# Faraday rotation in thin metal films bounded by topological insulator

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Electromagnetic excitations arising in a metal slab surrounded by an insulator with a non trivial topology associated to time reversal symmetry (topological insulator) are described in the frame of axion electrodynamics. It is shown that the rotation of the polarization plane induced by the non trivial topology is a linear function of the topological term for all slab thicknesses but strongly depends on the dispersion relations of bonding and antibonding surface modes. Numerical calculations show that for the bonding modes the presence of the topological term leads to an interval of in−plane wave vector with decreasing rotation angle, which becomes more pronounced for thin slabs. On the other hand, the rotation angle corresponding to antibonding surface modes increases with the in−plane wave vector and its magnitude is lower than that corresponding to bonding modes.

## Introduction

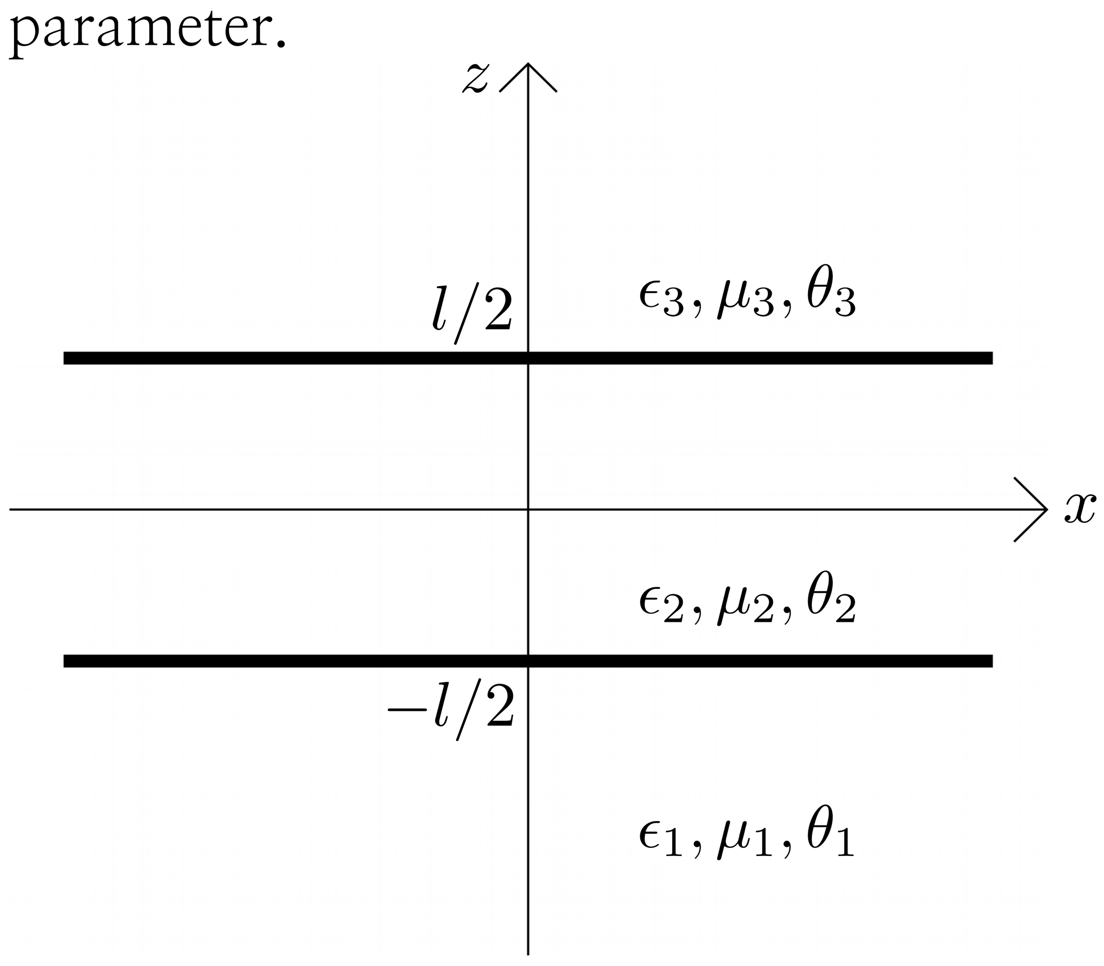
Topological insulators (TI) are materials that are insulators in the bulk but have gapless surface states protected against time−reversal symmetry.

