Universidad de los Andes

Séptima Escuela de Física Matemática

Topological quantum matter: from theory to applications

SIMULATION OF UNIVERSAL LOGIC QUANTUM GATES BY USING A TORIC CODE MODEL

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ABSTRACT

Currently a topic of high scientific interest is the quantum simulation of many-body spin interactions. Toric code model is highlighted, in which the spins are located at the edges of a square lattice on the surface of a torus. In this paper the Hamiltonian of the system is initially dentified. If the lattice is of dimension $k \times k$, the Hamiltonian is a matrix of dimension $2^{2k^2} \times 2^{2k^2}$. The many-qubit time evolution operator of the system is then obtained. Further to this, universal logic quantum gates are constructed. This work may have relevance to establish a representation of anyons by using the toric code model, which would simulate topological quantum gates.

Keywords: Many-electron systems, Quantum computation, Anyons.

Introduction

Quantum computing → Requires a fault tolerant mechanism

Errors of the physical states

Errors in the memory material

Errors during qubit dynamics

Surface code ↓ Kind of quantum computing highly fault tolerant

Stabilizer formalism of quantum mechanics → Allows to import classical binary code and using it as a quantum code.

↓ In this formalism it is convenient to express a quantum state as a single simultaneous eigenvector of commuting operators

$X \otimes X$ $Z \otimes Z$

↓ Stabilizers operators

Key: Proper Manipulation of

Toric Code Model

The toric code is a quantum error correction code and an example of stabilizer code defined on a two-dimensional network of spins with periodic boundary conditions.

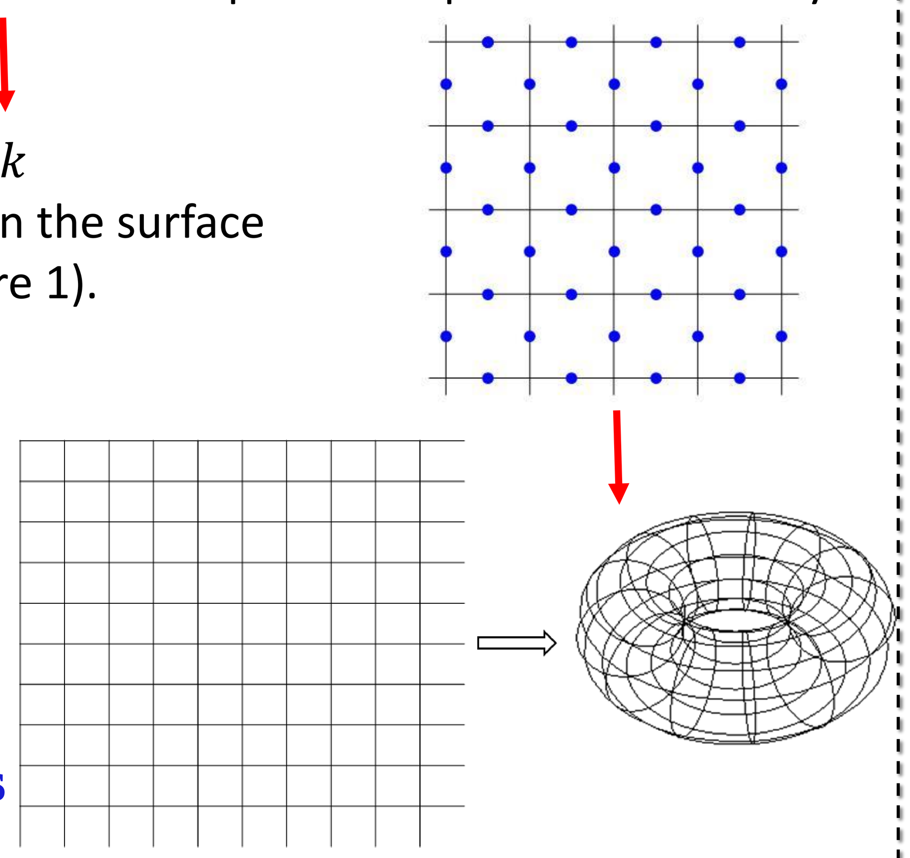
Consider a $k \times k$ square lattice on the surface of a torus (Figure 1).

