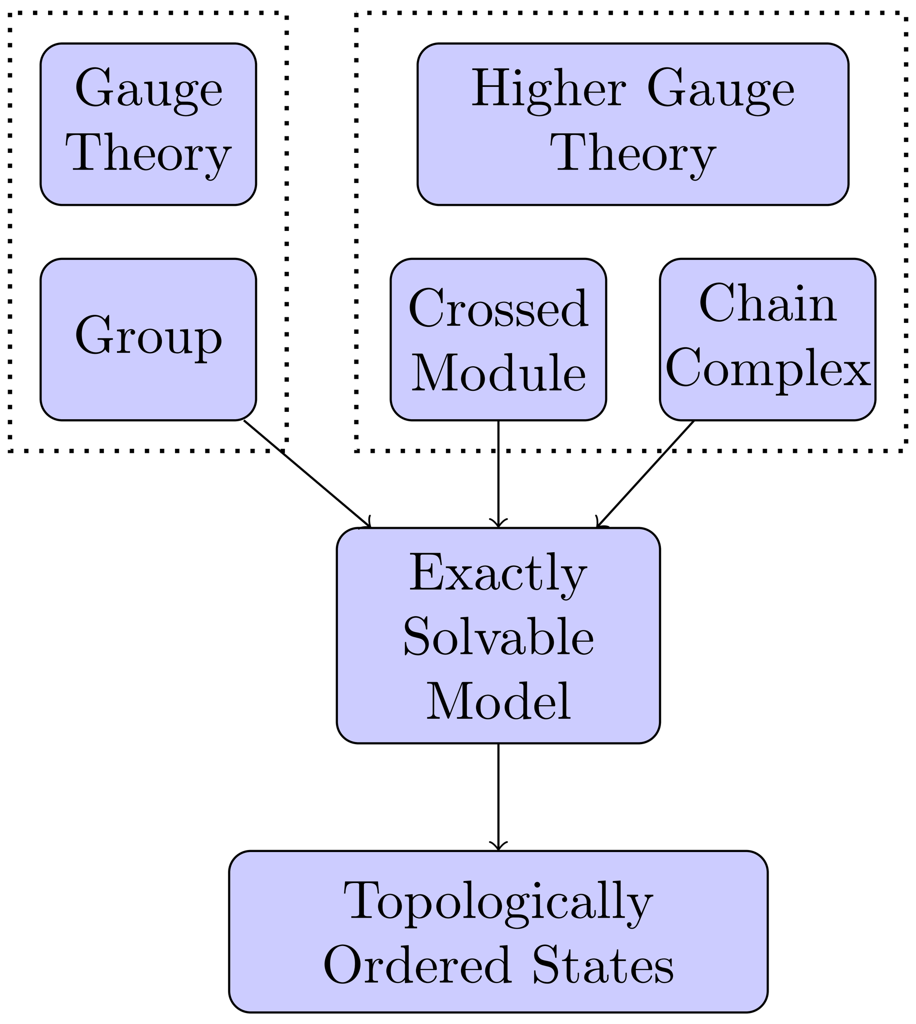
Topological Degeneracy from Higher Gauge Theory

J. P. Ibieta-Jimenez, R. Costa de Almeida, J. Lorca Espiro, P. Teotonio-Sobrinho
pibieta@if.usp.br
Departamento de Física Matemática, Universidade de São Paulo, Brazil.

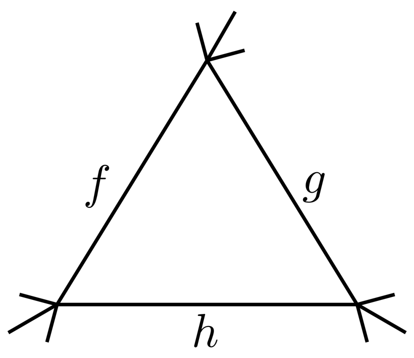
General Strategy



Gauge theory

Gauge field configurations:

$l_1, l_2, l_3 \dots \in K_1$
(Lattice edges)
$\mapsto$
$f, g, h, \dots \in G$
(Gauge group)



Hamiltonian Model:

