of models.

Short Communications

Iván Mauricio Burbano Aldana

DEPARTAMENTO DE FÍSICA

UNIVERSIDAD DE LOS ANDES, BOGOTÁ, COLOMBIA

KMS States and Tomita-Takesaki Theory

The lack of an axiomatic framework for Quantum Field Theory has brought forward attention to the algebraic formulation of physical theories. The power of such a description lies on the clearness of the physical and mathematical interpretation of the objects involved. Indeed, the description of thermal equilibrium states finds in the algebraic setting a natural framework through the KMS condition. Here we will discuss this mathematical formulation of equilibrium, its relationship to Gibbs states, and the dynamical invariance it provides. We will then make use of the modular theory of Tomita-Takesaki to show that for every normal faithful state on a von Neumann algebra there is a unique dynamical law such that the state satisfies the KMS condition. We thus conclude that KMS states induce canonical dynamics.

La falta de un marco axiomático para la Teoría Cuántica de Campos ha aumentado la relevancia de la formulación algebraica de las teorías físicas. Esta descripción es prometedora debido a la transparencia de la interpretación física y matemática de los objetos involucrados. En efecto, la descripción de estados de equilibrio térmico tiene una formulación natural en el marco algebraico denominada la condición KMS. Acá vamos a discutir esta formulación del equilibrio, su relación con los estados de Gibbs y la invarianza dinámica que provee. Luego vamos a utilizar la teoría modular de Tomita-Takesaki para mostrar que para todo estado normal y fiel sobre una álgebra de von Neumann hay una única ley dinámica tal que el estado satisface la condición KMS. Por lo tanto, concluimos que los estados KMS inducen dinámica de manera canónica.

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Nicolás Escobar

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An Application of the Classification of Lagrangian Algebras in a Group Theoretical Category to Topological Quantum Computation

We study representations of the braid groups from braiding gapped boundaries of Dijkgraaf-Witten theories and their twisted generalizations, which are (twisted) quantum doubled topological orders in two spatial dimensions. We show that the braid representations associated to Lagrangian algebras are all monomial with respect to some specific bases. We give explicit formulas for the monomial matrices and the ground state degeneracy of the Kitaev models that are Hamiltonian realizations of Dijkgraaf-Witten theories. Our results imply that braiding gapped boundaries alone cannot provide universal gate sets for topological quantum computing with gapped boundaries.

Joint work with César Galindo and Zhenghan Wang.

César Galindo

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Acyclic anyon models, thermal anyon error corrections, and braiding universality

In topological quantum computing (TQC), information is encoded in the ground state manifolds of topological phases of matter which are error correction codes. Therefore, TQC is intrinsically fault-tolerant against local errors. But at any finite temperature $T > 0$,

thermal anyon pairs created from the vacuum due to thermal fluctuations can diffuse and braid with computational anyons to cause errors, the so-called thermal anyon errors. In practice, thermal anyon creations are suppressed by the energy gap Δ and low temperature T as $\alpha e^{-\Delta T}$ for some positive constant α , so it might not pose a serious challenge. But if the suppression by gap and temperature is not enough, then thermal anyon errors could become a serious issue for long quantum computation. In [G. Dauphinais and D. Poulin. Fault-tolerant quantum error correction for nonabelian anyons. Communications in Mathematical Physics, 355, Oct 2017], the authors found an error correction scheme for acyclic anyon models.

In this talk, we will characterize acyclic anyon models as anyon models with nilpotent fusion rules. We obtain several general results on acyclic anyon models and find many more acyclic anyon models such as $SO(8)_2$, which has Property F . Our characterization of acyclic anyon models raise the question if the restriction to acyclic anyon models is a deficiency in the current protocol or could it be intrinsically related to the computational power of non-abelian anyons.

This talk is based on joint work with Eric Rowell and Zhenghan Wang.

