Confinement and deconfined regimes from non–perturbative propagators in 3D gauge systems

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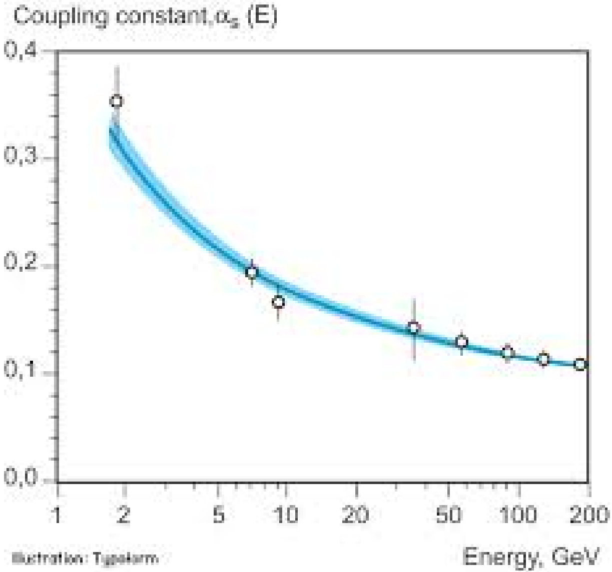
Introduction

- Three of the four fundamental forces of nature. Namely, electromagnetism, weak and strong interactions can be described by quantum gauge theories with a great success, theoretical and experimentally.
- In QCD for instance, thanks to the property of asymptotic freedom, in the high energy regime it is possible to observe in LHC and RHIC, after controlled heavy ion collisions, the short-live exotic state of quark–gluon plasma, where these particles are identified as fundamental particles.
- On the contrary, *i.e*, in the low energy regime, quarks and gluons can't be considered as asymptotic states of the S–matrix anymore, being dynamically removed from the physical spectrum, giving rise to the confinement regime.
- Looking at the QCD beta function, we can see the behaviour of the coupling constant in terms of the scale energy

$$g^2(\mu) = \frac{1}{\frac{11N}{16\pi^2} ln \frac{\mu^2}{\Lambda^2_{QCD}}} \tag{1}$$



Coupling constant, α_s (E)
0,4
0,3
0,2
0,1
0,0
1
2
5
10
20
50
100
200
Energy, GeV
Illustration: Typoform

The Faddeev–Popov action

$$S_{YM} = \frac{1}{4}\int d^4x F^a_{\mu\nu}F^a_{\mu\nu}, \tag{2}$$

Where $F_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$.
This action is invariant under gauge transformations

$$\tilde{A}_\mu = U^\dagger A_\mu U + U^\dagger \partial_\mu U \tag{3}$$

Where U is an element of $SU(N)$
In order to quantize the theory in a covariant way, within the path integral approach, we use the Landau gauge condition ($\partial_\mu A^a_\mu = 0$),

$$\mathcal{Z} = \mathcal{N}\int DA_\mu \delta(\partial A)\det(\mathcal{M}^{ab})e^{S_{YM}(A)}, \tag{4}$$

$\mathcal{M}^{ab} = \partial_\mu(\partial_\mu \delta^{ab} + f^{abc}A^c_\mu)$ is the Faddeev–Popov operator.
To localize the action, we introduce FP ghosts.

$$\mathcal{Z} = \int DADcD\bar{c}Dbe^{S_{FP}}$$

BRST Symmetry

$$S_{FP} = \int d^4x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \bar{c}^a \partial_\mu D^{ab}_\mu c^b + b^a \partial_\mu A^a_\mu \right) \tag{5}$$

c^a , $\bar{c}^a$ are the FP ghosts and b^a is the Nakanishi Laudrup field which enforce the gauge condition in the action.
Clearly, the Faddeev–Popov action is not gauge invariant. However, one finds a new symmetry which encodes the information of the gauge symmetry at quantum level, the so called BRST symmetry:

$$\begin{aligned} sA^a_\mu &= -D^{ab}_\mu c^b \equiv -\partial_\mu(\delta^{ab} c^b + g f^{acb} A^c_\mu c^b), \\ sc^a &= \frac{1}{2} g f^{abc} \bar{c}^b \bar{c}^c, \\ s\bar{c}^a &= b^a, \qquad sb^a = 0. \end{aligned}$$

The BRST operator s is nilpotent

$$s^2 = 0 \quad com \quad sS_{FP} = 0$$

- The operator s define trivial and non-trivial cohomology classes. The non-trivial BRST cohomology ckass characterize the observable content of the theory.
- Moreover, BRST symmetry define unitarity and ensures la renormalizability of the theory.