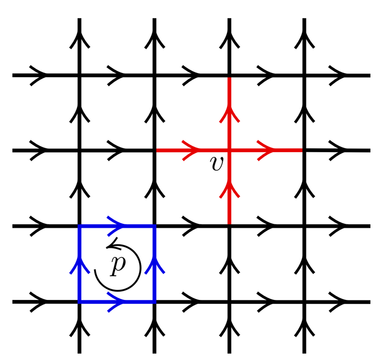
$$\mathcal{H} = \bigotimes_{l \in K_1} \mathbb{C}[G]$$

$$H = -\sum_{v \in K_0} A_v - \sum_{p \in K_2} B_p$$

Famous Example: Toric Code ($G = \mathbb{Z}_2$)

$$A_v = \frac{1}{2}\left(\mathbb{1} + \sigma^x_{l_1}\sigma^x_{l_2}\sigma^x_{l_3}\sigma^x_{l_4}\right)$$

$$B_p = \frac{1}{2}\left(\mathbb{1} + \sigma^z_{l_1}\sigma^z_{l_2}\sigma^z_{l_3}\sigma^z_{l_4}\right)$$



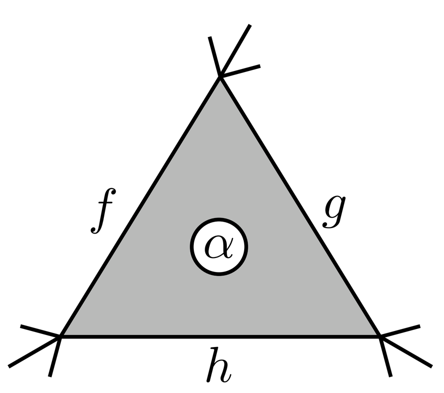
Some interesting features:

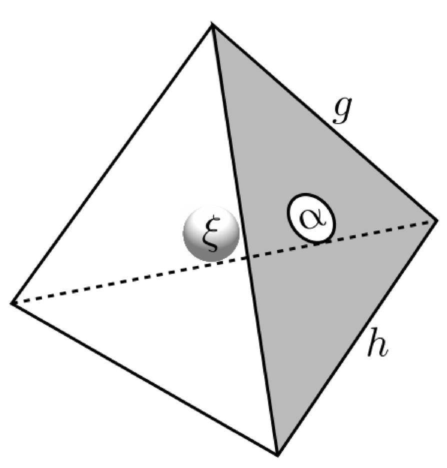
- Topological Degeneracy: GSD $= |H^1(\Sigma_g, \mathbb{Z}_2)| = 2^{2g}$
- Long Range Entanglement
- Anyonic Statistics
- Quantum Error Correction

Higher Gauge Theory

Generalization of Gauge Theory
$\Longrightarrow$
New TQFT's and Hamiltonian models?

Technical issue: poorly understood algebraic structures(n-groups) make it hard to study the corresponding models.





Models from Abelian Higher Gauge Theory

Key Idea:

- Use chain complex of abelian groups $(G_\bullet, \partial_\bullet)$ (abelian version of a gauge n-group)
- Not the most general setting but can be studied in detail using homological techniques
- Chain complex $(C_\bullet, \partial_\bullet)$ obtained from the lattice K can be used to keep track of the geometry
- Unified language that simplifies the analysis and provides an explicit procedure for obtaining ground states

$$\begin{array}{ccccccccc} \cdots & \xrightarrow{\partial_{n+2}} & C_{n+1} & \xrightarrow{\partial_{n+1}} & C_n & \xrightarrow{\partial_n} & C_{n-1} & \xrightarrow{\partial_{n-1}} & \cdots \\ & & \downarrow f_{n+1} & & \downarrow f_n & & \downarrow f_{n-1} & & \\ \cdots & \xrightarrow[\partial_{n+2}]{} & G_{n+1} & \xrightarrow[\partial_{n+1}]{} & G_n & \xrightarrow[\partial_n]{} & G_{n-1} & \xrightarrow[\partial_{n-1}]{} & \cdots \end{array}$$

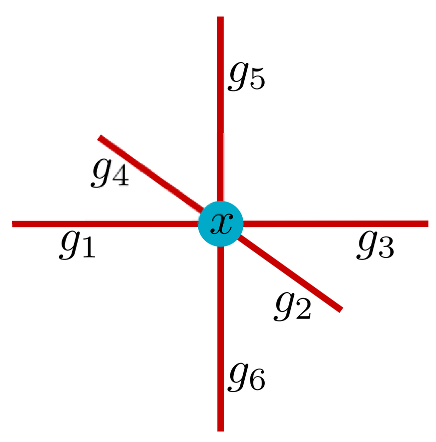
- Field configurations: $f \in \hom(C,G)^0 \implies$ states $|f\rangle = \bigotimes_n \bigotimes_{x \in K_n} |f_n(x)\rangle$
- Gauge transformations: $t \in \hom(C,G)^{-1} \implies$ operators $A_t\,|f\rangle := |f + \delta t\rangle$
- Holonomy measurement: $m \in \hom(C,G)_1 \implies$ operator $B_m\,|f\rangle := \chi_m(\delta f)\,|f\rangle$

