New SU(3) and SU(4) Fractional Quantum Hall States

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Contents

- Why are the fractional quantum Hall liquids amazing!
- Abelian quantum Hall liquids: Laughlin and the Composite Fermions
- Quantum Hall states in the sphere and the "Shift"
- The old story at filling $\nu = 2/3$ for $SU(2)$
- The surprise we found at $\nu = 2/3$ for $SU(3)$ and $SU(4)$:

Ground states in torus and sphere, which are $SU(3)$ and $SU(4)$ singlets, are not composite fermion states!

- Summary

The A team

The A team

Thierry Jolicoeur
Universite Paris-Sud

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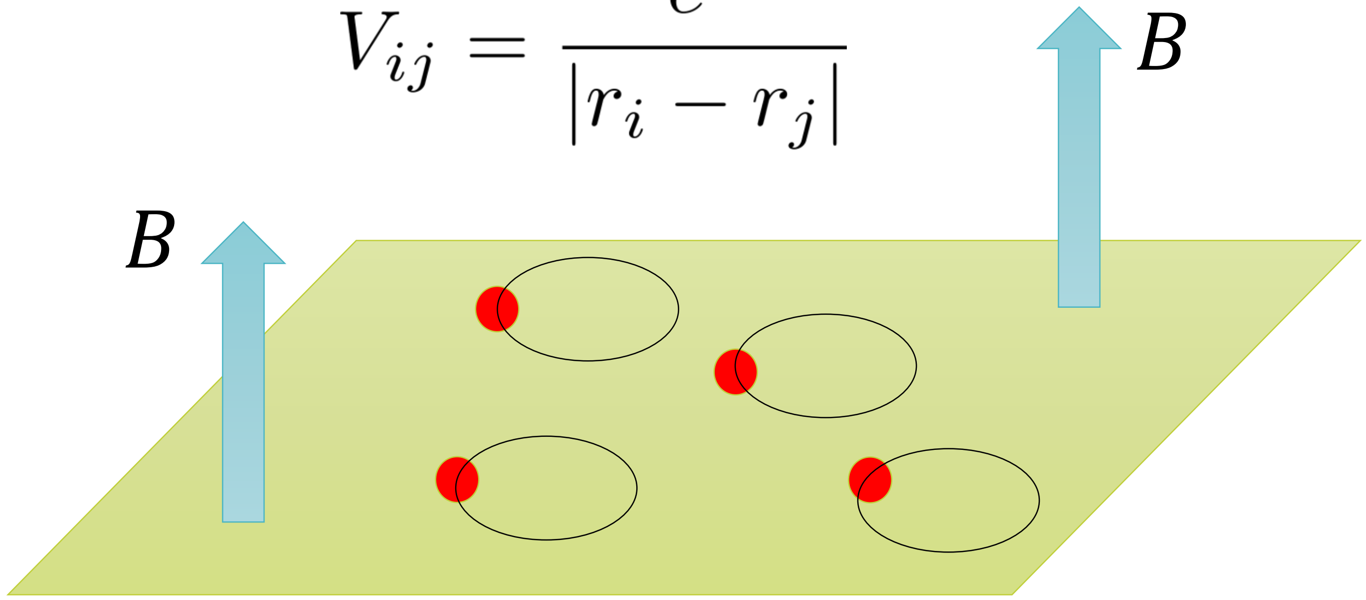
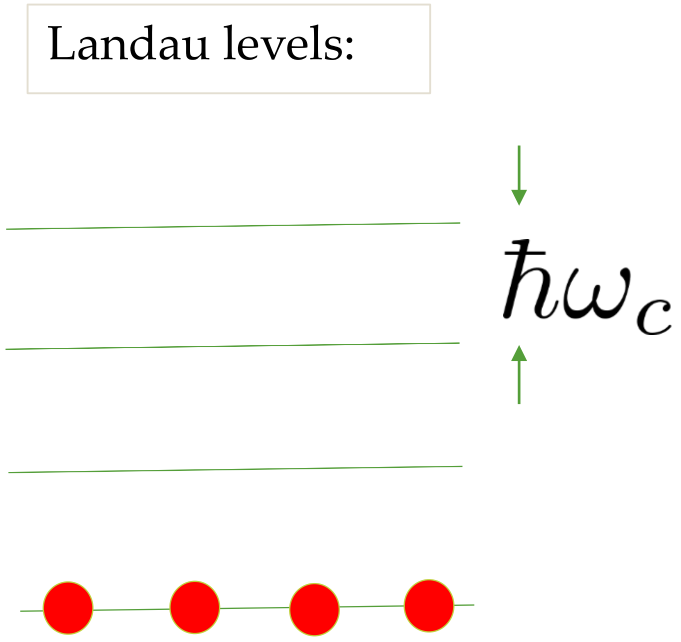
Grad Student U Texas
Soon to be in postdoc market!

Why fractional quantum Hall is amazing

Electrons in two-dimensions and super strong magnetic fields

$$V_{ij} = \frac{e^2}{|r_i - r_j|}$$



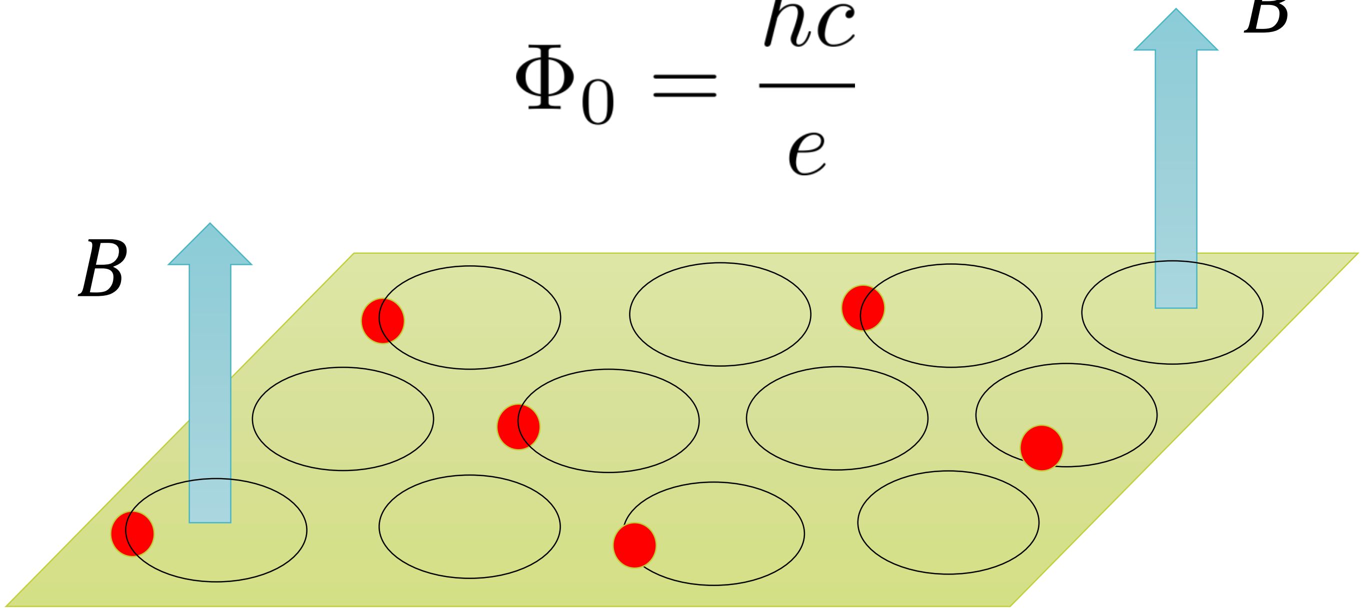


Why fractional quantum Hall is amazing

The number of states available equals the number of magnetic flux quanta:

$$N_\phi = \frac{\int da\ B}{\Phi_0}$$

$$\Phi_0 = \frac{hc}{e}$$



Filling fraction:

$$\nu = \frac{N_e}{N_\phi}$$

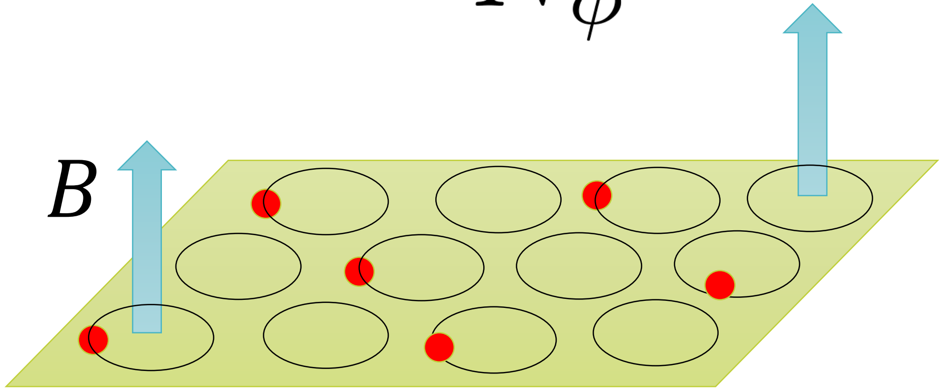
$$\nu = \frac{1}{2}$$

Why fractional quantum Hall is amazing

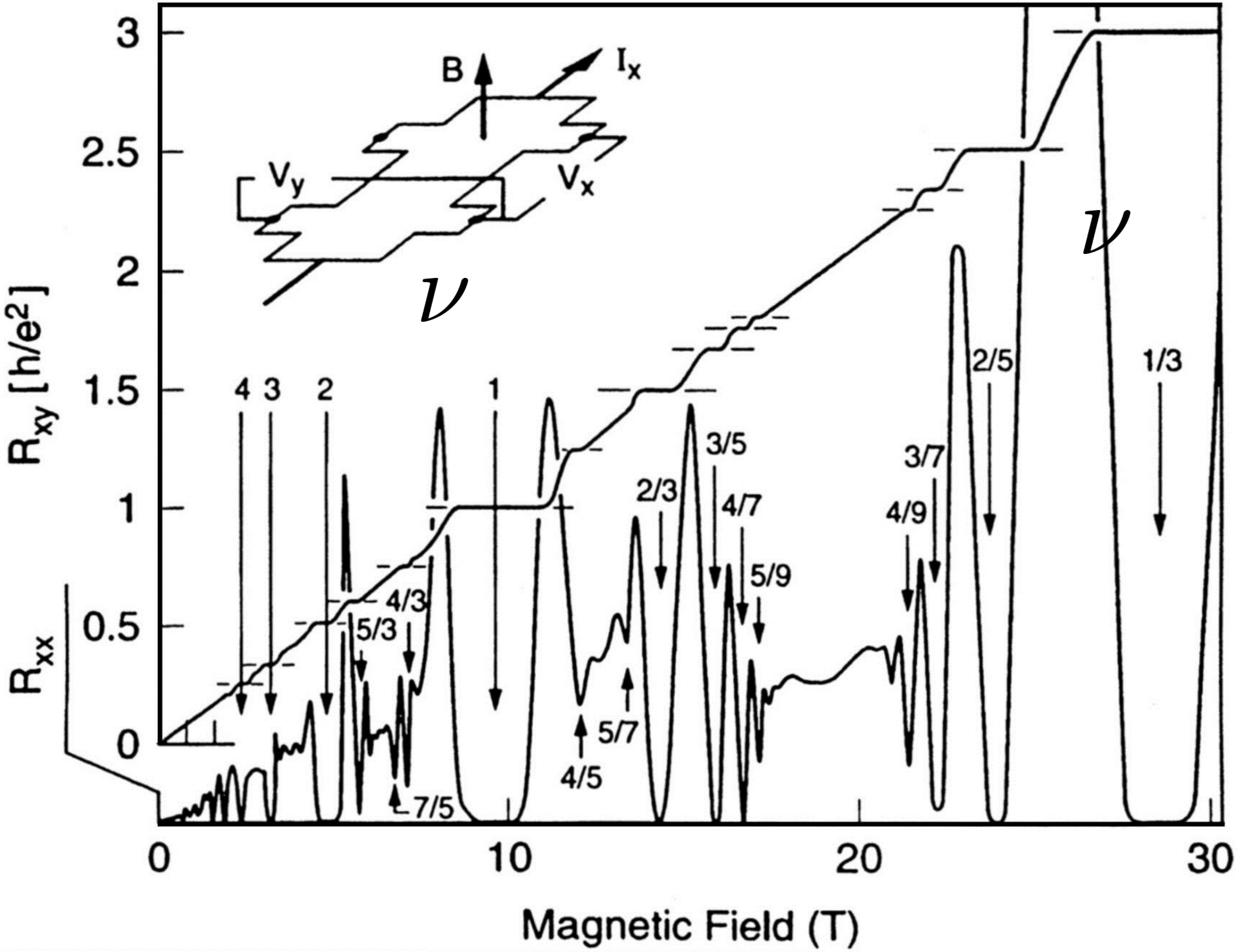
A zoo of correlated liquids

At certain rational fillings:

$$\nu = \frac{N_e}{N_\phi}$$



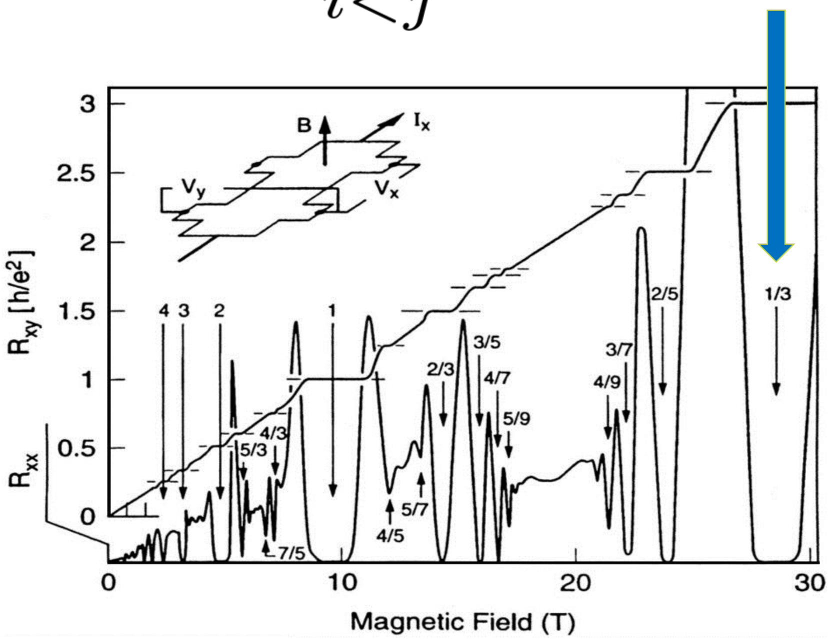
Stormer, Tsui, & Gossard, RMP (1999)



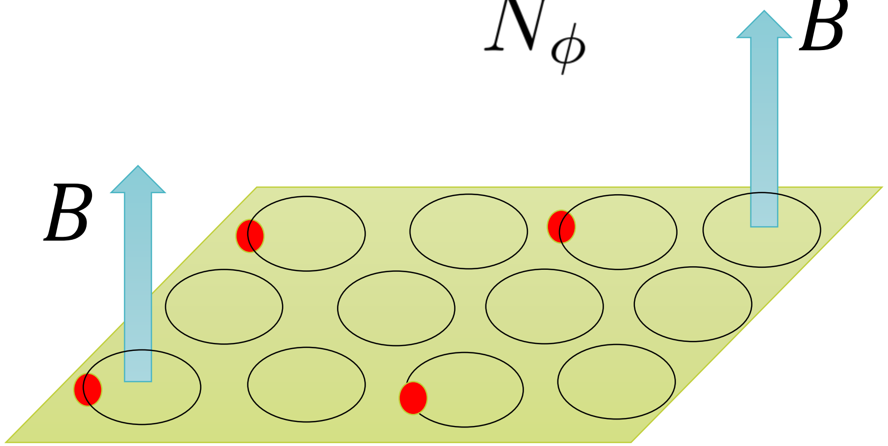
Laughlin state as a paradigm

A very stable correlated state at $\nu = 1/3$

$$\Psi_{\nu=1/3} = \prod_{i<j} (z_i - z_j)^3 e^{-\frac{|z_i|^2}{4l^2}}$$



$$\nu = \frac{N_e}{N_\phi}$$



$$\nu = \frac{1}{3}$$

Laughlin, PRL (1983)