Gribov copies

The Gribov ambiguity is a strong remark on the Faddeev–Popov gauge fixing method. The condition for the existence of copies of the gauge field is

$$\tilde{A}_\mu = U^\dagger A_\mu U + U^\dagger \partial_\mu U \tag{6}$$

$$\partial_\mu A_\mu = \partial_\mu \tilde{A}_\mu = 0. \tag{7}$$

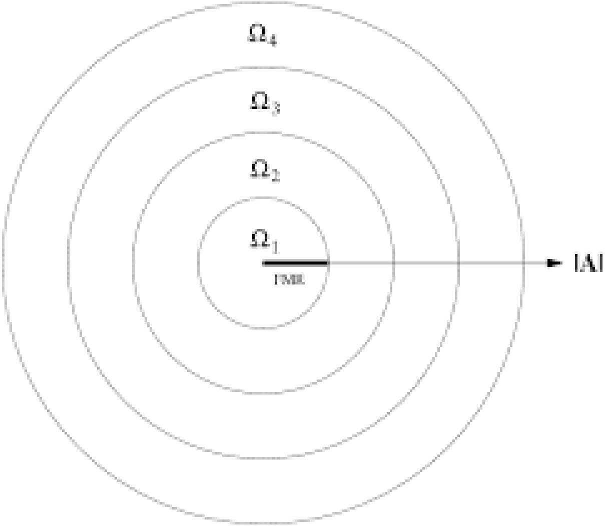
Taking these eqs. and making an infinitesimal transformation $U = 1 + \omega$, we get

$$\partial_\mu \left(\partial_\mu \omega + [A_\mu, \omega] \right) = 0 \tag{8}$$

Thus, this condition is equivalent to the existence of zero modes in the Faddeev–Popov operator. In order to get rid of these zero modes, Gribov restrict the functional space to the region

$$\Omega \equiv \Omega_0 = \left\{ A^a_\mu \ ; \ \partial_\mu A^a_\mu = 0, \mathcal{M}^{ab} > 0 \right\}$$

- As we shall see, this result has deep consequences in the infrared regime of the theory, instead, the Faddeev–Popov action gives a correct description on only in the UV perturbative regime.


Ω_4
Ω_3
Ω_2
Ω_1
FMR
|A|

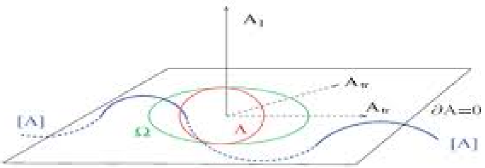


FIG. 2: A schematic representation of the configuration space of gauge fields, the "hyperplane" of transverse gauge fields, the first Gribov region and the fundamental modular region.

Restriction to the functional space

- In order to eliminate the zero modes of the FP operator, we parametrize the two-point ghost form factor in the Fourier space $\mathcal{G}^{ab}{=}(\mathcal{M})^{ab}$ as

$$\mathcal{G}(k;A)=\frac{1}{k^2}\frac{1}{1-\sigma(k,A)}, \tag{9}$$

  where $\sigma(k,A)$ is the ghost form factor
- Taking $k\longrightarrow 0$ we impose the no–pole condition $\sigma(0,A)<0$, where

$$\sigma(0;A)=\frac{Ng^2}{N^2-1}\frac{1}{4}\int\frac{d^4p}{(2\pi)^3}\frac{A^a_\mu(p)A^a_\mu(-p)}{p^2} \tag{10}$$