Recently, special attention has been devoted to the study of collective excitations arising at topolocally nontrivial interfaces, where the topological parameter jumps.

The aim of this work is to show some consequences in the rotation of the polarization plane of an electromagnetic wave that passes through a metal film with topologically nontrivial interfaces.

## Model and general relations

We consider a metal film of thickness $l$ surrounded by two materials in principle with different electric permittivity, magnetic permeability and topological parameter.




Due to the presence of the topological term, there is a breaking in the time−reversal symmetry, so the constitutive relations for a topological insulator read

$$\mathbf{D} = \epsilon\mathbf{E} + \epsilon_0\alpha\frac{\theta}{\pi}(c_0\mathbf{B})$$

$$c_0\mathbf{H} = \frac{c_0\mathbf{B}}{\mu} + \alpha\frac{\theta}{\pi}\frac{\mathbf{E}}{\mu_0}$$

We can take the direction of propagation of the wave to be the $x$ direction, so we have for magnetic and electric field

$$\begin{pmatrix} E_y^{(1,3)} \\ H_y^{(1,3)} \end{pmatrix} = \begin{pmatrix} E_0^{(1,3)} \\ H_0^{(1,3)} \end{pmatrix} e^{ik_z^{(1,3)}z} e^{i(k_x x-\omega t)}$$

$$\begin{pmatrix} E_y^{(2)} \\ H_y^{(2)} \end{pmatrix} = \left[ \begin{pmatrix} E_0^{(2)} \\ H_0^{(2)} \end{pmatrix} e^{ik_z^{(2)}z} + \begin{pmatrix} F_0^{(2)} \\ I_0^{(2)} \end{pmatrix} e^{-ik_z^{(2)}z} \right] e^{i(k_x x-\omega t)}$$

where $(k_z^{(j)})^2 = \frac{\omega^2}{c_0^2}\epsilon_j\mu_j - k_x^2$ for $j = 1, 2, 3$.

From now on $\epsilon_1 = \epsilon_3,\ \mu_1 = \mu_3$ and $\theta_1 = \theta_3$.

Using boundary conditions for the fields in the interfaces $z = \pm l/2$ leads us to the following dispersion relation

$$\left[\frac{\mu_1}{k_z^{(1)}} + \frac{\mu_2}{k_z^{(2)}} S\left(k_z^{(2)}\frac{l}{2}\right)\right]\left[\frac{\epsilon_1}{k_z^{(1)}} + \frac{\epsilon_2}{k_z^{(2)}} S\left(k_z^{(2)}\frac{l}{2}\right)\right] = \left(\alpha\frac{\Delta\theta}{\pi}\right)^2 \frac{\epsilon_0}{\mu_0}\frac{\mu_1}{k_z^{(1)}}\frac{\mu_2}{k_z^{(2)}} S\left(k_z^{(2)}\frac{l}{2}\right)$$

where $S\left(k_z^{(2)}\frac{l}{2}\right) = tan\left(k_z^{(2)}\frac{l}{2}\right)$ for bonding modes and $S\left(k_z^{(2)}\frac{l}{2}\right) = cot\left(k_z^{(2)}\frac{l}{2}\right)$ for antibonding modes.