Gustavo Melgarejo

UNIVERSIDAD CENTRAL DE VENEZUELA
CARACAS, VENEZUELA

Formulación canónica de un modelo alternativo de gravedad masiva linealizada

Se estudia la formulación canónica aplicada a la acción de Morand y Solodukhin en un espacio-tiempo plano, esta acción representa un modelo dual para una teoría de gravedad masiva linealizada, de allí el interés para estudiarla.

Se hace un estudio de las ecuaciones de movimiento del sistema para así determinar cuántos grados de libertad tiene la teoría y para hallar algunas propiedades sobre los campos que aparecen en ésta.

Esta es una teoría singular, y por lo tanto el espacio de fases contiene variables sin relevancia alguna, aplicando el método de Dirac se consigue determinar cuales son las variables físicas del sistema, además obtenemos un álgebra consistente en términos de los corchetes de Dirac, y mostramos que esta reproduce las ecuaciones de movimiento obtenidas a nivel

Lagrangiano.

Palabras Clave: Método de Dirac, Lagrangiano singular, formulación Hamiltoniana, cantidades de primera y de segunda clase.

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Corrientes persistentes de carga y de espín en siliceno usando la técnica de las Funciones de Green en el equilibrio.

La presencia del acoplamiento Espín-Órbita afecta las corrientes persistentes en anillos mesoscópicos. Se analizaron las corrientes persistentes de carga y de espín para anillos mesoscópicos de cuatro sitios basado en siliceno sujeto a interacciones Espín-Órbita tipo intrínseca y Rashba extrínseca e intrínseca, atravesado por un flujo de campo magnético de forma perpendicular al plano del anillo. Para el cálculo de las corrientes persistentes implementamos el formalismo de las funciones de Green en el equilibrio y la técnica de la ecuación de movimiento. En presencia de todas las interacciones EO, observamos que las interacciones intrínseca y Rashba extrínseca proceden en el comportamiento de la corriente de carga como una sola interacción, predominando ante el acoplamiento Rashba intrínseco. Por último calculamos cada componente vectorial de la corriente de espín. En la componente \hat{z} , observamos que la corriente de espín en función del flujo magnético cuando sólo se tiene presente el acoplamiento EO intrínseco se satura al variar la temperatura y cuando sólo se tiene presente los acoplamientos EO Rashba, la componente \hat{z} se anula. La corriente de carga y la corriente de espín para todos los casos disminuyen con el aumento de la temperatura y para los casos cuando sólo se tiene interacciones EO intrínseca y Rashba extrínseca los resultados son similares al caso de un anillo de grafeno en presencia de interacciones EO intrínseca y Rashba extrínseca.

Palabras Clave: Siliceno, corrientes persistentes, acoplamiento Espín-Órbita, funciones de Green.

Juan Gabriel Ramírez

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Ultrafast spectroscopy as a tool to understand quantum materials

The discovery of novel phases of matter is at the core of modern physics. In quantum materials, subtle variations in atomic-scale interactions can induce dramatic changes in macroscopic properties and drive phase transitions. Despite their importance, the mesoscale processes underpinning phase transitions often remain elusive because of the vast differences in timescales between atomic and electronic changes and thermodynamic transformations. In this talk, I will show pump-probe experiments (Femto- and Pico-second resolution at X-ray and THz wavelengths) in quantum materials that undergoes first order phase transitions (VO₂ and V₂O₃). Understanding the time scales of their interactions allowed us to identify the role of their degrees of freedom. Furthermore, I will show how new phases of matter can emerge dynamically.

Work done with multiple collaborations at UC San Diego, Cornell University and SLAC-LCLS. The work at UCSD was supported by the AFOSR grant #FA9550-16-1-0026 and a UC collaborative grant MRPI MR-15-328-528.