This implies modifications in the Faddeev–Popov measure

$$\begin{aligned} d\mu_{FP} &= \mathcal{D}A\delta(\partial A)\det\left(\mathcal{M}^{ab}\right)e^{-S_{YM}} \\ &\longrightarrow \mathcal{D}A\delta(\partial A)\det\left(\mathcal{M}^{ab}\right)\theta(1-\sigma(0,A))e^{-S_{YM}}, \end{aligned}$$

The presence of the function $\theta(1-\sigma(0,A))$ implements the restriction in the path integral

$$\mathcal{Z}=\int\mathcal{D}A\frac{d\beta}{2\pi i\beta}\delta(\partial A)\det\left(\mathcal{M}^{ab}\right)e^{-S_{YM}}e^{\beta(1-\sigma(0,A))}. \tag{11}$$

Modified propagators

The quadratic part of the action is

$$\mathcal{P}^{ab}_{\mu\nu} = \delta^{ab}\left(k^2\delta_{\mu\nu} + \left(1 - \frac{1}{\alpha}\right)k_\mu k_\nu + \frac{\gamma^4}{k^2}\right) \tag{12}$$

With $\gamma^4 = \frac{g^2 N}{2(N^2-1)}\beta^*$
Inverting the above expression we obtain the gluon propagador

$$\left\langle A^a_\mu(q)A^b_\nu(-q)\right\rangle = \frac{q^2}{q^4+\gamma^4}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\delta^{ab} \tag{13}$$

- The propagator displays complex poles, removing the gluon of the physical spectrum.
- Let us note that in the limit $\gamma \to 0$ we recover the usual UV behaviour.

gap equation

The parameter γ is not free, being determined in a self–consistent way by a suitable gap equation. Consider the quadratic part of the action in the path integral

$$Z_{quad} = \int \frac{d\beta}{2\pi i\beta}\, e^{-f(\beta)} \,, \tag{14}$$

where after integrating out the fields, $f(\beta)$ is given by

$$f(\beta) = \frac{1}{2} Tr \ln P^{ab}_{\mu\nu} + \ln\beta - \beta \tag{15}$$

$$Z_{quad} \simeq \; e^{-f(\beta^*)} \,, \tag{16}$$

where β^* corresponds to the stationary point of $f(\beta)$

$$\frac{\partial f(\beta)}{\partial\beta}\Big|_{\beta=\beta^*} = 0 \,, \tag{17}$$

Finally we get

$$\frac{3Ng^2}{4} \int \frac{d^4 q}{(2\pi)^3} \frac{1}{q^4+\gamma^4} = 1 \,. \tag{18}$$

2°part: Examples in 3D

- 3D is simpler in terms of renormalizability, but still non-trivial since it has degrees of freedom.
- There is examples of confining models such as gauge-Higgs systems
- Naturally, in 3D arises the topological contribution to the action encoded in the Chern–Simons term.

Because of the presence of the parameters of the systems involved , for instance de VEV of the Higgs field ν and the Chern–Simons mass M and also the Yang–Mills coupling constant g we shall find this behaviour of the correlation functions of the gauge field

$$\langle A^a_\mu(q)A^b_\nu(-q)\rangle \equiv \mathcal{G}^{ab}_{\mu\nu}(q) \tag{19}$$

$$= \mathcal{G}^{ab}_{\mu\nu}(q)_{phys} + \mathcal{G}^{ab}_{\mu\nu}(q)_{unphys} \tag{20}$$

Where $\mathcal{G}^{ab}_{\mu\nu}(q)_{phys}$ represents the existence of physical poles.

Adjoint representation of the Higgs field: The Georgi-Glashow model

$$S = \int d^3x \left(\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + \frac{1}{2} D^{ab}_\mu \varphi^b D^{ac}_\mu \varphi^c + \frac{\lambda}{2} \left(\varphi^a \varphi^a - \nu^2 \right)^2 + b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) \tag{21}$$

The covariant derivative is real, defined by

$$(D_\mu \varphi)^a = \partial_\mu \varphi^a + g \epsilon^{abc} A^b_\mu \varphi^c \tag{22}$$