Composite Fermions Hierarchy

$$\nu = \frac{N_e}{N_\phi}$$

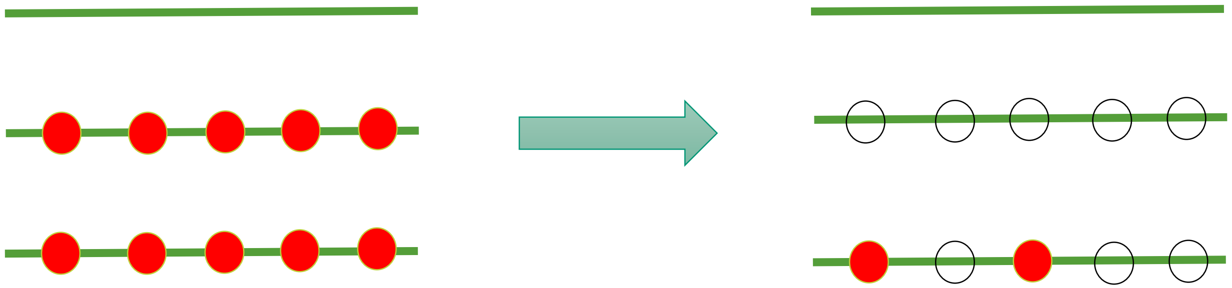
Typically most robust states show up at Jain's sequence:

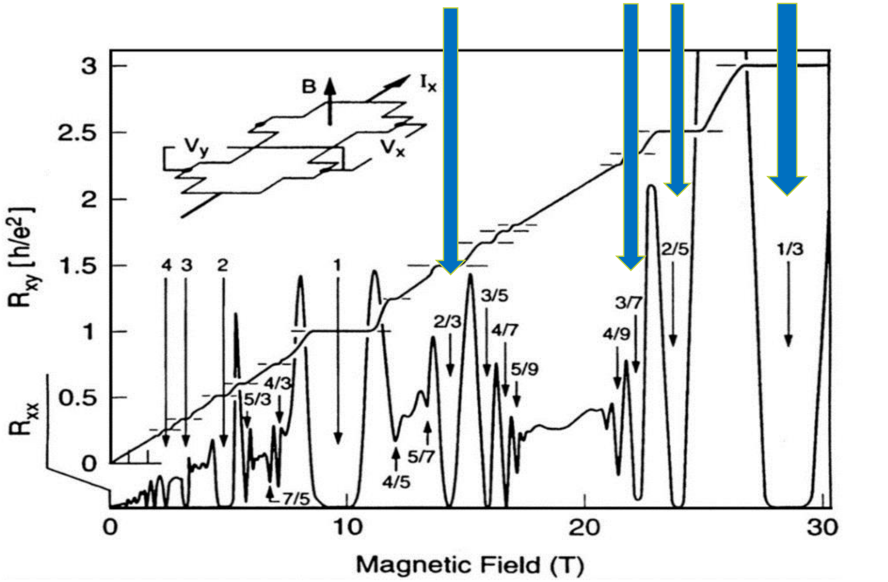
Integer quantum Hall states bound to 2 flux quanta:

$$\nu = \frac{n}{2n \pm 1}, \quad n = 1, 2, \ldots$$

$$\nu^* = 2 \qquad\qquad \nu = \frac{2}{5}$$

$$N_\phi^* = \frac{1}{2} N_e \qquad\qquad N_\phi = \left(2 + \frac{1}{2}\right) N_e$$





Jain, PRL (1989)

Composite Fermions Hierarchy

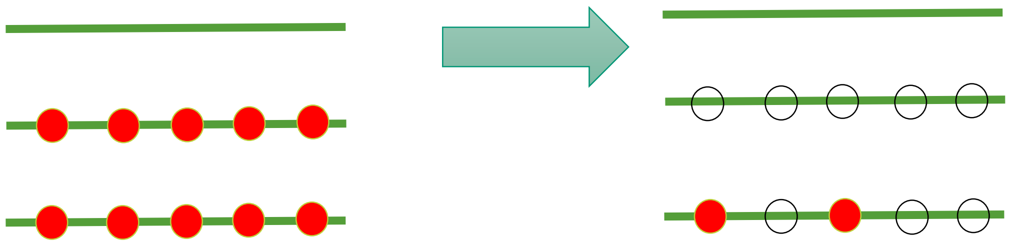
Composite Fermions describe abelian topological states.

Topological properties of composite fermions agree with other Hierarchy constructions and with Chern-Simons. They represent the same phase.

I believe the Hierarchy is a form of spontaneous symmetry breaking of indistinguishability (permutation symmetry).

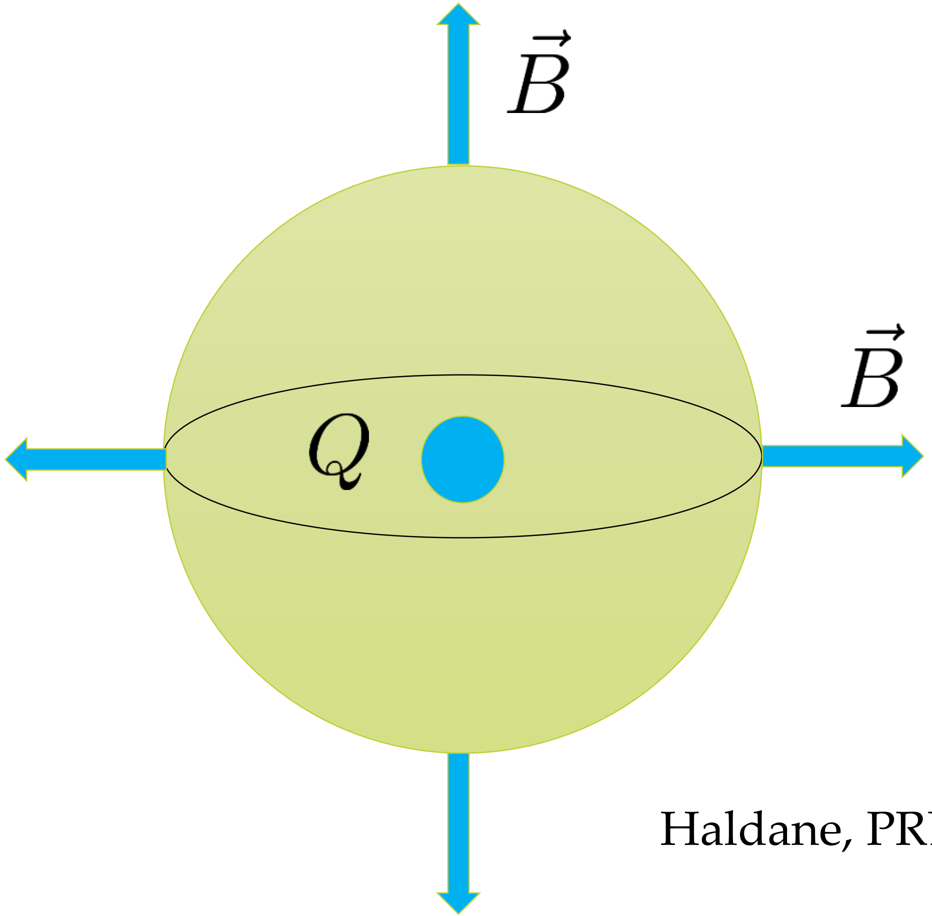
$$\nu^* = 2 \qquad \nu = \frac{2}{5}$$

$$N_\phi^* = \frac{1}{2} N_e \qquad N_\phi = \left(2 + \frac{1}{2}\right) N_e$$



Quantum Hall States on curved surfaces

- A sphere with a magnetic charge (monopoles):



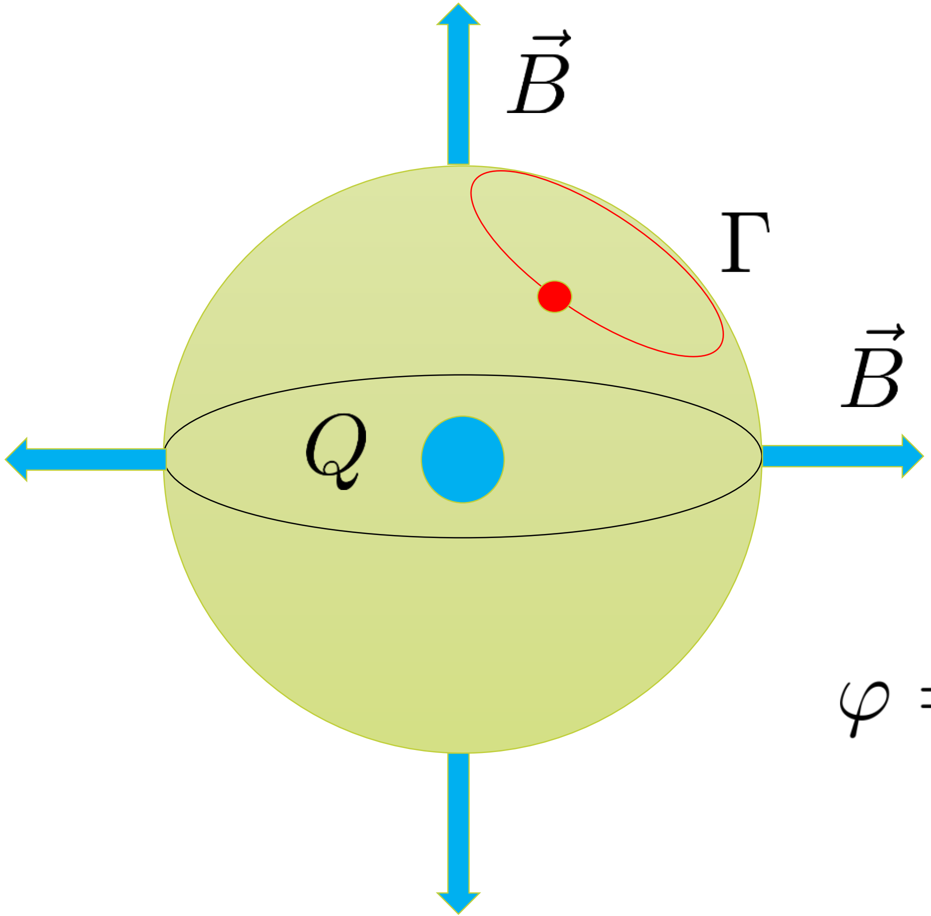
$$\nabla \cdot B = \frac{\rho_B}{\epsilon_B}$$