Ling Sequera

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Topological characterization of phase transitions

In this work we study phase transitions from the point of view of topology. Topological invariants (of index-theory type) can characterize phases of matter for some of quantum, as well as classical systems. A paradigmatic example is given by the (classical) Ising model, for which the phase transition can be characterized in terms of basis projections and related \mathbb{Z}_2 -index, as shown by H. Araki and D. Evans (1983) [AE83]. In joint work with S. Tabban and A.F. Reyes-Lega, we have established an alternative characterization of the phase transition in terms of the \mathbb{Z}_2 -index that naturally arises in the context of the Shale-Stinespring theorem [GBVF00]. This approach allows us to make an explicit connection between the current approaches to topological quantum matter (based on the study of topological invariants) and the operator-algebraic approach of Araki and Evans.

On the other hand, for some kind of topological systems it is possible to express the relevant invariants through local formulas (i.e Chern numbers) using K -theory elements. Recently, Kauffman et al. (2016) [LKWK16b], [LKWK16a], have proposed a formula of this type for theories in the frame of Clifford algebras using quaternionic K -theory. We verify the validity of this local index formula for the particular case of the Ising model, taking into account the connection with the CAR and Clifford algebras.

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Souad M. Tabban Sabbagh

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Quantum Entropic Ambiguities and Tomita-Takesaki Theory

Given an algebra of observables \mathcal{A} and a state ω , a density matrix ρ_ω acting on \mathcal{H}_ω can be obtained through the GNS construction $(\mathcal{H}_\omega, \pi_\omega)$ that gives rise to the same expectation values as ω for elements $a \in \mathcal{A}$, that is, $\omega(a) \equiv \text{Tr}_{\mathcal{H}_\omega}(\rho_\omega \pi_\omega(a))$. An entropy can be assigned to the state ω by computing the von Neumann entropy of the density matrix $S(\rho_\omega) = -\text{Tr}_{\mathcal{H}_\omega}(\rho_\omega \log \rho_\omega)$. This has proved to be useful, e.g., in the study of entanglement properties of identical particles [ATdQAA13]. However, there are situations for which this density matrix is not unique, thus leading to an entropy ambiguity. This occurs whenever the irreducible components of the representation π_ω appear in \mathcal{H}_ω with multiplicities [AdQAS13]. In the present work, we develop an interpretation of this phenomenon as a gauge symmetry arising from the action of unitaries in the commutant of the representation

via Tomita-Takesaki modular theory. In the finite-dimensional case, a complete characterization of the ambiguity can be given in terms of the modular data, and a physical interpretation can be obtained in terms of an equivalent description of the system as a bipartite system. We extend our analysis to the case of group transformation C^* -algebras describing the algebras of observables of quantum systems obtained by quantization on configuration spaces of the form G/H , with G a compact Lie group and H a finite, non-abelian subgroup.

This is part of a joint work with A.P. Balachandran, I. Burbano Aldana and A.F. Reyes-Lega.

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Posters

David Armendáriz

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Problemas y aplicaciones de valores en la frontera para funciones monogénicas fraccionarias

En el presente trabajo se presenta una breve discusión sobre la construcción de funciones monogénicas fraccionarias para las denominadas álgebras de Clifford y cómo éstas pueden resolver un problema de Dirichlet de valores en la frontera utilizando el operador fraccionario de Riemann-Liouville.

Joint work with N. di Teodoro (Departamento de Matemáticas, Universidad San Francisco de Quito, Quito) y J. Ceballos (Departamento de Matemáticas, Universidad de las Américas, Quito) .



Diego Fernando Cardona Pineda

DEPARTAMENTO DE FÍSICA
UNIVERSIDAD TECNOLÓGICA DE PEREIRA, COLOMBIA.

Renormalización de la energía Casimir mediante la función Zeta de Riemann para configuraciones toroidales

A partir de un campo escalar sin masa, confinado en una variedad espacio-temporal toroidal, se encontraron las energías de Casimir en función de las dimensiones para diferentes condiciones de frontera [Eli89]. Para la parte temporal, se tomó una condición periódica con periodo β [Haw77]. En los cálculos, se consideran temperaturas bajas para el sistema, donde dicho parámetro se define como $T=1/\beta$. En particular, se trabajó sobre los casos T^3 y $T^3 \times \mathbb{R}^1$ y se obtuvieron las energías finitas [Kir10] mediante la técnica de renormalización definida por la función zeta de Riemann. Los valores de estas energías, reflejan la dependencia del resultado de la renormalización en la topología de la variedad.