The vacuum configuration for the potential of the spontaneous symmetry breaking

$$\varphi^a \varphi^a = \nu^2 \tag{23}$$

We choose the vacuum expectation value for the scalar field in the third direction of the internal space

$$\langle \varphi^a \rangle_0 = \nu \delta^{a3} \tag{24}$$

Defining shift fields

$$\begin{aligned} \Phi^a &= \varphi^a + \langle \varphi^a \rangle_0 \\ \varphi^a &= \Phi^a - \nu \delta^{a3} \end{aligned} \tag{25}$$

Making use of the decomposition for the quadratic part of the action

$$S_{quad} = \int d^3x \left(\frac{1}{4} \left(\partial_\mu A^a_\nu - \partial_\nu A^a_\mu\right) + b^a \partial_\mu A^a_\mu + \frac{g^2\nu^2}{2} \left(A^1_\mu A^1_\mu + A^2_\mu A^2_\mu\right) \right) \qquad (26)$$

From this, one should argue that the non-diagonal gauge fields $A^{\alpha\beta}_\mu$ with $\beta = 1, 2$ acquires a mass $m^2_H = g^2\nu^2$

$$\left\langle A^\alpha_\mu(p) A^\beta_\nu(-p) \right\rangle = \frac{\delta^{\alpha\beta}}{p^2 + m^2_H} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \qquad (27)$$

while the third component A^a_μ naively should be massless

$$\left\langle A^3_\mu(p) A^3_\nu(-p) \right\rangle = \frac{1}{p^2} \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \qquad (28)$$

However, the famous result of Polyakov (Nucl. Phys. B120 (1977) 429-458) establish that in the weak coupling regime ($g^2 << \nu^2$) the component A^3_μ remains confined due to monopole contributions, which condensation give rise to an area law for the Wilson loop.

The effect of the Gribov horizon

Because of the split of the color matrix it is necessary two ghost form factors to implement the restriction to the Gribov horizon.

$$\sigma_{off}(0;A) \leq 1 \tag{29}$$
$$\sigma_{diag}(0;A) \leq 1. \tag{30}$$

Condition implemented by a step function

$$Z = \int [DA_\mu]\delta(\partial A)(det\mathcal{M})\theta(1-\sigma_{diag})\theta(1-\sigma_{off})e^{-S_{YM}}. \tag{31}$$

Expression that in the momentum space is

$$Z_{quad} = \int \frac{d\beta e^\beta}{2\pi i\beta}\frac{d\omega e^\omega}{2\pi i\omega} DA_\mu^\alpha DA_\mu^3 e^{-\frac{1}{2}\int\frac{d^3q}{(2\pi)^3}A_\mu^\alpha(q)\mathcal{P}_{\mu\nu}^{\alpha\beta}A_\nu^\beta(-q)-\frac{1}{2}\int\frac{d^3q}{(2\pi)^3}A_\mu^3(q)\mathcal{Q}_{\mu\nu}A_\nu^3(-q)}, \tag{32}$$

with

$$\mathcal{P}_{\mu\nu}^{\alpha\beta} = \delta^{\alpha\beta}\left(\delta_{\mu\nu}\left(q^2+\nu^2g^2\right)+\left(\frac{1}{\xi}-1\right)q_\mu q_\nu+2\frac{g^2}{3}\left(\beta+\frac{\omega}{2}\right)\frac{1}{q^2}\delta_{\mu\nu}\right) \tag{33}$$
$$\mathcal{Q}_{\mu\nu} = \delta_{\mu\nu}\left(q^2-2\frac{\omega g^2}{3}\frac{1}{q^2}\right)+\left(\frac{1}{\xi}-1\right)q_\mu q_\nu \tag{34}$$

Inverting this we find the propagators

$$\left\langle A^3_\mu(q) A^3_\nu(-q) \right\rangle = \frac{q^2}{q^4 + \frac{2\omega g^2}{3}} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \tag{35}$$

$$\left\langle A^\alpha_\mu(q) A^\beta_\nu(-q) \right\rangle = \delta^{\alpha\beta} \frac{q^2}{q^2\left(q^2 + g^2 v^2\right) + g^2\left(\frac{2\beta}{3} + \frac{\omega}{3}\right)} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \tag{36}$$

