Topological Insulators in Nanowires

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Abstract

In this work we review the main characteristics of a topological insulator and some applications in the study of nanowires. In particular, we summarize some results obtained for the energy spectrum in nanowires of circular and rectangular cross-section. In these systems the surface states show an energy gap associated with the Berry phase. Finally, we discuss how to control this gap using magnetic or exchange fields and the possible applications in spintronics.

Surface states in topological insulators nanowires

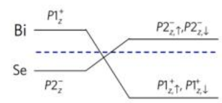
The 3D topological insulators of the Bi_2Se_3 family in the ***k•p*** approximation are described by a four band continuum Hamiltonian [1]:

$$H^{3D} = E(\mathbf{k}) + \begin{pmatrix} \mathcal{M}(\mathbf{k}) & A_1 k_z & 0 & A_2 k_- \\ A_1 k_z & -\mathcal{M}(\mathbf{k}) & A_2 k_- & 0 \\ 0 & A_2 k_+ & \mathcal{M}(\mathbf{k}) & -A_1 k_z \\ A_2 k_+ & 0 & -A_1 k_z & -\mathcal{M}(\mathbf{k}) \end{pmatrix}$$

$$\mathcal{M}(\mathbf{k}) = M_0 - B_2(k_x^2 + k_y^2) - B_1 k_z^2$$

$$E(\mathbf{k}) = C + D_1 k_z^2 + D_2(k_x^2 + k_y^2)$$

$$k_\pm = k_x \pm i k_y$$



With a flat boundary (e.g x-y plane), it has two bound-surface eigenstates which are evanescent along normal direction (z), and vanish at the surface [2]:

$$u(k_x, k_y, \lambda_{1,2})\, e^{i(k_x x + k_y y)} \left(e^{\lambda_1 z} - e^{\lambda_2 z}\right)$$

These states have energies within the bulk gap, and are degenerate. By projecting the 3D Hamiltonian in this basis, it reduces to a 2D effective Dirac Hamiltonian [3]:

$$H^{\pm\hat{z}} = \frac{D_1}{B_1} M_0 \pm A_2 \sqrt{1 - \frac{D_1^2}{B_1^2}} \begin{pmatrix} 0 & i k_x + k_y \\ -i k_x + k_y & 0 \end{pmatrix}$$

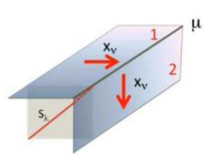
$$v_F = A_2 \sqrt{1 - \frac{D_1^2}{B_1^2}} \qquad \varepsilon = \frac{D_1}{B_1} M_0 \pm v_F \sqrt{k_x^2 + k_y^2}$$

For a closed curved boundary (e.g cylinder of radius R and axis z), the projected 2D effective Hamiltonian has de form [4,5]:

$$H_{2D} = \begin{bmatrix} 0 & -iBk_z + \frac{A}{R}\left(-i\frac{\partial}{\partial\theta} + \frac{1}{2}\right) \\ iBk_z + \frac{A}{R}\left(-i\frac{\partial}{\partial\theta} + \frac{1}{2}\right) & 0 \end{bmatrix}$$

$$E_{\parallel} = \pm\sqrt{A^2 k_\phi^2 + B^2 k_z^2}$$

Matching conditions for joined flat surfaces



For a prism-shaped boundary, bound states of different surfaces are related at the corners, in first approximation, by the change of basis (e.g x-z planes junction) [3] :

$$\Psi^{(x)} = e^{ik_y y} \sum_n e^{ik_z^{(n)} z} c_n^{(x)} \chi_n^{(x)}(x) \qquad \Psi^{(z)} = e^{ik_y y} \sum_n e^{ik_x^{(n)} x} c_n^{(z)} \chi_n^{(z)}(z)$$

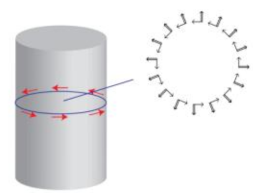
$$c_n^{(x)} = \sum_m < x, n | z, m > c_m^{(z)}$$

$$< x, n | z, m > = \int_{\lambda=0}^{\lambda=1} ds_\lambda e^{-ik_z^{(n)} z_\lambda + ik_x^{(m)} x_\lambda} < \chi_n^{(x)}(x_\lambda) | \chi_m^{(z)}(z_\lambda) >$$

where the integration path is chosen such that the projected Hamiltonian of the "flattened" corner surface will be Hermitian, and so the current will be conserved across the junction.