$$\int da \cdot B = \Phi = \frac{Q}{\epsilon_B}$$

Haldane, PRL (1983)

Quantum Hall States on curved surfaces

Aharonov-Bohm phase of electric test charge on surface:

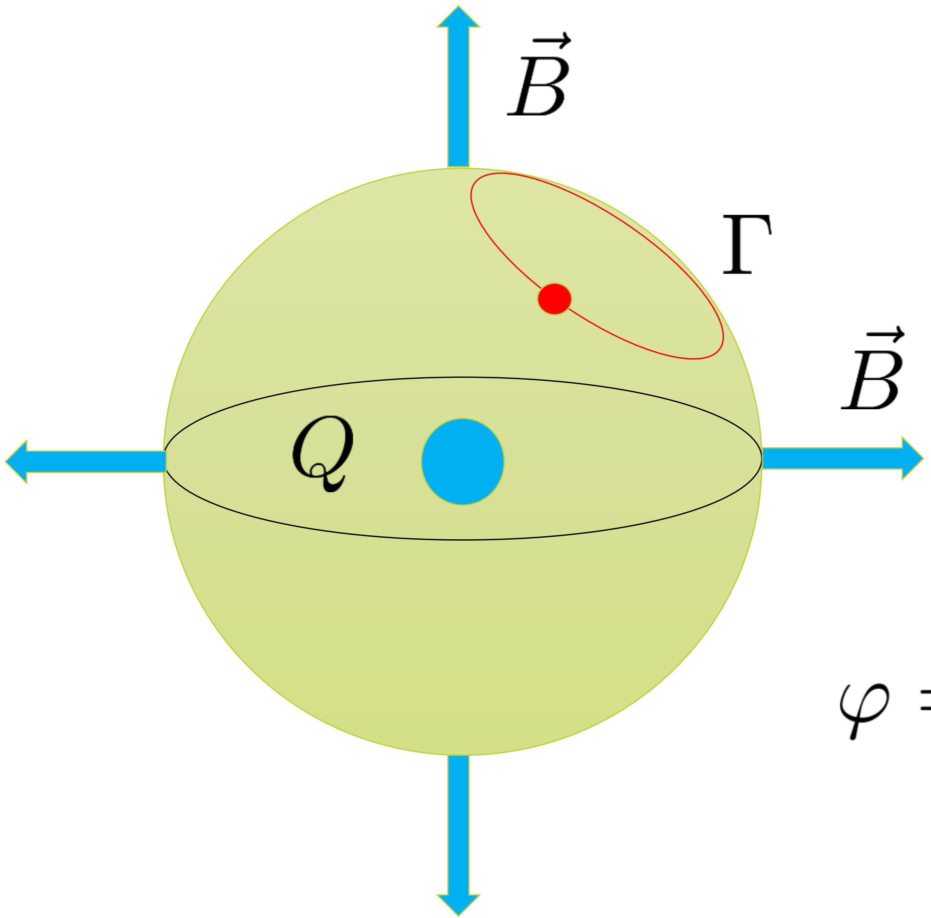


$$\int da \cdot B = \Phi = \frac{Q}{\epsilon_B}$$

$$\varphi = -\frac{e}{\hbar c}\oint_\Gamma A \cdot d\vec{l} = -\frac{e}{\hbar c}\frac{Q}{\epsilon_B}\frac{\mathcal{A}_\Gamma}{\mathcal{A}_{\text{sphere}}}$$

Quantum Hall States on curved surfaces

Aharonov-Bohm phase of electric test charge on surface:



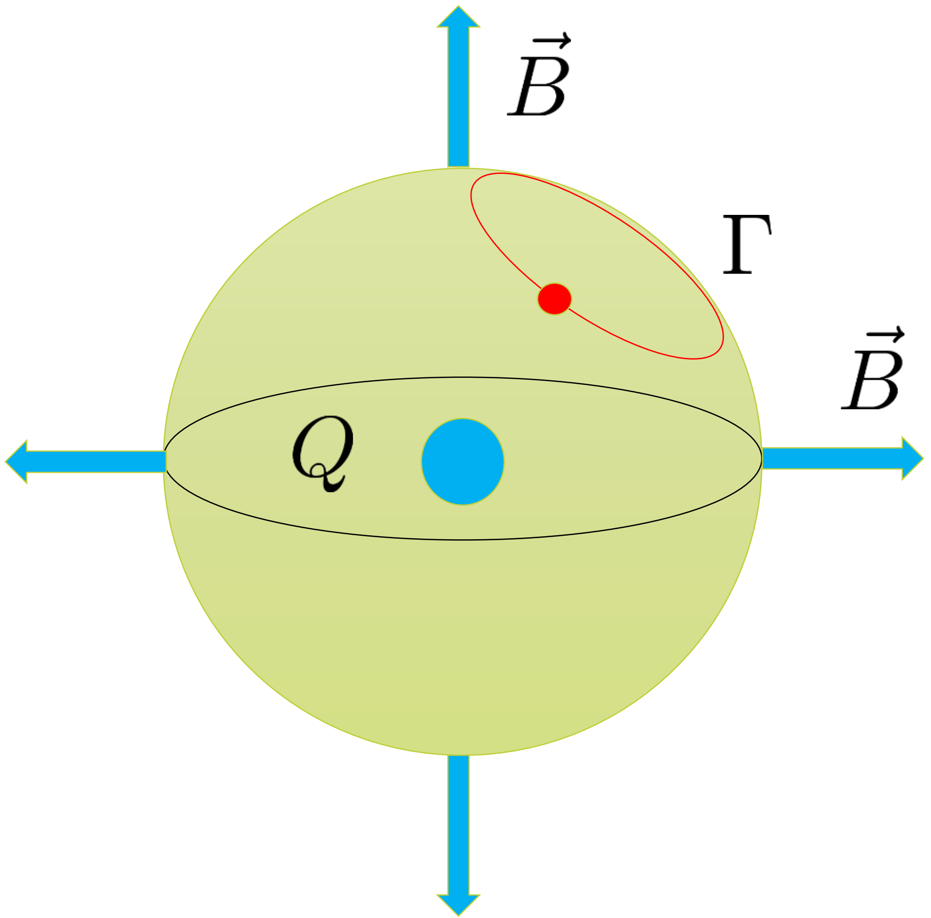
$$\int da \cdot B = \Phi = \frac{Q}{\epsilon_B}$$

$$\Phi_0 = \frac{hc}{e}$$

$$\varphi = -\frac{e}{\hbar c}\oint_\Gamma A \cdot d\vec{l} = -2\pi \frac{\Phi}{\Phi_0} \frac{\mathcal{A}_\Gamma}{\mathcal{A}_{\text{sphere}}}$$

Quantum Hall States on curved surfaces

Aharonov-Bohm phase of electric test charge on surface:



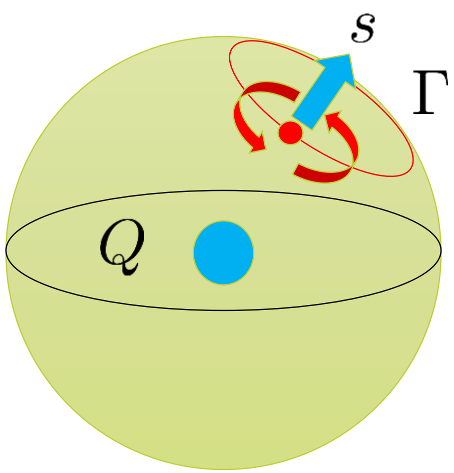
$$\int da \cdot B = \Phi = \frac{Q}{\epsilon_B}$$

$$\varphi = -\frac{e}{\hbar c} \oint_\Gamma A \cdot d\vec{l} = -2\pi \frac{\Phi}{\Phi_0} \frac{\mathcal{A}_\Gamma}{\mathcal{A}_{\text{sphere}}}$$

$$\Gamma \to \text{Sphere} \quad \Rightarrow \quad \frac{\Phi}{\Phi_0} \in \mathbf{Z}$$

Quantum Hall States on curved surfaces

Aharonov-Bohm like phase of spinning particles on curved surface:

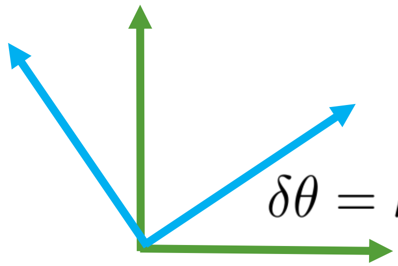


$$\int da\ \kappa = \Phi_\kappa = 2\pi(2-2g)$$

Gauss-Bonnet

$$g_{\text{sphere}} = 0$$

$$\varphi_s = -\frac{s}{\hbar}\oint_\Gamma \omega \cdot d\vec{l} = -\frac{s}{\hbar}\int_\Gamma \kappa da$$

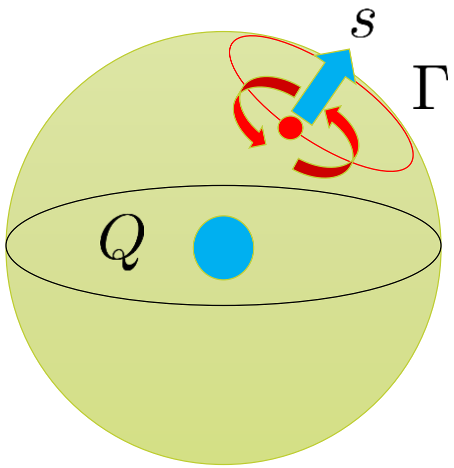


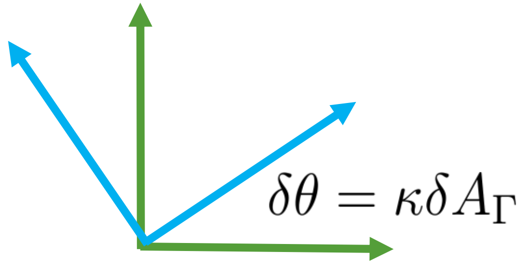
$$\delta\theta = \kappa\delta A_\Gamma$$

$$\kappa = \pm\frac{1}{R_1 R_2}$$

Quantum Hall States on curved surfaces

Aharonov-Bohm like phase of spinning particles on curved surface:





$$\kappa = \pm \frac{1}{R_1 R_2}$$

$$\int da\ \kappa = \Phi_\kappa = 2\pi(2-2g)$$

Gauss-Bonnet

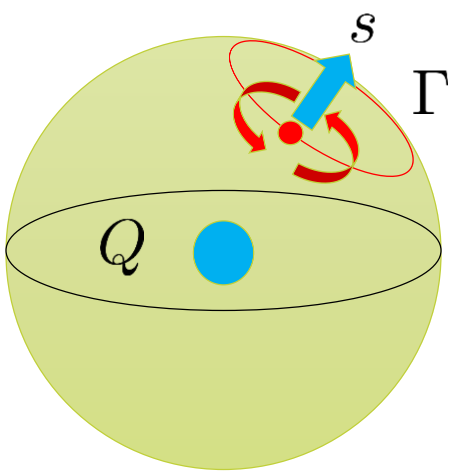
$$g_{\text{sphere}} = 0$$

$$\varphi_s = -\frac{s}{\hbar}\oint_\Gamma \omega \cdot d\vec{l} = -\frac{2\pi s(2-2g)}{\hbar}\frac{\mathcal{A}_\Gamma}{\mathcal{A}_{\text{sphere}}}$$

$$\varphi_B = -\frac{e}{\hbar c}\oint_\Gamma A \cdot d\vec{l} = -2\pi\frac{\Phi}{\Phi_0}\frac{\mathcal{A}_\Gamma}{\mathcal{A}_{\text{sphere}}}$$

Quantum Hall States on curved surfaces

Aharonov-Bohm like phase of spinning particles on curved surface:



$$\int da\ \kappa = \Phi_\kappa = 2\pi(2-2g)$$

Gauss-Bonnet

$$g_{\text{sphere}} = 0$$

$$N_\kappa = \frac{s(2-2g)}{\hbar}$$

$$\varphi_s = -\frac{s}{\hbar}\oint_\Gamma \omega \cdot d\vec{l} = -\frac{2\pi s(2-2g)}{\hbar}\frac{\mathcal{A}_\Gamma}{\mathcal{A}_{\text{sphere}}}$$

$$N_\kappa^{sphere} = \frac{2s}{\hbar}$$

$$\varphi_B = -\frac{e}{\hbar c}\oint_\Gamma A \cdot d\vec{l} = -2\pi\frac{\Phi}{\Phi_0}\frac{\mathcal{A}_\Gamma}{\mathcal{A}_{\text{sphere}}}$$

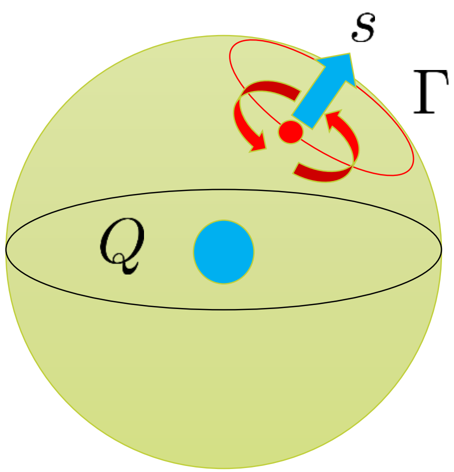
$$N_\kappa^{torus} = 0$$

Quantum Hall States on curved surfaces

Laughlin state on torus: $N_\phi^B = 3N_e$

Laughlin state on sphere: $N_\phi^B + N_\phi^\kappa = 3N_e$

$$N_\phi^\kappa = \frac{2s}{\hbar}$$



$$N_\phi^B + 3 = 3N_e$$

$$s_{\text{Laughlin}} = \frac{3}{2}\hbar$$

Emergent orbital
spin of composite boson

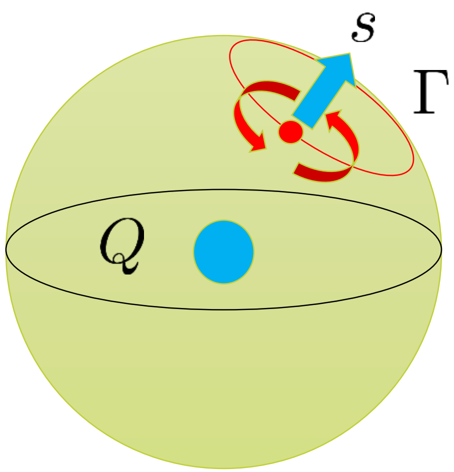
Wen & Zee, PRL (1992)

Shift (S) of states on sphere

State on torus: $$N_\phi^B = \nu^{-1} N_e$$

State on sphere: $$N_\phi^B + N_\phi^\kappa = \nu^{-1} N_e$$

$$N_\phi^\kappa = \frac{2s}{\hbar}$$



$$N_\phi^B + S = \nu^{-1} N_e$$

$$s_\nu = \frac{S}{2}\hbar$$

Emergent orbital spin

Wen & Zee, PRL (1992)

SU(2) states at 2/3 (“old news”)

Two states compete: “ferromagnet” and a 2-component singlet.