The non–perturbative parameters β and ω must be fixed in terms of the coupling constant g and the Higgs VEV ν through the corresponding Gap equations.

$$Z_{quad} = \int \frac{d\beta}{2\pi i \beta} \frac{d\omega}{2\pi i \omega} e^\beta e^\omega \left(\det \mathcal{Q}_{\mu\nu}\right)^{-\frac{1}{2}} \left(\det \mathcal{P}^{\alpha\beta}_{\mu\nu}\right)^{-\frac{1}{2}} \tag{37}$$

$$\left(\det \mathcal{Q}_{\mu\nu}\right)^{-\frac{1}{2}} = \exp\left[-\int \frac{d^3q}{(2\pi)^3} \ln\left(q^2 + \frac{2\omega g^2}{3} \frac{1}{q^2} \right) \right] \tag{38}$$

$$\left(\det \mathcal{P}^{\alpha\beta}_{\mu\nu}\right)^{-\frac{1}{2}} = \exp\left[-2 \int \frac{d^3q}{(2\pi)^3} \ln\left((q^2 + g^2 v^2) + g^2\left(\frac{2\beta}{3} + \frac{\omega}{3}\right) \frac{1}{q^2} \right) \right] \tag{39}$$

Solving the Gap equations: Confinement and Higgs phases

The gap equation are

$$\left(\frac{g^2}{2}\right)\int\frac{d^3q}{(2\pi)^3}\left(\frac{1}{q^4+\omega^*\frac{2g^2}{3}}\right) = 1 \qquad (40)$$

$$\left(\frac{g^2}{2}\right)2\int\frac{d^3q}{(2\pi)^3}\left(\frac{1}{q^2(q^2+g^2v^2)+(\beta^*\frac{2g^2}{3}+\omega^*\frac{g^2}{3})}\right) = 1 \qquad (41)$$

Solving directly

$$\omega^* = \frac{3g^6}{2^{11}\pi^4} \qquad (42)$$

$$\beta^*\frac{2g^2}{3}+\omega^*\frac{g^2}{3} = \left[\frac{1}{2}g^2\nu^2-\frac{g^4}{32\pi^2}\right]^2 \qquad (43)$$

Phases from propagators

Let us analyse the analytic structure of the propagators

- The diagonal component of the propagator $\left\langle A^3_\mu(q) A^3_\nu(-q)\right\rangle$ is independent of the Higgs VEV ν and of the Gribov type. This imples that this field is always confined forall values of g and ν. This result is in agreement with the Polyakov result confinement of the electric charge by the condensation of monopoles.

Concerning to the non–diagonal gauge fields

$$\left\langle A^\alpha_\mu(q) A^\beta_\nu(-q)\right\rangle = \delta^{\alpha\beta} \frac{q^2}{q^2\left(q^2+g^2v^2\right)+g^2\left(\frac{2\beta}{3}+\frac{\omega}{3}\right)}\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) \quad (44)$$

$$= \delta^{\alpha\beta}\left(\frac{\mathcal{R}_+}{q^2+m_+^2}-\frac{\mathcal{R}_-}{q^2+m_-^2}\right)\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) \quad (45)$$

with $m_+^2 = \frac{-g^2v^2+\sqrt{g^4v^4-4\tau}}{2}$, $m_-^2 = \frac{-g^2v^2-\sqrt{g^4v^4-4\tau}}{2}$
and $\mathcal{R}_+ = \frac{m_+^2}{m_+^2-m_-^2}$, $\mathcal{R}_- = \frac{m_+^2}{m_+^2-m_-^2}$
Observamos que cuando $\tau < \frac{g^2\nu^2}{4}$ ambas masas m_+^2, m_-^2 son reales, positivas y diferentes, Así como también las cantidades $\mathcal{R}_+$ and $\mathcal{R}_-$

confined and Higgs regimes

- when $g^2 < 32\pi^2\nu^2$, the off diagonal propagator has a physical mode with real positive mass m_+^2. Due to the fact that the diagonal field is confined, this phase is called the U(1) symmetric phase. It is also worth observing that, for the particular value $g^2 = 16\pi^2\nu^2$, the unphysical mode dissapears , as $\mathcal{R}_- = 0 = m_-^2$. Thus, for that particular value of the gauge coupling, the off-diagonal propagator reduces to a single physical Yukawa mode

$$\left\langle A^\alpha_\mu(q)A^\beta_\nu(-q)\right\rangle = \delta^{\alpha\beta}\left(\frac{1}{q^2+16\pi^2 v^4}\right)\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right) \tag{46}$$