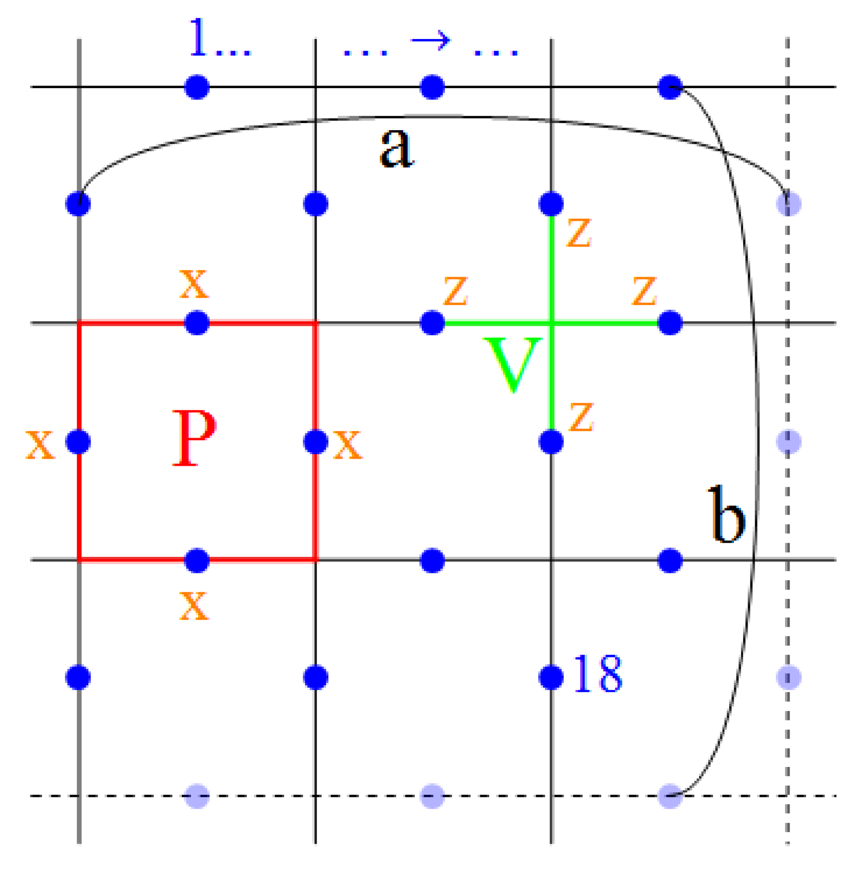
qubits

$$n = 2k^2$$

To Figure 1

$n = 18$ **qubits**





Figurr 1. Scheme of an 3x3 arrangement of the toric code. The blue dots represent qubits, ***P*** and ***V*** represent a plaquette and a vertex of the network respectively.

Toric Code Model

Stabilizers Operators

$$A_V = \prod_{i \in V} \sigma_i^x \qquad B_P = \prod_{i \in P} \sigma_i^z$$

$i \in V$ ↓ Indicates the edges touching the vertex V

$i \in P$ ↓ Indicates the edges surrounding the plaquette P

Properties:

- Commute with each other because plaquettes and vertices have either 0 or 2 edges in common.
- A_V and B_P contain Pauli's operators representing four-body spin interaction .
- A_V and B_P are also Hermitian and have eigenvalues 1 and −1. Therefore, they constitute an Abelian subgroup of the Pauli group of n qubits that is a stabilizer group.

Protected subespace

$\mathcal{H}$: Hilbert space of the $n = 2k^2$ qubits.