Also we can compute the rotation angle of the polarization plane due to the topologically non trivial interfaces that is

$$\frac{E_0^{(1)}}{E_1^{(1)}} = \left(\alpha\frac{\Delta\theta}{\pi}\right)\frac{c_1}{c_0\mu_0}\frac{\mu_1\mu_2 S\left(k_z^{(2)}\frac{l}{2}\right)}{\frac{\mu_1}{\kappa} + \mu_2 S\left(k_z^{(2)}\frac{l}{2}\right)}$$

where $\kappa := \frac{k_z^{(1)}}{k_z^{(2)}}$ and $\mu_j H_0^{(j)} := -\frac{E_1^{(i)}}{c_j}$

## Results

We use, for the metal film the typical form of the permittivity for a metal, that is the one described in terms of a free electron gas which is

$$\frac{\epsilon(\omega)}{\epsilon_0} = 1 - \frac{\omega_p^2}{\omega(\omega + i\gamma)}$$ where $\omega_p$ is the plasma frequency.

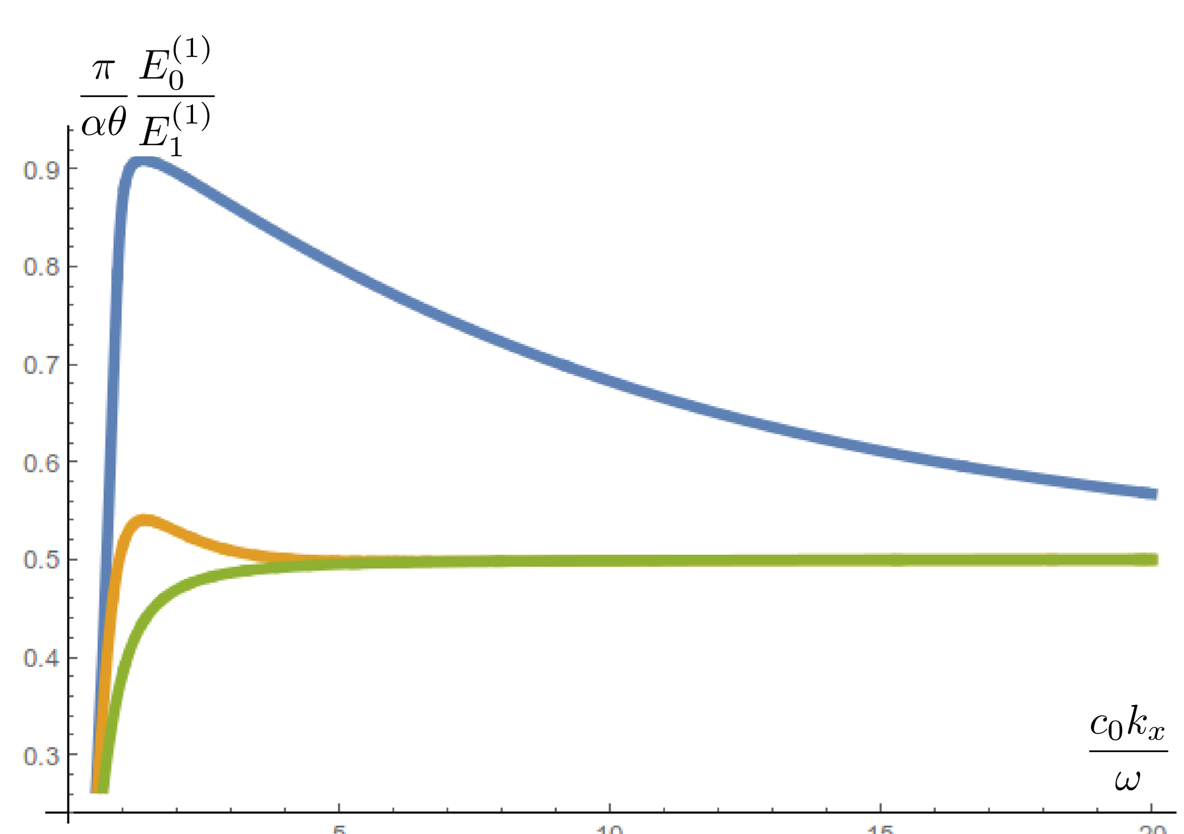



*Figure 1. Rotation angle of the polarization plane as a function of the wave vector for three different thicknesses* (blue→0.1 orange→1 green→10 normalized to the plasma frequency) in the case of bonding surface modes.