- when $g^2 > 32\pi^2\nu^2$, corresponding to $\tau > \frac{g^2\nu^2}{4}$, all masses become complex and the off diagonal propagator becomes of the Gribov type with two complex poles. This region corresponds to a phase in which all gauge modes are confined. Which is referred in the Lattice literature as the SU(2) confined phase
- It is interesnting to mention that, apparently, the transition between the totally confined phase and the partially deconfined phase is performed in a continous way. The existence of the Yukawa ghost seems to be necessary, however it is an open point.

Topologically massive Yang–Mills theory

In this case, the starting action is

$$S_M = S_{CS} + S_{FP} \quad (47)$$

$$S_M = -iM \int d^3x \, \epsilon_{\mu\rho\nu} \left(\frac{1}{2} A_{\mu a} \partial_\rho A^a_\nu + \frac{1}{3!} g f^{abc} A^a_\mu A^b_\rho A^c_\nu \right) \quad (48)$$

$$+ \frac{1}{4} \int d^3x \, F^a_{\mu\nu} F^a_{\mu\nu} + \int d^3x \left(b^a \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu c^b \right) \quad (49)$$

M is the Chern–Simons mass, b^a a Lagrange multipler that enforce the Landau gauge condition, $\partial_\mu A^a_\mu = 0$, y $(\bar{c}^a, c^a)$ are the usual FP ghosts

- A remarkable feature of this model is that the topological contribution has the effect of generates a deconfined massive mode, as the gauge field propagator shows

$$\left\langle A^a_\mu(q) A^b_\nu(-q) \right\rangle = \frac{\delta^{ab}}{(q^2 + M^2)} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + M \epsilon_{\mu\rho\nu} \frac{q_\rho}{q^2} \right) \quad (50)$$

Thus, the question that naturally arises is if this physical excitation will survives the confining effect of the Gribov horizon.

Infrared Propagator

$$Z_{quad} = \int \frac{d\beta}{2\pi i \beta} \, [DA] \, e^{\beta} \, e^{-\frac{1}{2}\int \frac{d^3q}{(2\pi)^3} A^a_\mu(q) Q^{ab}_{\mu\nu} A^b_\nu(-q)} \,, \tag{51}$$

with

$$Q^{ab}_{\mu\nu} = \delta^{ab}\left(q^2\delta_{\mu\nu} + \left(\frac{1}{\alpha} - 1\right) q_\mu q_\nu + \frac{\gamma^4}{q^2}\delta_{\mu\nu} - M\epsilon_{\mu\rho\nu}q_\rho \right) \,, \tag{52}$$

The parameter γ stands for the Gribov mass parameter

$$\gamma^4 = \frac{2}{3}\frac{Ng^2}{N^2-1}\beta \,. \tag{53}$$

Inverting $Q^{ab}_{\mu\nu}$, we get the gauge field propagator

$$\left\langle A^a_\mu(q) A^b_\nu(-q) \right\rangle = \delta^{ab} \frac{q^2\left(q^4+\gamma^4\right)}{(q^4+\gamma^4)^2 + M^2 q^6} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{q^4+\gamma^4} M\epsilon_{\mu\lambda\nu}q_\lambda \right) \,. \tag{54}$$

It is worth to note that removing the horizon *i.e.* setting $\gamma = 0$ we recover the original massive propagator of the Yang–Mills–Chern–Simons theory. Analogously, when $M = 0$, the el Gribov propagator for Yang–Mills is obtained

Gap equation of the parameter γ

$$Z_{quad} = \int \frac{d\beta}{2\pi i\beta}\, e^{-f(\beta)}\,, \tag{55}$$

Due to the topological nature of the Chern–Simons term, it doesn't couples to the spacetime metric, so it doesn't contribute to the vaccum energy. It turns out that the Gap equation is independent of M.