$\mathcal{L}$: Protected subspace

$$\mathcal{L} = \{|\xi\rangle \in \mathcal{H}: A_V|\xi\rangle = |\xi\rangle, |B_P|\xi\rangle = |\xi\rangle, \forall V, P\}$$

A_V and B_P satisfy $\prod_V A_V = 1, \prod_P B_P = 1$ and for that reason there are $2k^2 - 2$ independent stabilizers operators.

Hamiltonian

Spin-1/2 particles interact on the square lattice through the Hamiltonian

$$H_{TC} = -\sum_v A_v - \sum_p B_p$$

The size of the Hilbert space of the system defined on a $k \times k$ square lattice is $N = 2^{2k^2}$, thus H_{TC} has dimension $2^{2k^2} \times 2^{2k^2}$

- Stabilizers operators constitute a complete set of operators which commute with H_{TC}.
- The dimension of the protected space $\mathcal{L}$ is $2^{2k^2}/2^{2k^2-2} = 4$, which corresponds to the 4-fold degenerate ground state.

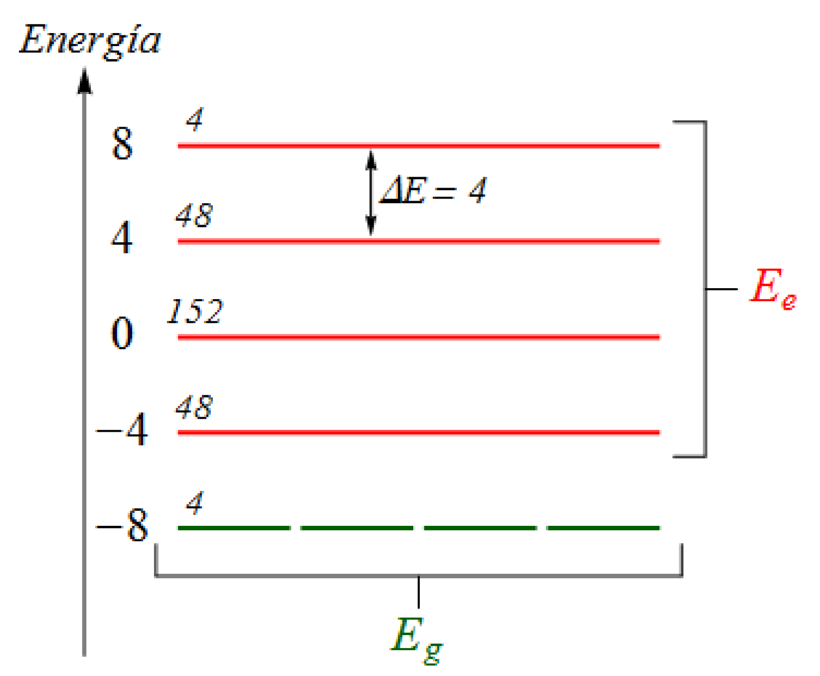
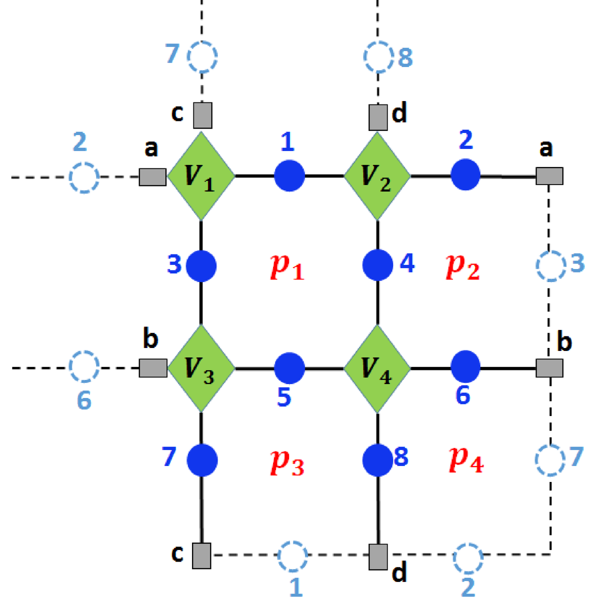


Figure 2. Schematic spectrum of the toric code Hamiltonian of an 2×2 arrangement case. ***Eg*** represents the 4-fold degenerate ground state, whereas ***Ee*** represents the excited states

Application of the model

$k = 2$ → $n = 8$; 8 stabilizer operators, 4 vertices, 4 plaquettes. $N = 256$, H_{TC} 256×256 matrix. There are 252 excited states..



