Entanglement and Weak Values: Application to Regularization

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Abstract

In this work, we present the framework of weak values and the surprising quantum effects they originate. In particular, we address the problem of simultaneous measurement of non-commuting observables, and show that it can be solved with appropriate pre and post-selection states that are entangled with an ancillary system. Then we apply the previous results to develop a regularization method in quantum field theory. We provide a regularization of the ultraviolet divergences in the Feynman propagator, both in the bosonic and fermionic cases, by using weak values and entanglement.

1. INTRODUCTION

In 1988 Aharonov, Albert and Vaidman introduced the concept of weak value of an observable A between a pre-selection state $\langle\psi_i\rangle$ *and a post-selection state* $\langle\psi_f\rangle$ *[1].*

$$A_w = \frac{\langle\psi_f|A|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \tag{1}$$

This new quantity can give place to unusual or surprising results if measurements that minimize the interaction of the system with the measurement device (weak measurements) and pre and post-selected ensembles are considered [2]. Those unusual situations or quantum miracles have to deal with the fact that weak values can afford values outside of the conventional range of expectation values of observables [3], but keep hidden "under the noise" if post-selection is not considered.

As a particular surprising effect due to weak values, it has been shown [4] that it is possible to assign simultaneous weak values to a family of operators, given that some linearity restrictions are satisfied and that the pre and post-selections involve entangled states. This fact is related with the problem of inference of a simultaneous measurement of two observables that do not commute, say for example x *and* p *as we will see in the next section.*

2. SIMULTANEOUS MEASUREMENT OF NON-COMMUTING OBSERVABLES

In this section we consider the simultaneous measurement of two non-commuting observables A_1 *and* A_2*. This measurement can be seen as an interaction of the system with two independent measuring instruments, where each one is an accurate probe for the respective observable. Our goal is to find conditions such that the initial state of the measurement instruments* $\hat{\rho}$ *is related with the final state* $\hat{\rho}'$ *via a unitary, deterministic Kraus operator*

$$\hat{\rho}' = \hat{K}\hat{\rho}\hat{K}^\dagger \qquad \hat{K} = e^{i(\alpha_1\hat{q}_1+\alpha_2\hat{q}_2)} \tag{2}$$

This Kraus operator can be shown to be proportional to the matrix element

$$\hat{K} = e^{i(\alpha_1\hat{q}_1+\alpha_2\hat{q}_2)} \propto \langle\psi_f|\hat{U}(q_1,q_2)|\psi_i\rangle$$

From this we see that the inference problem is equivalent to finding a pair of states $(|\psi_i\rangle, |\psi_f\rangle)$ *such that weak values* $e^{i(\alpha_1\hat{q}_1+\alpha_2\hat{q}_2)}$ *can be assigned to the family of operators* $\{\hat{U}(q_1,q_2)|(q_1,q_2)\in\mathbb{R}^2\}$*. More precisely,*

$$K = \frac{\langle\psi_f|e^{i(A_1q_1+A_2q_2)}|\psi_i\rangle}{\langle\psi_f|\psi_i\rangle} \tag{3}$$

Let us tackle the problem by considering also an ancillary system described by canonical variables x_a, p_a*. Define the conjugate pairs* $(\hat{x}_\pm, \hat{p}_\pm)$ *as*

$$\hat{x}_\pm = \frac{1}{\sqrt{2}}(\hat{x}\pm\hat{x}_a) \qquad \hat{p}_\pm = \frac{1}{\sqrt{2}}(\hat{p}\pm\hat{p}_a) \tag{4}$$

If we consider a pre-selection in the state $|x_-,p_+\rangle$ *and a post-selection performed by the state* $|x_+,p_-\rangle$*, we have that the Kraus operator is*

$$\hat{K} = \frac{\langle x_+,p_-|e^{i(\hat{q}_1\hat{x}+\hat{q}_2\hat{p})}|x_-,p_+\rangle}{\langle x_+,p_-|x_-,p_+\rangle} = e^{i(\hat{q}_1x+\hat{q}_2p)} \tag{5}$$

And the inference problem is solved, with the additional condition that the states in the PPS ensemble are entangled states.