$$\gamma = \lambda^{1/4} g^2\,, \qquad \lambda^{1/4} = \frac{\sqrt{2}N}{12\pi}\,. \tag{56}$$

So, the gauge field propagator takes the form

$$\left\langle A^a_\mu(q) A^b_\nu(-q)\right\rangle = \delta^{ab}\frac{q^2\left(q^4+\lambda g^8\right)}{\left(q^4+\lambda g^8\right)^2 + M^2 q^6}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} + \frac{q^2}{q^4+\lambda g^8} M\epsilon_{\mu\lambda\nu} q_\lambda\right) \tag{57}$$

Let us start by splitting the propagator (57) in two parts, a parity conserved, and a parity violating one, namely

$$\left\langle A^a_\mu(q) A^b_\nu(-q) \right\rangle = \mathcal{G}^{ab}_{\mu\nu}(q)\Big|_{par} + \mathcal{G}^{ab}_{\mu\nu}(q)\Big|_{par-viol} \tag{58}$$

with

$$\mathcal{G}^{ab}_{\mu\nu}(q)\Big|_{par} = \delta^{ab} \frac{q^2\left(q^4+\lambda g^8\right)}{\left(q^4+\lambda g^8\right)^2+M^2q^6} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) , \tag{59}$$

$$\mathcal{G}^{ab}_{\mu\nu}(q)\Big|_{par-viol} = \delta^{ab} \frac{q^4}{\left(q^4+\lambda g^8\right)^2+M^2q^6} M\epsilon_{\mu\lambda\nu} q_\lambda . \tag{60}$$

Using partial fraction decomposition,

$$\mathcal{G}^{ab}_{\mu\nu}(q)\Big|_{non-par} = \delta^{ab} \left(\frac{\mathcal{R}_1}{q^2+m_1^2} + \frac{\mathcal{R}_2}{q^2+m_2^2} + \frac{\mathcal{R}_3}{q^2+m_3^2} + \frac{\mathcal{R}_4}{q^2+m_4^2} \right) M\epsilon_{\mu\lambda\nu} q_\lambda \tag{61}$$

$$\mathcal{G}^{ab}_{\mu\nu}(q)\Big|_{par} = \delta^{ab} \left(\frac{\mathcal{F}_1}{q^2+m_1^2} + \frac{\mathcal{F}_2}{q^2+m_2^2} + \frac{\mathcal{F}_3}{q^2+m_3^2} + \frac{\mathcal{F}_4}{q^2+m_4^2} \right) \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \tag{62}$$

where the residues are given by

$$\mathcal{R}_1 = \frac{m_1^4}{(m_2^2-m_1^2)(m_3^2-m_1^2)(m_4^2-m_1^2)} , \tag{63}$$

$$\mathcal{R}_2 = -\frac{m_2^4}{(m_2^2-m_1^2)(m_3^2-m_2^2)(m_4^2-m_2^2)} , \tag{64}$$

$$\mathcal{R}_3 = \frac{m_3^4}{(m_1^2-m_3^2)(m_2^2-m_3^2)(m_4^2-m_3^2)} , \tag{65}$$

$$\mathcal{R}_4 = -\frac{m_4^4}{(m_4^2-m_1^2)(m_4^2-m_3^2)(m_4^2-m_2^2)} \tag{66}$$

and

$$\mathcal{F}_1 = \frac{m_1^2\left(G+m_1^4\right)}{(m_2^2-m_1^2)(m_3^2-m_1^2)(m_4^2-m_1^2)} , \tag{67}$$

$$\mathcal{F}_2 = -\frac{m_2^2\left(G+m_2^4\right)}{(m_2^2-m_1^2)(m_3^2-m_2^2)(m_4^2-m_2^2)} , \tag{68}$$

$$\mathcal{F}_3 = \frac{m_3^4\left(G+m_3^4\right)}{(m_1^2-m_3^2)(m_2^2-m_3^2)(m_4^2-m_3^2)} , \tag{69}$$

$$\mathcal{F}_4 = -\frac{m_4^2\left(G+m_4^4\right)}{(m_1^2-m_4^2)(m_3^2-m_4^2)(m_4^2-m_2^2)} . \tag{70}$$