Figurae 3. Arrangement of spins in a 2×2 square lattice of the toric code . The blue balls represent the spins, green diamonds represent the 4 vertices, and p_i represent the 4 plaquettes. The scheme emphasizes that the square lattice is on the surface of a torus to the indicate the connections **a**, **b**, **c**, **d**.

Vertex operators and plaquette operators

$$A_{V1} = \sigma_2^x\sigma_1^x\sigma_7^x\sigma_3^x$$
$$A_{V2} = \sigma_1^x\sigma_2^x\sigma_8^x\sigma_4^x$$
$$A_{V3} = \sigma_6^x\sigma_5^x\sigma_3^x\sigma_7^x$$
$$A_{V4} = \sigma_5^x\sigma_6^x\sigma_4^x\sigma_8^x$$
$$B_{P1} = \sigma_1^z\sigma_3^z\sigma_4^z\sigma_5^z$$
$$B_{P2} = \sigma_2^z\sigma_4^z\sigma_3^z\sigma_6^z$$
$$B_{P3} = \sigma_5^z\sigma_7^z\sigma_8^z\sigma_1^z$$
$$B_{P4} = \sigma_6^z\sigma_8^z\sigma_7^z\sigma_2^z$$

Initial State

$$|\psi(0)\rangle = \sqrt{Z}|\psi_g\rangle = (1/\sqrt{2})(|\psi_g\rangle + |\psi_e\rangle)$$

Quantum Gate

The initial state $|0\rangle$ arises from applying $\sqrt{Z}$ operation on the ground state $|\psi_g\rangle$ in the Hilbert space. In other words, $\sqrt{Z}$ operation is a rotation of 45° counterclockwise. When the initial state evolves by the evolution operator $U(t)$ we get an state $|\psi(t)\rangle$, such that, to the make an analysis of its time dependent components, can be established a set T of values of t as $T = \left\{t_k = \left(\frac{\pi}{4}\right)(2k+1) : |\psi(t = t_k)\rangle = |1\rangle, \forall k \in \mathbb{Z}\right\}$, where $|1\rangle$ is orthogonal to $|0\rangle$ and allow therefore, define effective spin states of the complete system, providing the possibility of simulate a NOT operation. To determine a CNOT operation, we must devise a way to identify a state $|0\rangle$ and a state $|1\rangle$ for two spin systems.

For example, the original lattice and the dual lattice (They have the original torus lattice) can form the two systems. Ather possibility is the original network plus the second one displaced $\pi/4$. These proposals are necessary because there are vertex and plaquette operators depending interaction, as well as interaction between systems. Having states $|0\rangle$ and $|1\rangle$ for each system we proceed to determine states $|00\rangle$, $|01\rangle$, $|10\rangle$ and $|11\rangle$ which provide the possibility of simulate a CNOT operation.

CONCLUSIONS

Quantum computing certainly gives new physical problems that have great conceptual richness, and that as few areas of physics, can be studied simultaneously at theoretical and experimental level. The study of surface code as a type of fault-tolerant quantum computing, requires models involving many-body interactions, and requires the construction of new types of logic gates. Toric code model, which represents four-body interactions in a two-dimensional lattice, by their collective spin states, allow the construction of an initial state such that the evolution by the operator U(t) of the system, allows the determination of a set T of values of t, to simulate a NOT operation. To study and analyze in detail how to simulate a CNOT operation, a mechanism is devised for construction of states $|i\rangle \otimes |j\rangle = |ij\rangle$, where $|i\rangle$ and $|j\rangle$ represent orthogonal states of two different systems

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