Joint work with Héctor Iván Arcos Velazco.

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Daniel Castro

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Goldstone bosons and geometry

We review the general aspects of broken global symmetries in the framework of quantum field theory [Wei13, PS95, CL84]. Considering a general continuous symmetry \mathcal{G} broken (via an arbitrary mechanism) to a subgroup \mathcal{H} , we interpret the Nambu-Goldstone bosons (NGB) as the coordinates of the coset-space \mathcal{G}/\mathcal{H} [Dob97], which turns to be a Riemann space, and we exploit its topological and symmetry properties [Nab00, Nak03, Tap10]. Killing fields and its associated equations can be defined and allow us to construct general lagrangians which describe the dynamics of the NGB, thus purely mathematical assumptions about the coset space can (in principle) describe the dynamics of physical particles identified with the NGB. We apply the above construction to the particular case of the non-linear σ model coupled to the electroweak standard model $SU(2) \times U(1)$, in order to describe the low energy interactions of pions, and we also verify the Goldstone boson equivalence theorem for processes involving the standard model W bosons [DP94a, DP94b].

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Jhon M. Córdoba-Pareja y Servio T. Pérez-Merchancano

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Mixture of particles and strong interactions in the optical lattice

In this work the behavior of a finite particle numbers is studied (bosons and fermions) that interact strongly generating different states mixed between small groups that permute. Such particles are subject to a harmonic potential in an optical lattice, which is very low temperatures. For this analysis an energy band is constructed using the Hubbard-fermi formalism that determines the fundamental energy of the different particle mixtures. Subsequently, the matrix diagonalization method for quantum mechanical states is implemented with the purpose of observing the behavior of these energies employing the Bose-Fermi-Hubbard Hamiltonian which takes into account the symmetry of configuration of the fundamental state through of a boson function and the interaction between neighboring particles. This energy shows the behavior of bosons and fermions trapped as a function of the possible quantum correlations exhibiting different critical states that occur in the optical lattice that are explained by the mean field theory.

Joint work with L. E. Bolívar-Marínez (Departamento de Física, Universidad del Cauca, Popayán, Colombia).

Héctor Efrén Guerrero Mora

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A new way to determine the vector of position of curves in the space

The curves parameterized in space are objects of great interest and with them we can model and analyze problems that appear in different fields of physics and other sciences. In the classical differential geometry of curves it is well known that there are two geometric invariants that are: curvature and torsion; which determine the behavior of the curve in space. The fundamental theorem of the local theory of curves in space states that if two differentiable $\kappa = \kappa(s)$ and $\tau = \tau(s)$ functions are given and positive $\kappa = \kappa(s)$ shows that there is a curve whose curvature is $\kappa = \kappa(s)$ and whose torsion is $\tau = \tau(s)$ and any other curve that meets the conditions that its curvature is $\kappa = \kappa(s)$ and its torsion is $\tau = \tau(s)$ differs from the previous curve by a rigid movement.

The demonstration of this result, which appears in most texts of differential geometry, is done considering a system of nine differential equations and using the existence and uniqueness theorem of ordinary differential equations.

The problem of explicitly finding the position vector of a differentiable curve in space with the arc length parameter knowing only its curvature $\kappa = \kappa(s)$ and its torsion $\tau = \tau(s)$ is a problem that is still open in the classical differential geometry of the curves. Only in some very specific cases this problem is solved.

In this talk we show a new proof of the theorem of the local theory of curves in space considering a differential equation of non-linear second order:

$$\frac{d}{ds} \left\{ \frac{1}{\kappa} \frac{d\omega}{ds} \right\} = -\kappa\omega + \tau \sqrt{1 - \omega^2 - \left(\frac{1}{\kappa} \frac{d\omega}{ds} \right)^2}.$$

Some properties of this nonlinear differential equation are shown.