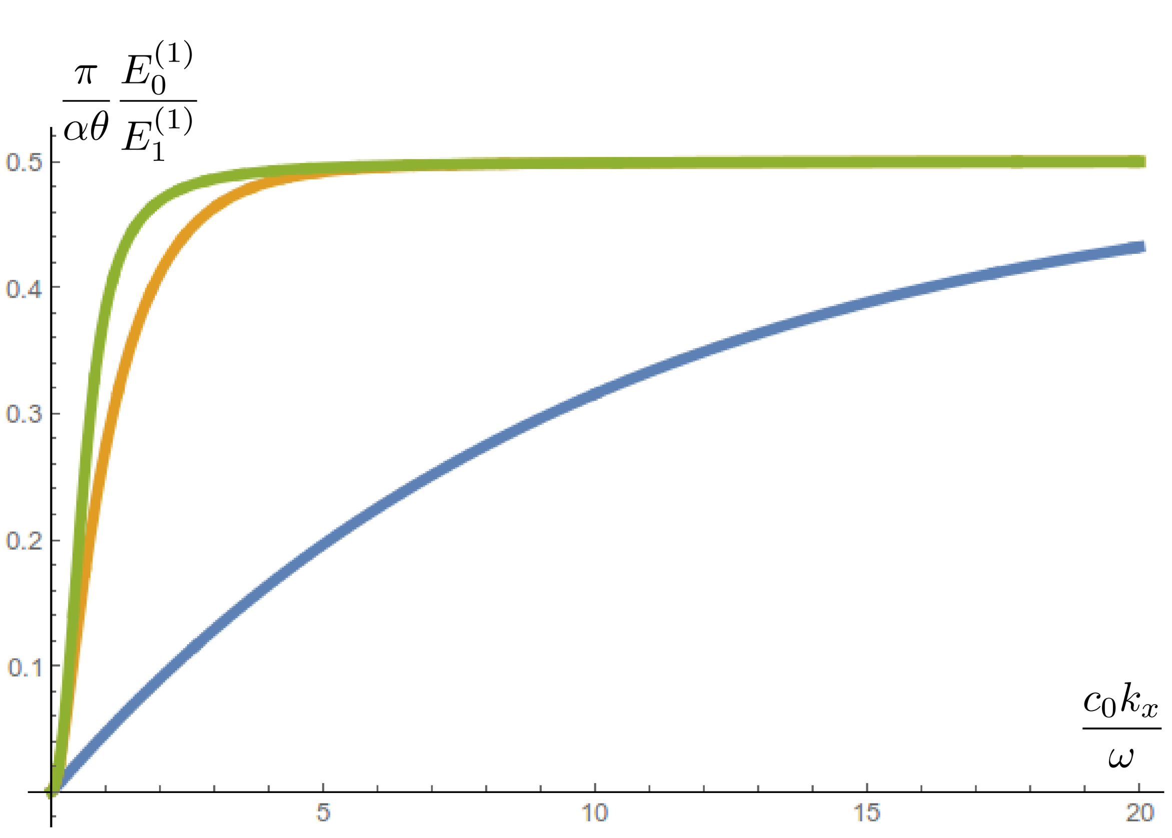



*Figure 2. Rotation angle of the polarization plane as a function of the wave vector for three different thicknesses* (blue→0.1 orange→1 green→10 normalized to the plasma frequency) in the case of antibonding surface modes.

## Conclusions

The presence of topologically non trivial interfaces leads to a rotation of the polarization plane of the wave that depends on the specific mode excited in the surface (bonding or antibonding).

A natural extension of this work could be the study of the rotation of the polarization plane in systems of multiple interfaces, for example photonic crystals.

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## References

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