# Séptima Escuela de Fsica Matemática. Topological quantum matter: from theory to applications. Excercises

## SESSION 2

### Problem 1 - Plasma mapping of Laughlin states

Consider a Laughlin wavefunction for a filling factor $\nu = 1/m$ where $m$ is odd (even) for fermions (bosons):

$$\Psi_m = \prod_{i<j}(z_i - z_j)^m e^{-\frac{|z_i|^2}{4l^2}}, \tag{1}$$

(a) Consider the probability density, $|\Psi_m|^2$, and find $U(z_i)$ from the following expression:

$$|\Psi_m|^2 = e^{-\beta U(z_i)} \tag{2}$$

for $\beta = \frac{2}{m}$.

(b) Convice yourself that $U$ describes the potential energy of a classical system of point particles of charge $m$ interacting with the two-dimensional Coulomb potential (logarithmic in distance) in the pressence of a uniform background charge density $n_b = -\frac{1}{2\pi l^2}$. This system is called the classical Coulomb plasma. By invoking the physical idea that particles have to neutralize the background in order find the minimum free energy configuration, convince yourself that the density of particles is uniform density and it must be $n = -\frac{n_b}{m}$. (This "proofs" the Laughlin state has uniform density in the bulk and filling $1/m$).

(c) Consider now the wavefunction of a Laughlin quasihole located at $z_0$:

$$\Psi_m^{qh}(z_0) = \prod_i (z_i - z_0) \prod_{i<j}(z_i - z_j)^m e^{-\frac{|z_j|^2}{4l^2}}, \tag{3}$$

Find in the same way the corresponding $U_{qh}(z_0; z_i)$ as in (a). Imagine the Laughlin quasihole as point like test charge impurity embedded in the plasma, so that its position $z_0$ is just a fixed parameter. What is the charge of this impurity?. By invoking perfect screening assumption so that the plasma responds to the impurity to retain local charge neutrality convince yourself that the screening cloud of the true electrons around the impurity is a $1/m$ missing fraction of a true electron.

### Problem 2 - Pentagon and hexagon identities

(a) Derive the Pentagon and Hexagon identities for the F and R matrices describing the fusion and exchange rules of anyons. Section 4.1.5 of Jiannis' book.

(b) Convince yourself of the spin-statistics theorem for a pair of anyon and anti-anyon. Section 4.1.6 of Jiannis' book.

### Problem 3 - Ising anyons fusions and braiding rules

Calculate the exact form of the F and R matrices for the Ising anyons. Section "4.3 Example I: Ising anyons" of Jiannis' book.