This technique allows you to find the vectors of position of a wide variety of curves in space, only knowing its curvature function and its function of torsion.

In this talk we will show how by means of this method the position vector of some curves like the general helices, the slant helices, the rectifying curves, among others, are obtained in a very easy way.

Juan Pablo Ibierta Jiménez
DEPARTAMENTO DE FÍSICA MATEMÁTICA
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Topological Order from Abelian Higher Gauge Theories

In recent years, attempts to generalize lattice gauge theories to model topological order have been carried out through the so called 2-gauge theories. These have opened the door to interesting new models and new topological phases which are not described by previous schemes of classification. In this paper we show that we can go beyond the 2-gauge construction when considering chain complexes of abelian groups. Based on elements of homological algebra we are able to greatly simplify already known constructions for abelian theories under a single all encompassing framework. Furthermore, this formalism allows us to systematize the computation of the corresponding topological degeneracies of the ground states and establishes a connection between them and a known cohomology, which conveniently characterizes them with a suitable set of quantum numbers.

Nicolás Medina
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State-Based Grothendieck Topologies and Entropy Paths for the Category of Projection Operators on a Hilbert Space

Emma Mora
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Persistent charge currents in a mesoscopic ring based on graphene

We study the persistent charge currents in a graphene model consisting of a one-dimensional mesoscopic ring affected by the presence of spin-orbit (SO) coupling. The ring is crossed by a magnetic flow perpendicular to the plane's ring. The general framework is the formalism of Green functions in thermodynamic equilibrium, these Green functions are determined

from the equations of motion. The main advantage of this technique is that it avoids the diagonalization of the Hamiltonian. We reproduced the calculations of the persistent charge currents for a mesoscopic ring based on graphene without SO coupling, then for a ring with intrinsic coupling of graphene, and another ring with Rashba coupling; noting great changes in the magnitudes and behavior of the persistent currents against the magnetic flux. Finally, we calculate the persistent currents for a ring in the presence of both SO interactions. We observe that these act as a single interaction, in which the behavior of the Rashba type coupling predominates. For all cases, the degradation of the current when the system temperature increases is observed .

Daniel Padilla-Gonzalez

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BOGOTÁ, COLOMBIA

Berry Phase: a Tool to Study Quantum Phase Transitions

The classical phase transitions as well as quantum phase transitions are usually classified as first and second kind transitions. This classification depends of the behavior of an *order parameter* as a function of thermodynamic or internal parameters of the system. However, this classification is not complete. In fact, there are some kind of phase transitions which cannot classify as a first or second order phase transitions as the *Kosterlitz-Thouless transition*. This new type phase transitions are characterized by some kind of *topological invariants* and the topological phases described by this invariants are robust against thermal effects or presence of impurities in the system. An example of this invariants is the Berry phase which is a non-trivial phase due to a smooth evolution of the system. In this work, we study the possibility of use the Berry phase to determine the critical points of quantum phase transition for some one dimensional systems. With aid of this topological invariant, we give a description of topological phase transitions in one dimensional models like Heisenberg model [HKH08] or the Su-Schrieffer-Heeger model for polyacetylene [YLSL16].

References

- [HKH08] T. Hirano, H. Katsura, and Y. Hatsuga. Topological classification of gapped spin chains: Quantized berry phase as a local order parameter. *Phys. Rev. B*, 77:094431, Mar 2008.
- [YLSL16] Wing Chi Yu, Yan Chao Li, P. D. Sacramento, and Hai-Qing Lin. Reduced density matrix and order parameters of a topological insulator. *Phys. Rev. B*, 94:245123, Dec 2016.