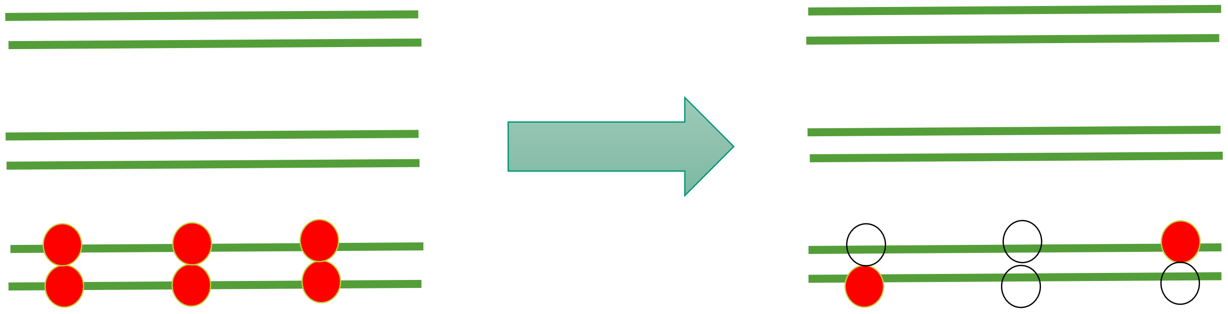
Singlet from composite fermions

$$\nu^* = -2$$

$$N_\phi^* = -\frac{1}{2} N_e$$

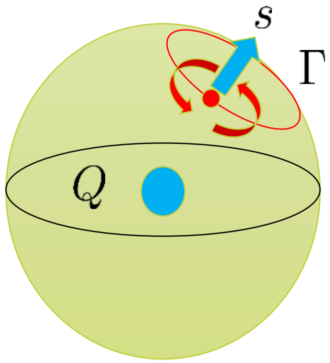
$$\nu = \frac{2}{3}$$

$$N_\phi = \left(2 - \frac{1}{2}\right) N_e$$



$$\mathcal{S} = 1$$

$$s = \frac{\hbar}{2}$$



SU(2) states at 2/3 (“old news”)

Two states compete: “ferromagnet” and a 2-component singlet.

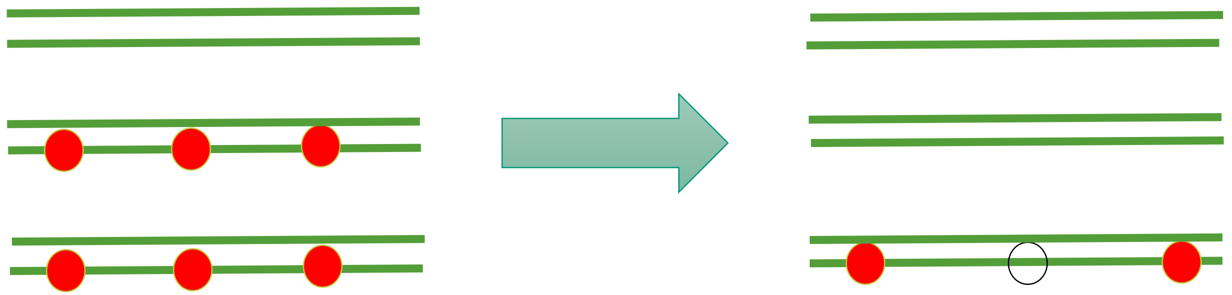
Ferromagnet from composite fermions

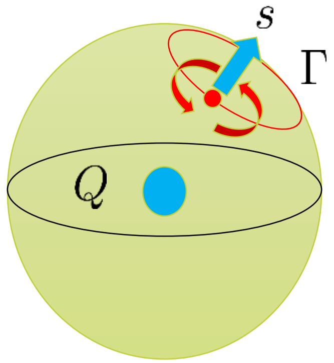
$$\nu^* = -2 \qquad \nu = \frac{2}{3}$$

$$N_\phi^* = -\frac{1}{2} N_e \qquad N_\phi = \left(2 - \frac{1}{2}\right) N_e$$

$$\mathcal{S} = 0$$

$$s = 0$$





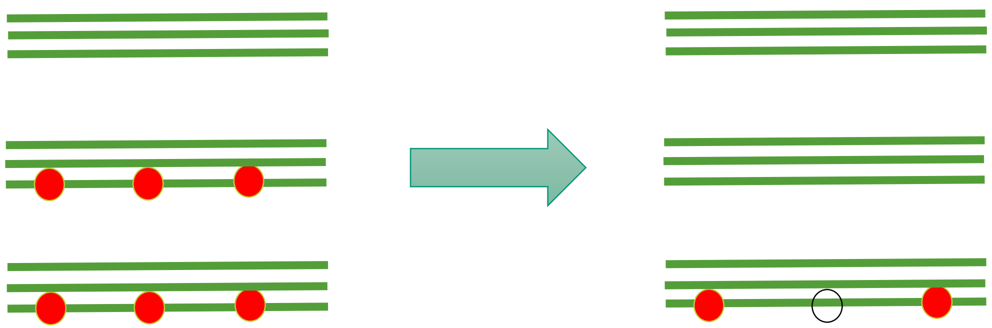
SU(3) & SU(4) composite fermions at 2/3

No new states are expected:

Ferromagnet

$$\nu^* = -2 \qquad \nu = \frac{2}{3}$$

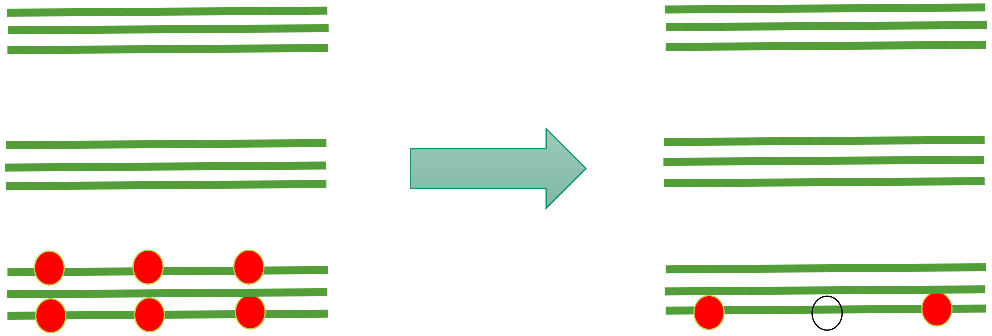
$$N_\phi^* = -\frac{1}{2} N_e \qquad N_\phi = \left(2 - \frac{1}{2}\right) N_e$$