3. REGULARIZATION VIA WEAK VALUES

Given that the pre and post-selected ensembles formalism is based on the choice of initial an final states to define the weak value, one can evidence a similarity with propagators in quantum field theory. Taking into account that quantum entanglement allows us to obtain arbitrary results in terms of weak values for the observables of a quantum system and the similarity of weak values and propagators, these techniques could be applied to regularize propagators that present divergencies. For that purpose we define the weak Feynman propagator as

$$D_F^{(w)}(x_1-x_2) = \frac{\langle f|T[\phi(x_1)\phi(x_2)]|i\rangle}{\langle f|i\rangle}. \tag{6}$$

After introducing expressing the fields in its frequency decomposition we get

$$\begin{aligned} D_F^{(w)}(x_1-x_2) &= \Theta(t_1-t_2)\int\frac{d^3k}{2\omega_k}\left[\frac{\langle f|\hat{a}_k\hat{a}_k^\dagger|i\rangle}{\langle f|i\rangle}e^{-ik(x_1-x_2)} + \frac{\langle f|\hat{a}_k^\dagger\hat{a}_k|i\rangle}{\langle f|i\rangle}e^{ik(x_1-x_2)}\right] \\ &+ \Theta(t_2-t_1)\int\frac{d^3k}{2\omega_k}\left[\frac{\langle f|\hat{a}_k\hat{a}_k^\dagger|i\rangle}{\langle f|i\rangle}e^{ik(x_1-x_2)} + \frac{\langle f|\hat{a}_k^\dagger\hat{a}_k|i\rangle}{\langle f|i\rangle}e^{-ik(x_1-x_2)}\right]. \end{aligned}$$

If we now define the weak value of the number operator as $\nu_k = \frac{\langle f|\hat{N}_k|i\rangle}{\langle f|i\rangle}$*, we obtain the following expression for the propagator*

$$\begin{aligned} \frac{\langle f|T[\phi(x_1)\phi(x_2)]|i\rangle}{\langle f|i\rangle} &= \Theta(t_1-t_2)\int\frac{d^3k}{2\omega_k}\left[2\nu_k\cos(k(x_1-x_2)) + e^{-ik(x_1-x_2)}\right] \\ &+ \Theta(t_2-t_1)\int\frac{d^3k}{2\omega_k}\left[2\nu_k\cos(k(x_1-x_2)) + e^{ik(x_1-x_2)}\right]. \end{aligned} \tag{7}$$

From which we see that the condition $\lim_{k\to\infty}\nu_k = -\frac{1}{2}$ *is a sufficient one for regularizing the propagator.*

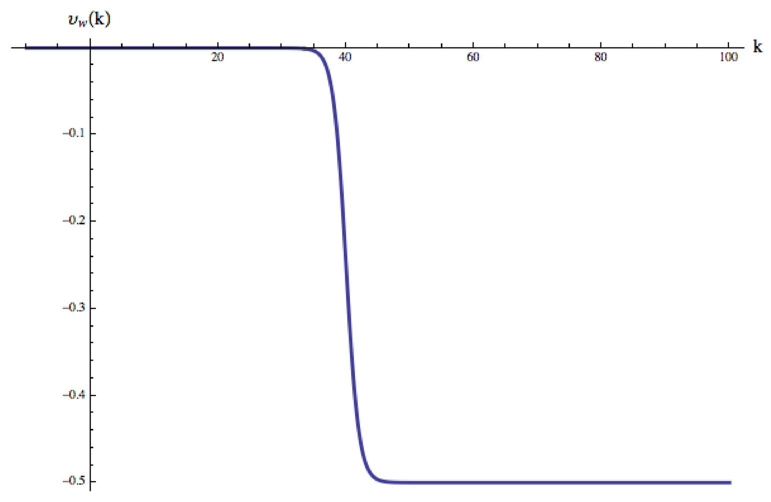

Figure 1: *Desired behaviour of the weak number value*

If we consider as in the previous inference problem, a composed system of the field and an ancillary field and the PPS ensemble given by the (entangled) states