Julián A. Sánchez

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Island Size Distribution with Hindered Aggregation

The study of the effect of hindered aggregation on the island formation process in one-dimensional epitaxial growth. In the proposed model the aggregation of monomers to stable islands is hindered by an additional aggregation barrier, ϵ_a which decreases the diffusion rate to those islands. As ϵ_a increases the system exhibits a crossover between two different kinds of processes, diffusion-limited aggregation (DLA) and attachment-limited aggregation (ALA). The island size distribution, $P(s)$, is calculated by using a self-consistent (SC) set of equations for the capture kernels. The results given by the analytical model are compared with those from numerical kinetic Monte Carlo simulations. We test our analytical model with extensive numerical simulations and previously established results. Since $P(s)$ is usually measurable experimentally, we can use it to actually calculate microscopic parameters of the model such ϵ_a .

Joint work with Manuel Camargo (CICBA, Universidad Antonio Nariño, Cali) y Diego Luis González (Departamento de Física, Universidad del Valle, Cali) .

Alejandra Torres Montas

UNIVERSIDAD AUTÓNOMA DE SAN LUIS POTOSÍ
MÉXICO

Dynamics in the quasi-homogeneous isosceles three body problem

In this talk we present a qualitative study of three body in which two equal point masses are confined to a horizontal plane, symmetrically disposed with respect to their common center of mass, and a third point mass is allowed to move only on the vertical axis perpendicular to the plane of the equal point masses through the center of mass. At any time, the configuration formed by the three mass points is that of an isosceles triangle and the only rotation allowed are with respect to the vertical axis with a mutual interaction given by the potential $U(r) = -\frac{A}{r^a} - \frac{B}{r^b}$ and $A, B > 0$. A particular case of our study is the Schwarzschild potential presented in [APCS14]. To the investigation we employed appropriate regularization transformations to analyze behaviour of the flow near triple collision [McG74].

References

- [APCS14] John A Arredondo, Ernesto Pérez-Chavela, and Cristina Stoica. Dynamics in the schwarzschild isosceles three body problem. *Journal of Nonlinear Science*, 24(6):997–1032, 2014.
- [McG74] Richard McGehee. Triple collision in the collinear three-body problem. *Inventiones mathematicae*, 27(3):191–227, 1974.

Jefferson Vaca

ESCUELA POLITÉCNICA NACIONAL DEL ECUADOR

Simulación Monte Carlo Metrópolis en el modelo-xy

En el presente trabajo se realiza un estudio a la transición de fase KT en el modelo xy utilizando una simulación con un método montecarlo metrópolis. Esta simulación esta principalmente programada en C. Con lo cual se ha logrado determinar la la energía, capacidad calorífica, y correlación entre spins para cada temperatura. Para determinar la temperatura de transición, se utiliza el maximal de la capacidad calorífica con lo que se ha determinado que la temperatura critica es $TKT=0.88$

Monday, May 21st (Room: B-202)

8:00 – 9:00 *Registration*

9:00 – 9:15 *Opening*

9:15 – 10:15 Nicolas Regnault: *Introduction to topological phases and entanglement I*

10:15 – 11:15 Norbert Schuch: *Topological order: a simple perspective from stabilizer codes and tensor networks I*

11:15 – 11:45 Break

11:45 – 12:45 Raoul Santachiara: *Bootstrapping in two and higher dimension I*

12:45 – 14:00 Lunch Break

14:00 – 15:00 Problem Session

15:00 – 15:30 Break

15:30 – 16:10 Gustavo Melgarejo: *Formulación canónica de un modelo alternativo de gravedad masiva linealizada*

16:10 – 16:50 Ling Sequera: *Topological characterization of phase transitions*

Tuesday, May 22nd (Room: B-202)

9:00 – 10:00 Nicolas Regnault: *Introduction to topological phases and entanglement II*

10:00 – 11:00 Norbert Schuch: *Topological order: a simple perspective from stabilizer codes and tensor networks II*