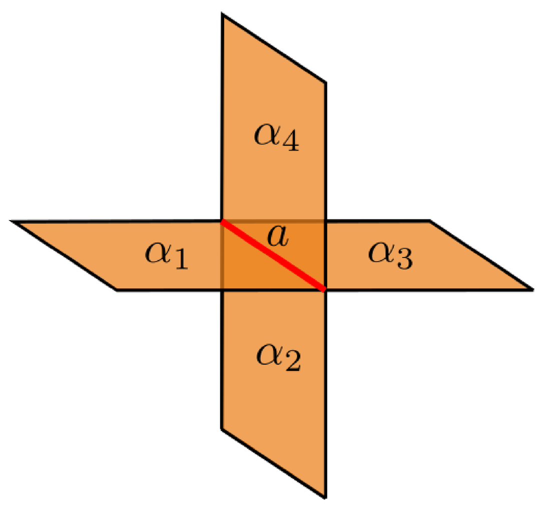
Hamiltonian Model:

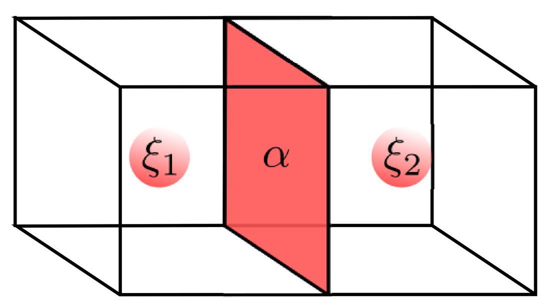
$$\mathcal{H} = \bigoplus_f \mathbb{C}\,|f\rangle = \bigotimes_n \bigotimes_{s \in K_n} \mathbb{C}[G_n]$$

$$H = -\sum_n \sum_{x \in K_n} A^0_x - \sum_n \sum_{y \in K_n} B^0_y$$

$$A^0_x = \frac{1}{|G_{n+1}|}\sum_{g \in G_{n+1}} A_{gx^*} \qquad B^0_x = \frac{1}{|G_{n-1}|}\sum_{r \in \hat{G}_{n-1}} B_{rx_*}$$







Ground State Degeneracy

Theorem. *The dimension of the ground state subspace $\mathcal{H}_0$ is given by:*

$$GSD = |H^0(C,G)| = \prod_n |H^n(C, H_n(G))|$$

The topological nature of the cohomology groups underlines the topological ordering of the ground states.
Each factor $H^n(C, H_n(G))$ provides quantum numbers for the ground states and are related to the different mechanisms and symmetries responsible for generating the degeneracy.

Some Remarks

- Large class of models which reproduce models in the literature and provide new ones
- Explicit formula for GSD for manifolds of arbitrary dimensions
- Possible applications to quantum error correction
- Extended excitations in higher dimensions (fractional statistics? Motion group reps?)
- Twisted versions?

References

[1] A. Kitaev. Anyons in an exactly solved model and beyond. *Annals of Physics*, 321:2–111, January 2006.

[2] Y. Hu, Y. Wan, and Y.-S. Wu. Twisted quantum double model of topological phases in two dimensions. *Physical Review B*, 87(12):125114, March 2013.

[3] A. Bullivant, M. Calçada, Z. Kádár, P. Martin, and J. F. Martins. Topological phases from higher gauge symmetry in 3 +1 dimensions. *Physical Review B*, 95(15):155118, April 2017.

[4] M. J. Bernabé Ferreira, P. Padmanabhan, and P. Teotonio-Sobrinho. 2D quantum double models from a 3D perspective. *Journal of Physics A Mathematical General*, 47:375204, September 2014.

[5] R. Costa de Almeida, J. P. Ibieta-Jimenez, J. Lorca Espiro, and P. Teotonio-Sobrinho. Topological order from a homological perspective. *In preparation.*