$$|\psi_i\rangle = \sum_{n=0}^{\infty} e^{-\frac{\beta n}{2}}|n\rangle_s|n\rangle_a \qquad |\psi_f\rangle = \sum_{n=0}^{\infty}(-1)^n e^{-\frac{\beta n}{2}}|n\rangle_s|n\rangle_a. \tag{8}$$

We obtain for the weak value of the number operator $\nu_k = -\frac{1}{e^{\beta}+1}$*, which actually tends to* $-1/2$ *as* $k\to\infty$ *and is zero for low energies. We apply the same procedure for the Dirac Feynman propagator, to obtain*

$$\begin{aligned} S_F^{(w)} &= (i\partial\!\!\!/ + m)\Theta(t_1-t_2)\left[\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\left(e^{-ip(x_1-x_2)}(1-\nu_{a,p}) - e^{ip(x_1-x_2)}\nu_{b,p}\right)\right] \\ &-(i\partial\!\!\!/ + m)\Theta(t_2-t_1)\left[\int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}\left(e^{-ip(x_1-x_2)}\nu_{a,p} - e^{ip(x_1-x_2)}(1-\nu_{b,p})\right)\right] \end{aligned} \tag{9}$$

Which implies the condition $\lim_{p\to\infty}(1-\nu_{a,p}-\nu_{b,p}) = 0$ *for regularization.*

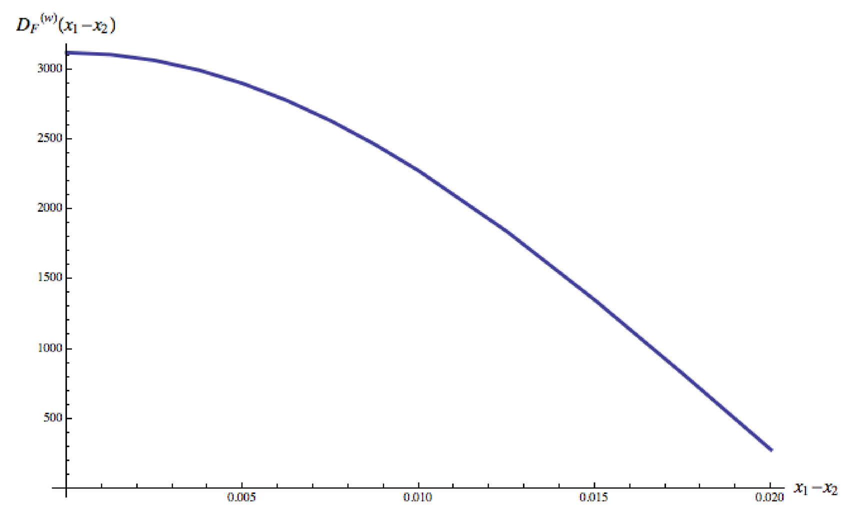

Figure 2: *Numerical solution for the weak bosonic propagator near the origin*

4. CONCLUSIONS

The idea of this work was to extend the framework of weak values to quantum field theory and study the problem of divergences in the theory. In particular we obtained necessary conditions for the regularization of the bosonic Feynman propagator by means of entanglement and weak values. We also proposed a pair of states that regularize the propagator and provided numerical realizations of this regularization. In addition we applied the regularization method to a Dirac field and found conditions for regularizing the propagator, taking into account that some additional assumptions are required in this case. We expect that this work enforces the idea that weak values are legitimate physical concepts that can appear in several branches of physics and that often lead us to new surprising results.

References

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