Energy spectrum and Berry phase

The energy spectrum of a circular and rectangular cross-section nanowires are obtained analytically. For the cylindrical isotropic wire we have [4,5] :



$$E(k_z, k_\phi) = \pm A\sqrt{k_z^2 + k_\phi^2}$$

$$k_\phi R = \pm 1/2, \pm 3/2, \cdots$$

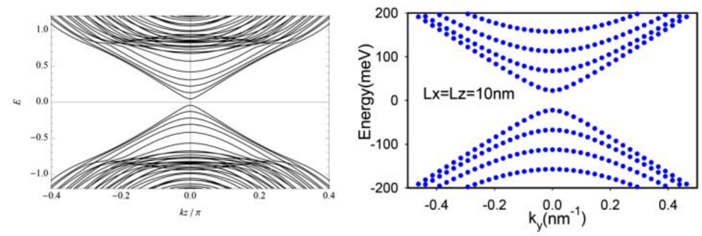
This quantization rule can be easily rationalized by including the π geometrical *Berry phase* acquired by an helical electron in a full circulation:

$$\psi(z, \phi + 2\pi) = -\psi(z, \phi)$$

A similar expression is obtained for the rectangular wire [3]:

$$E = \sqrt{(A_1 k_z)^2 + (A_2 k_y)^2} \qquad 2(k_x L_x + k_z L_z) + \pi = 2\pi n$$

$$\epsilon_{n,k_y} = \pm\sqrt{(A_2 k_y)^2 + \left(\pi \frac{A_1 A_2}{A_1 L_z + A_2 L_x}(n - \frac{1}{2})\right)^2}$$



Both energy spectrum have a finite gap associated with the Berry phase.

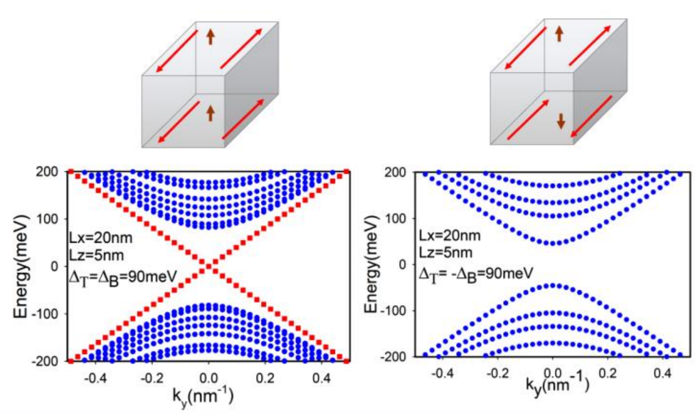
Controlling the Berry energy gap

The application of a precise magnetic flux $\Phi = -\pi/e$ along the wires, cancels the Berry phase, and so the Berry energy gap.

Placing the surface in proximity to a ferromagnetic insulator, the magnetization exchange fields enters as a mass term in the Dirac Hamiltonian (e.g x-y plane) [2,3]:

$$\widehat{\mathcal{H}} = v(\hat{p}_x \sigma_y - \hat{p}_y \sigma_x) - M\sigma_z$$

Now there is one protected chiral edge state (red). When equal exchange fields are applied to two opposite surfaces, these must *have the same direction* in order to allow a circulating chiral state in the lateral surfaces [3].



Selected references

[1] H. Zhang, et al., Nature Phys. 5, 438 (2009).
[2] M. Z. Hasan and C. L. Kane, Rev. Mod. Phys. 82, 3045 (2010).
[3] L. Brey and H. A. Fertig, Phys. Rev. B 89, 085305 (2014).
[4] R. Egger, A. Zazunov, and A. L. Yeyati, Phys. Rev. Lett. 105, 136403 (2010).
[5] K. Imura et al., Phys. Rev. B 84, 195406 (2011).