2-component singlet

$$\nu^* = -2 \qquad \nu = \frac{2}{3}$$

$$N_\phi^* = -\frac{1}{2} N_e \qquad N_\phi = \left(2 - \frac{1}{2}\right) N_e$$

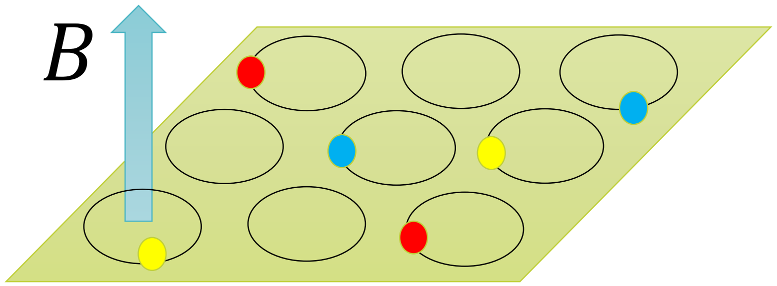


Surprise for SU(3) at 2/3

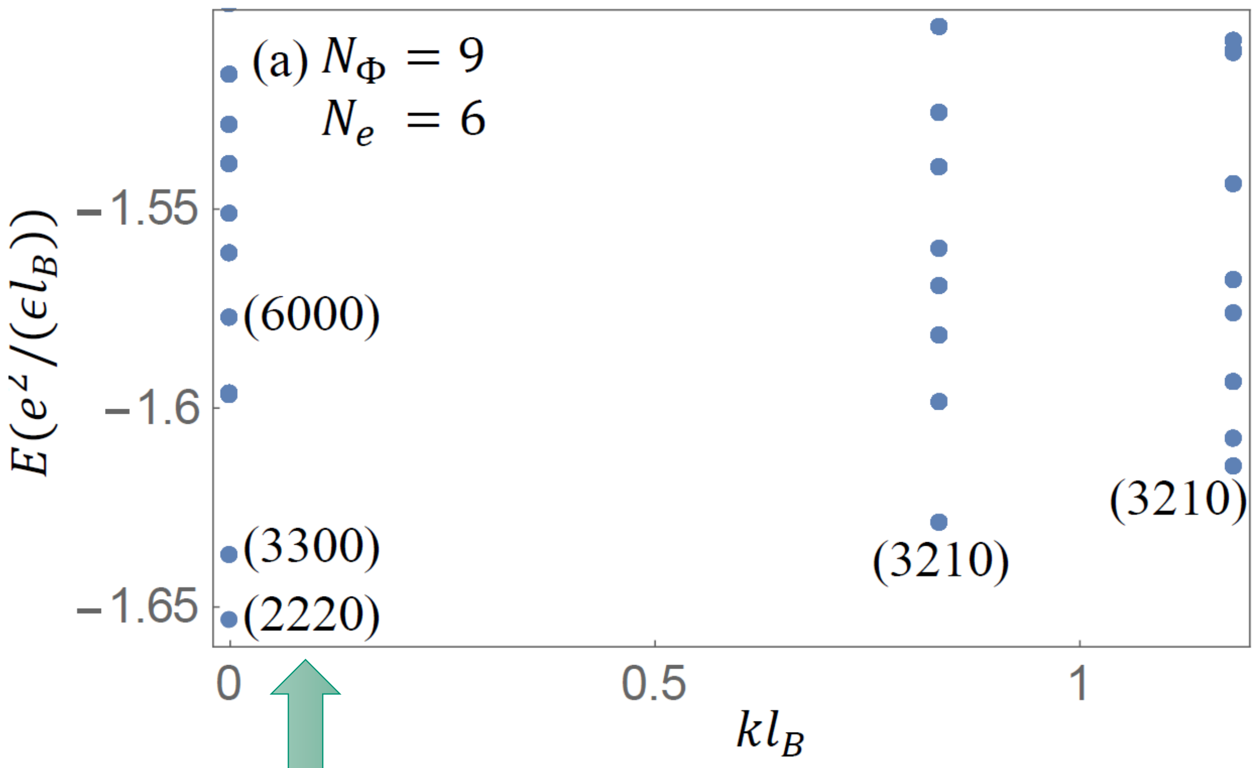
Exact diagonalization on torus

$$V_{ij} = \frac{e^2}{|r_i - r_j|}$$

Three colors of electrons:



$$\nu = \frac{2}{3} \quad \nu = \frac{N_e}{N_\phi}$$



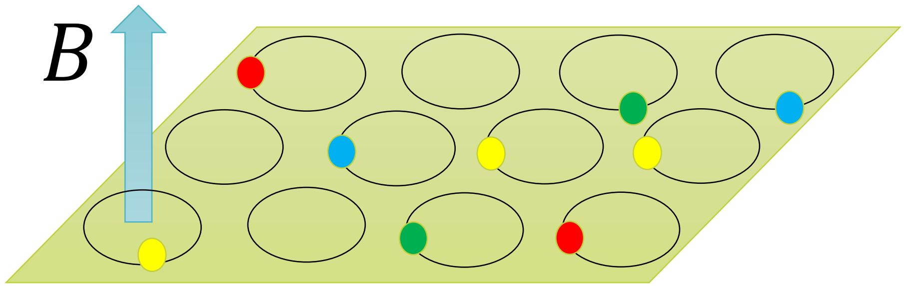
Electrons spontaneously choose an SU(3) singlet occupying all colors!!!

Surprise for SU(4) at 2/3

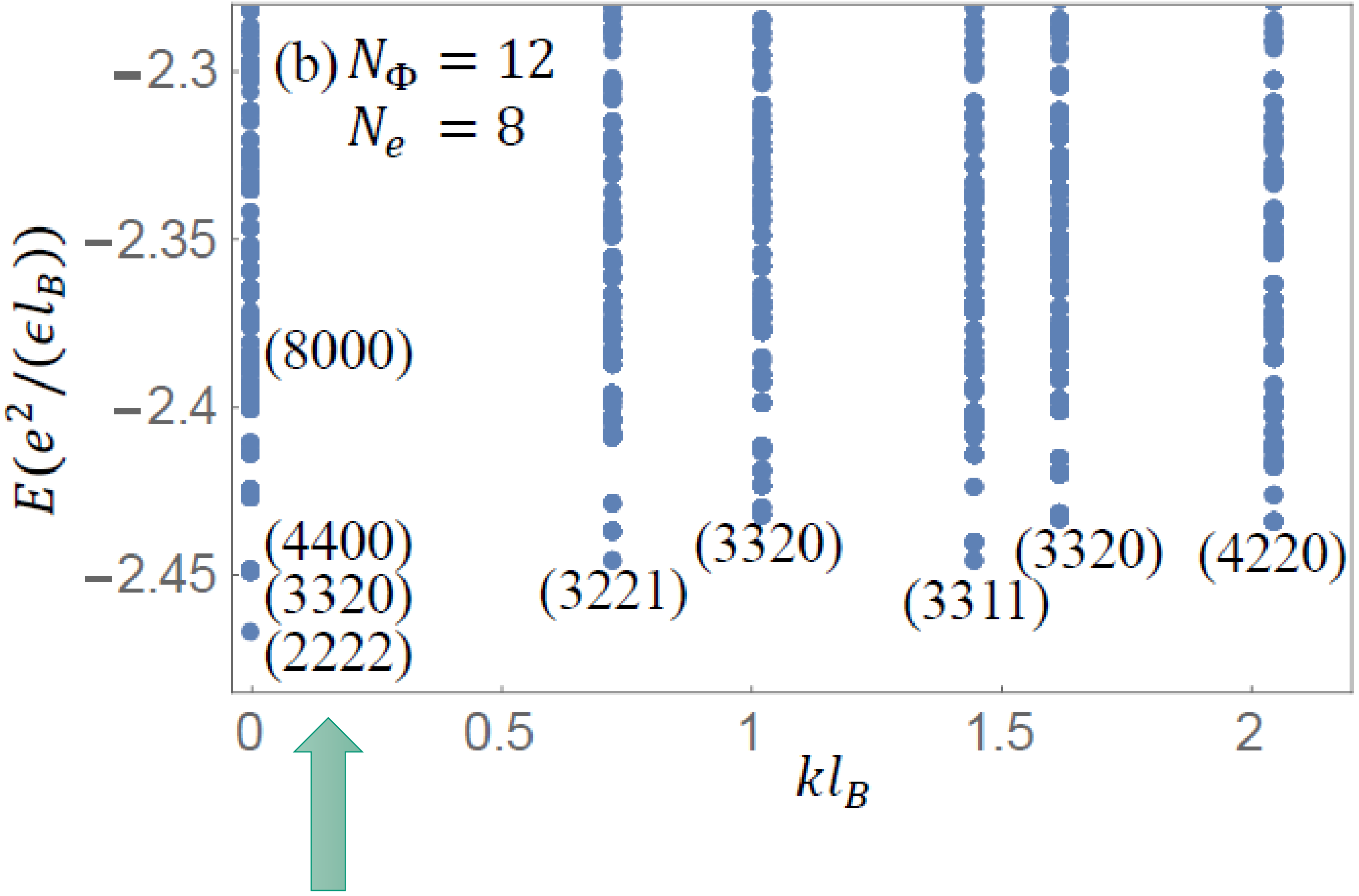
Exact diagonalization on torus

$$V_{ij} = \frac{e^2}{|r_i - r_j|}$$

Four colors of electrons:

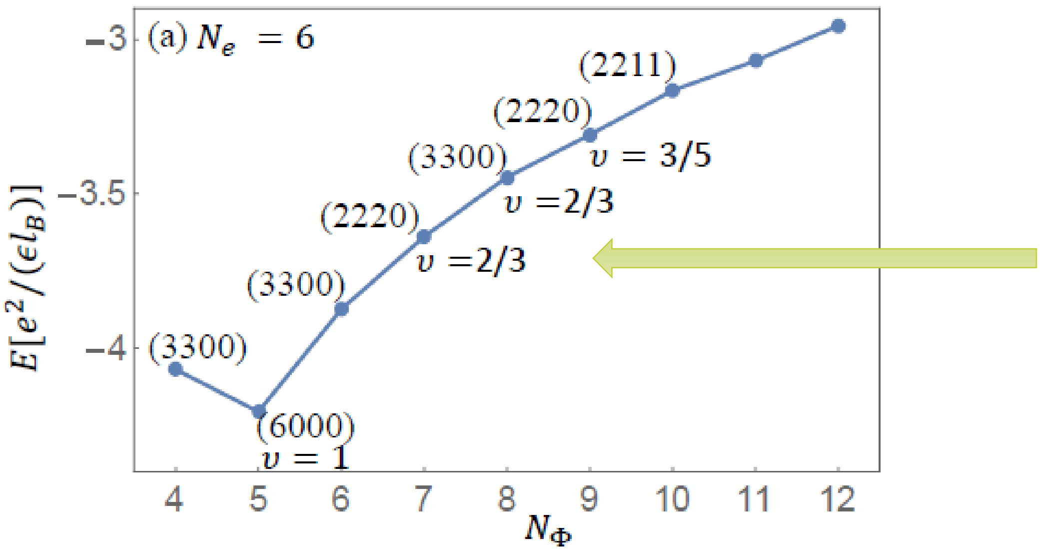


$$\nu = \frac{2}{3} \qquad \nu = \frac{N_e}{N_\phi}$$



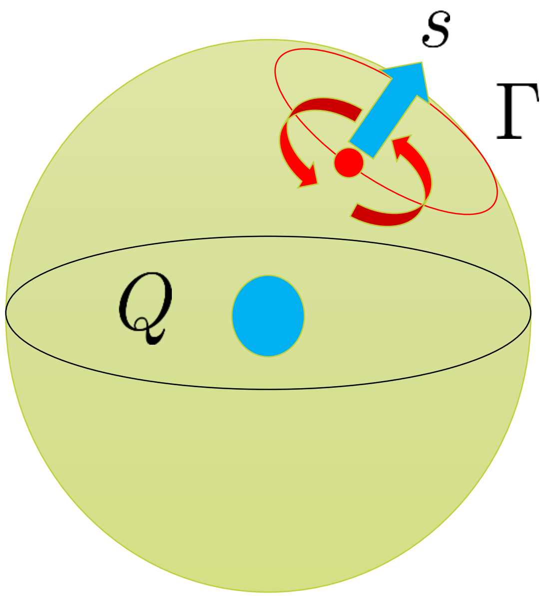
Electrons spontaneously choose an SU(4) singlet occupying all colors!!!

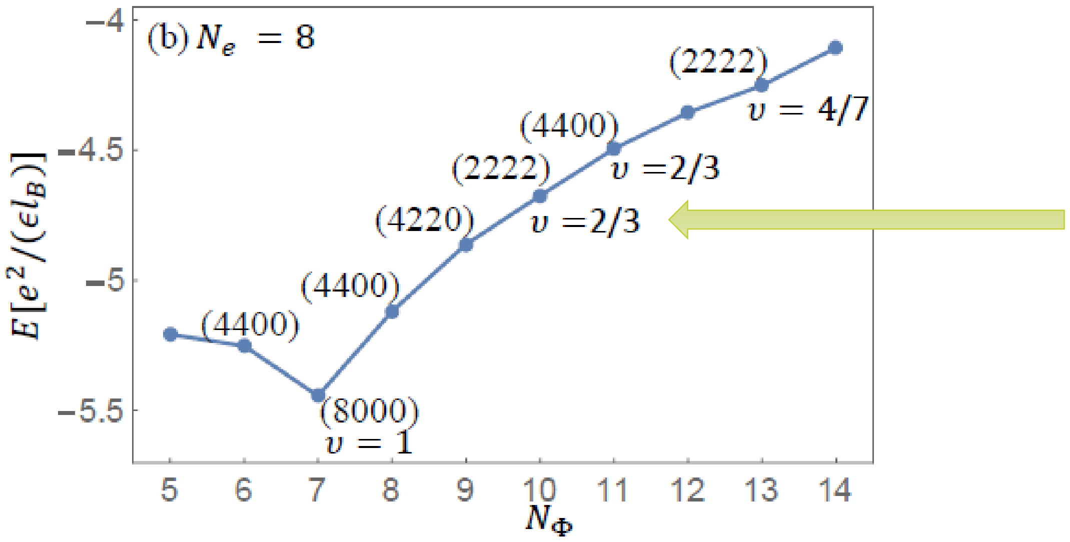
SU(3) and SU(4) in sphere



$SU(3)$

$\mathcal{S} = 2$





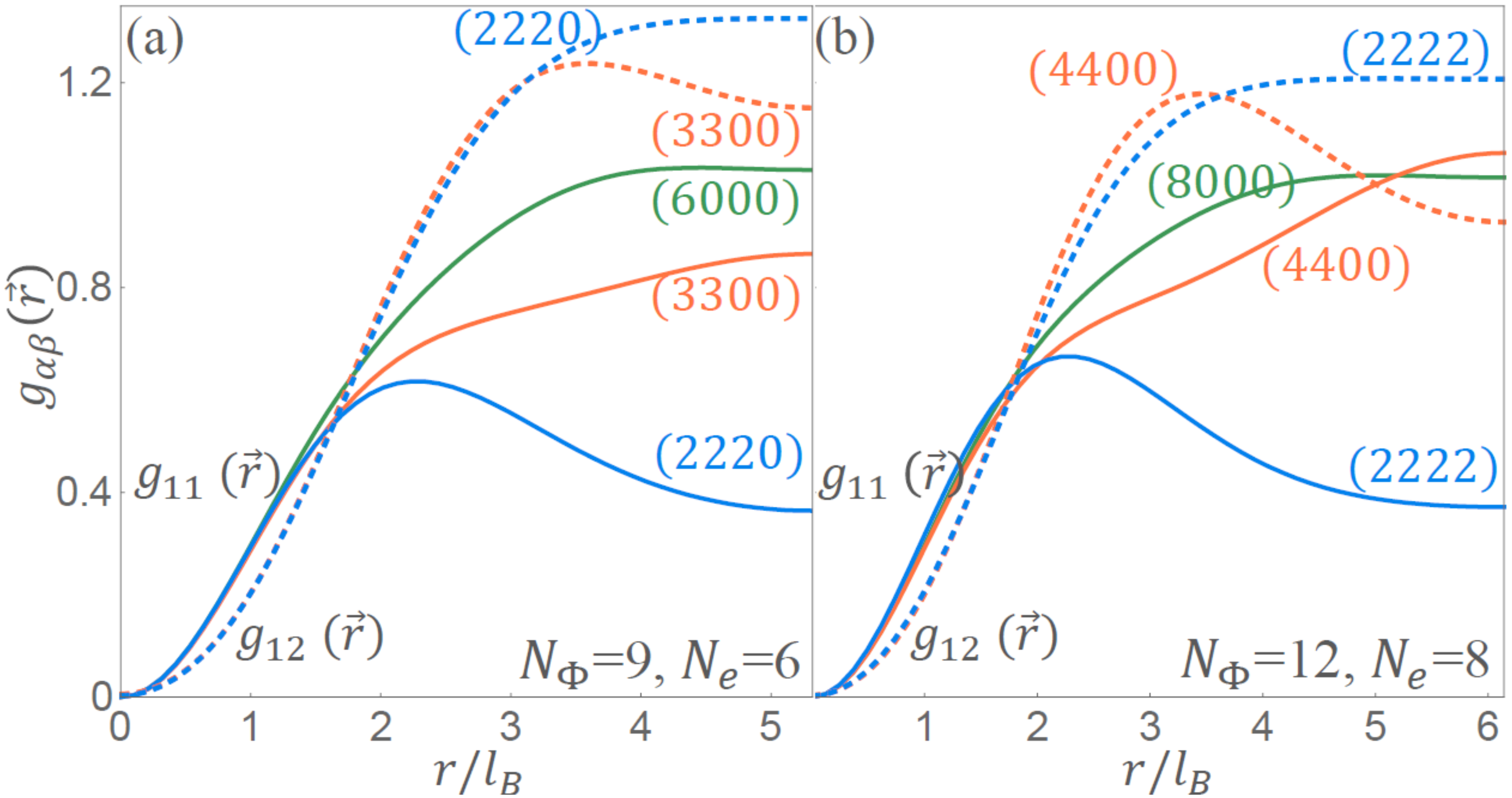
$SU(4)$

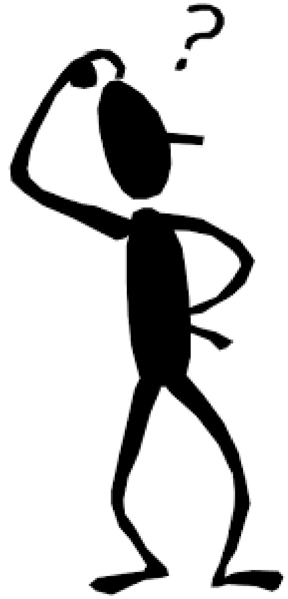
$\mathcal{S} = 2$

Unexpected pattern in correlations

Maybe the whole story at $\nu = 2/3$ needs to be re-thougth?

$N_e = 6,\ N_\phi = 9$ $\qquad$ $N_e = 8,\ N_\phi = 12$





Summary

- Fractional quantum Hall states are liquids with particle-like excitations which are a fraction of bare electrons and have fractional statistics: "topological order".
- Abelian fractional quantum Hall liquids are condensates of composite bosons (Chern-Simons boson) with an emergent orbital spin which couples to the curvature of space.
- Composite fermions *do not* describe the new SU(3) and SU(4) singlet states we have discovered at $\nu = 2/3$.
- Microscopic understanding is missing.