11:00 – 11:30 Break

11:30 – 12:30 Raoul Santachiara: *Bootstrapping in two and higher dimension II*

12:30 – 14:00 Lunch Break

14:00 – 14:40 Iván Burbano: *KMS States and Tomita-Takesaki Theory*

14:40 – 15:20 Souad Tabban: *Quantum Entropic Ambiguities and Tomita-Takesaki Theory*

15:20 – 15:40 Break

15:40 – 16:40 Problem Session

16:40 – 17:00 Break

17:00 – Poster Session & refreshments

Wednesday, May 23rd (Room: B-202)

9:00 – 10:00 Nicolas Regnault: *Introduction to topological phases and entanglement III*

10:00 – 11:00 Norbert Schuch: *Topological order: a simple perspective from stabilizer codes and tensor networks III*

11:00 – 11:30

Break

11:30 – 12:30 Raoul Santachiara: *Bootstrapping in two and higher dimension III*

Thursday, May 24th (Room: B-202)

9:00 – 10:00 Nicolas Regnault: *Introduction to topological phases and entanglement IV*

10:00 – 11:00 Norbert Schuch: *Topological order: a simple perspective from stabilizer codes and tensor networks IV*

11:00 – 11:30 Break

11:30 – 12:30 Raoul Santachiara: *Bootstrapping in two and higher dimension IV*

12:30 – 14:00 Lunch Break

14:00 – 14:40 César Galindo: *Acyclic anyon models, thermal anyon error corrections, and braiding universality*

14:40 – 15:20 Nicolás Escobar: *An Application of the Classification of Lagrangian Algebras in a Group Theoretical Category to Topological Quantum Computation*

15:20 – 15:50 Break

15:50 – 16:50 Problem Session

Friday, May 25th (Room: B-202)

9:00 – 10:00 Nicolas Regnault: *Introduction to topological phases and entanglement V*

10:00 – 11:00 Norbert Schuch: *Topological order: a simple perspective from stabilizer codes and tensor networks V*

11:00 – 11:30 Break

11:30 – 12:30 Raoul Santachiara: *Bootstrapping in two and higher dimension V*

12:30 – 14:00 Lunch Break

14:00 – 15:00 Problem Session

15:00 – 15:30 Break

15:30 – 16:10 Dayanna Pereira: *Corrientes persistentes de carga y de espín en siliceno usando la técnica de las Funciones de Green en el equilibrio.*

16:10 – 16:50 Juan Gabriel Ramírez: *Ultrafast spectroscopy as a tool to understand quantum materials*

17:00 – Entrega de certificados de asistencia

Contact Information

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	Monday	Tuesday	Wednesday	Thursday	Friday
8:00 – 9:00	Registration				
9:00 – 10:00	N. Regnault	N. Regnault	N. Regnault	N. Regnault	N. Regnault
10:00 – 11:00	N. Schuch	N. Schuch	N. Schuch	N. Schuch	N. Schuch
11:00 – 11:30	<i>Break</i>	<i>Break</i>	<i>Break</i>	<i>Break</i>	<i>Break</i>
11:30 – 12:30	R. Santachiara	R. Santachiara	R. Santachiara	R. Santachiara	R. Santachiara
12:30 – 14:00	<i>Break</i>	<i>Break</i>		<i>Break</i>	<i>Break</i>
	14:00 – 15:00 Problem Session	14:00 – 14:40 I. Burbano		14:00 – 14:40 C. Galindo T	14:00 – 15:00 Problem Session
	15:00 – 15:30 <i>Break</i>	14:40–15:20 S. Tabban		14:40-15:20 N. Escobar	15:00 – 15:30 <i>Break</i>
	15:30-16:10 G. Melgarejo	15:20 – 15:40 <i>Break</i>		15:20 – 15:50 <i>Break</i>	15:30-16:10 D. Pereira
	16:10-16:50 L. Sequera	15:40 – 16:40 Problem Session		15:20 – 15:50 Problem Session	16:10–16:50 J.G. Ramirez
		16:40–17:00 <i>Break</i>			17:00 Entrega de certificados
		17:00 Poster Session			