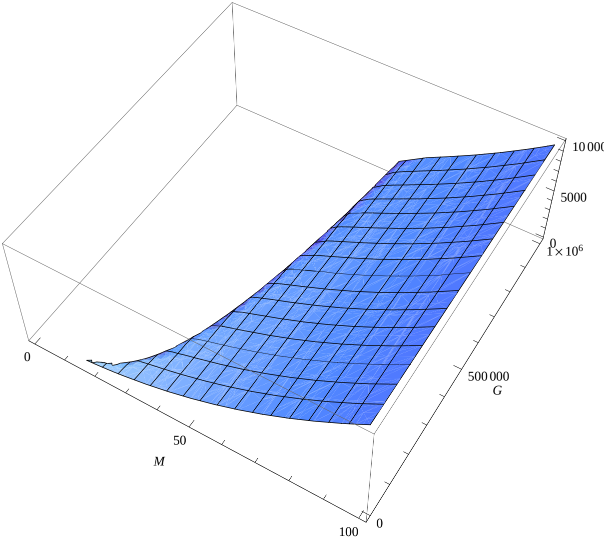
Confined and deconfined regimes

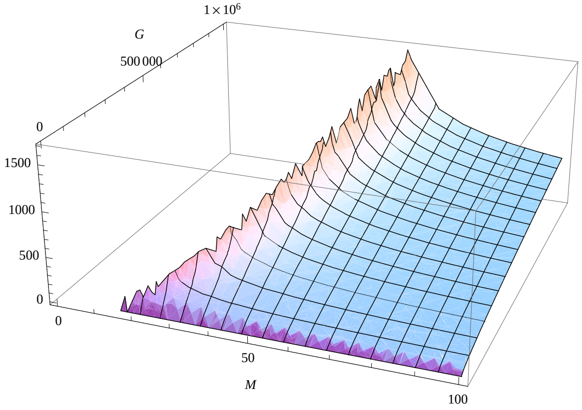
To study properly que regimes of the theory, we look at the discriminant of the roots's polynomia

$$\Delta = 256M^4G^5 - 27M^8G^4 \tag{71}$$

From the latter expression it is possible to distinguish two regimes:

- For ($\Delta > 0$ or $G > (3M/4)^4$) the four masses are complex, and physical propagating modes doesn‘t exist.
- When ($\Delta < 0$ or $G < (3M/4)^4$) two masses turns out to be real and positive and two remains as complex conjugate. Thus, the system shows physical excitations.
- This result is interpreted as following, for small values of the CS mass M and large values of the coupling constant g all the excitations are confined, while for large values of the CS mass and weak coupling real poles appears and the theory enters in a deconfined regime.


10 000
5000
0
1×10^6
500 000
G
0
0
50
M
100


1×10^6
G
500 000
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1500
1000
500
0
0
50
M
100

Summary

- In both models it is possible identify real poles in the non–perturbative propagators associated. In this way it is possible distinguish confined and observable regions in the parameter space of each theory
- In the Georgi Glashow mode the gauge field A^3_μ remains always confined cause the analytic structure of the correlation function shows to non–physical complex masses, in agreement with Polyakov result,
- This analysis is also aplicable to the fundamental representation case, where, in contrast with present case, the Higgs mechanism breaks all the generators of the group, this case turns out to be more simple, due to the absence of a massless gauge field. Thus, also we can find two phases, a confined and a Higgs phase.
- The present analysis can be quite relevant in the study of QCD at high temperatures since, in this case, the theory can be described with an effective three-dimensional theory in which the Chern-Simons term appear upon integrating out the fermions.
- The coupling constant of this kind of induced Chern-Simons term is proportional to the number of fermions flavours N_f. Hence, the present results imply that when the Yang-Mills coupling is very small compared with the flavours number then the theory is not in the confining phase while the Yang-Mills coupling is very large compared with N_f then the theory is in the confining phase. These conclusions are very satisfactory from the intuitive point of view since it is well known that adding fermions flavours to Yang-Mills action "decreases" the confining character of the theory