Phase transition of anyons confined in one-dimensional optical lattice

J. Arcila- Forero , R. Franco y J. Silva-Valencia

Departamento de Física.

Facultad de Ciencias. Universidad Nacional de Colombia.

Contacto: jfarcilaf@unal.edu.co

Abstract

Anyons are the third fundamental category of particles, for two anyons under particle exchange the wavefunction acquires a fractional phase $e^{i\theta}$, giving rise to fractional statistics with $0 < \theta < \pi$. The greatest interest for the anyons study emerged when the fractional quantum Hall effect observed experimentally had natural explanation in term of anyons. We study a Hubbard model for anyons equivalent to a variant of the Bose-Hubbard, we established an exact mapping between anyons and bosons in one-dimension. Using the density matrix renormalization group method we studied the system properties, we presented the phase diagram for density $\rho = 1$ with some angles, the quantum transition is from Mott insulator to Superfluid phase, the Mott lobe expands with increasing statistical angle.

We study the block von Neumann entropy, which was used to establish the critical points. For a fixed density we study the critical point evolution vs θ and find that the position of the critical point increasing with the angle. For fixing statistical we showed the dependency between the position of the critical point and the density, also the critical point decreases with increasing of the density, implying that the Mott region decreases.

Introduction

⋇ Anyons

Anyons—particles carrying fractional statistics that interpolate between bosons and fermions—have been conjectured to exist in low-dimensional systems. For two anyons under particle exchange the wavefunction acquires a fractional phase $e^{i\theta}$[1].

⋇ Model

The Anyon-Hubbard Model:

$$H = -t\sum_{j}^{L}\left(a_j^{\dagger}a_{j+1} + h.c\right) + \frac{U}{2}\sum_{j}^{L} n_j(n_j - 1) \tag{1}$$

Where t is the tunnelling amplitude connecting two neighbouring sites and U is the on-site interaction energy. We introduce an exact mapping between anyons and bosons in 1D. Let us define the fractional version of a Jordan–Wigner transformation,

$$a_j = b_j exp\left(i\theta\sum_{i=1}^{j-1} n_i\right), \tag{2}$$

with $n_i = a_i^{\dagger}a_i = b_i^{\dagger}b_i$ the number operator for both particle types. Provided that the particles of type b are bosons, $[b_j, b_j^{\dagger}] = \delta_{ji}$ and $[b_j, b_i] = 0$.

By inserting the Anyon–Boson mapping, equation (2), the Hamiltonian can be rewritten in terms of bosonic operators:

$$H = -t\sum_{j}^{L}\left(b_j^{\dagger}b_{j+1}e^{i\theta n_j} + h.c\right) + \frac{U}{2}\sum_{j}^{L} n_j(n_j - 1). \tag{3}$$

The mapped, bosonic Hamiltonian thus describes bosons with a occupation-dependent amplitude $e^{i\theta n_j}$[1].

⋇ Method

We used the density matrix renormalization group (DMRG) method with open boundary conditions [2]. We used the finite-size algorithm for sizes up to $L = 256$; we considered a truncated Hilbert space with five states by site and the density $\rho = N/L$. We kept up to $m = 200$ states per block and obtained a discarded weight around 10^{-8} or less.

Results

We present the phase diagram for conditional-hopping bosons for density $\rho = 1$

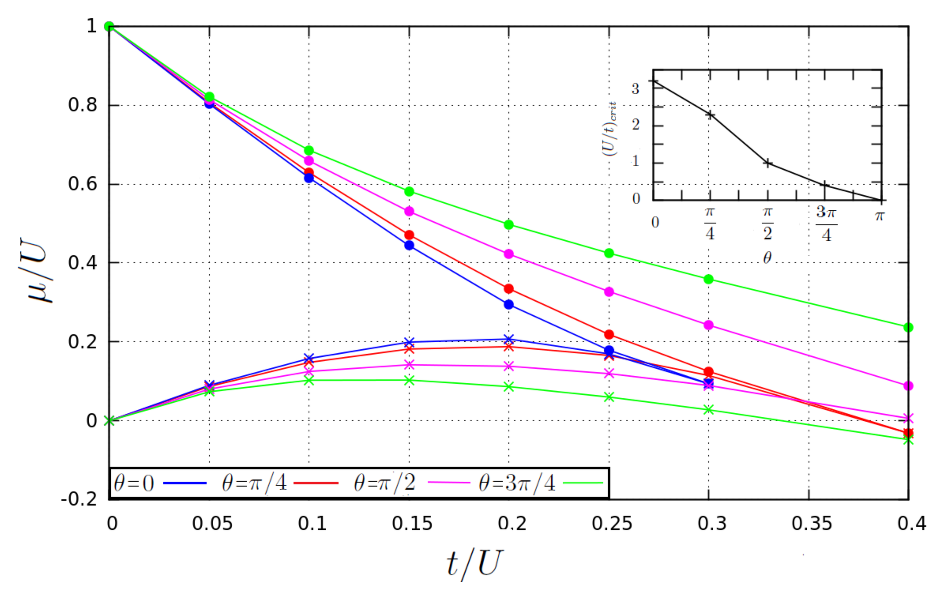

Fig 1: Phase diagram with different angles. Inset: Critical point dependence[1].

The quantum phase transition is from Mott insulator to Superfluid phase, the Mott lobe expands with increasing statistical angle. the conditional-hopping help to localize the particles.

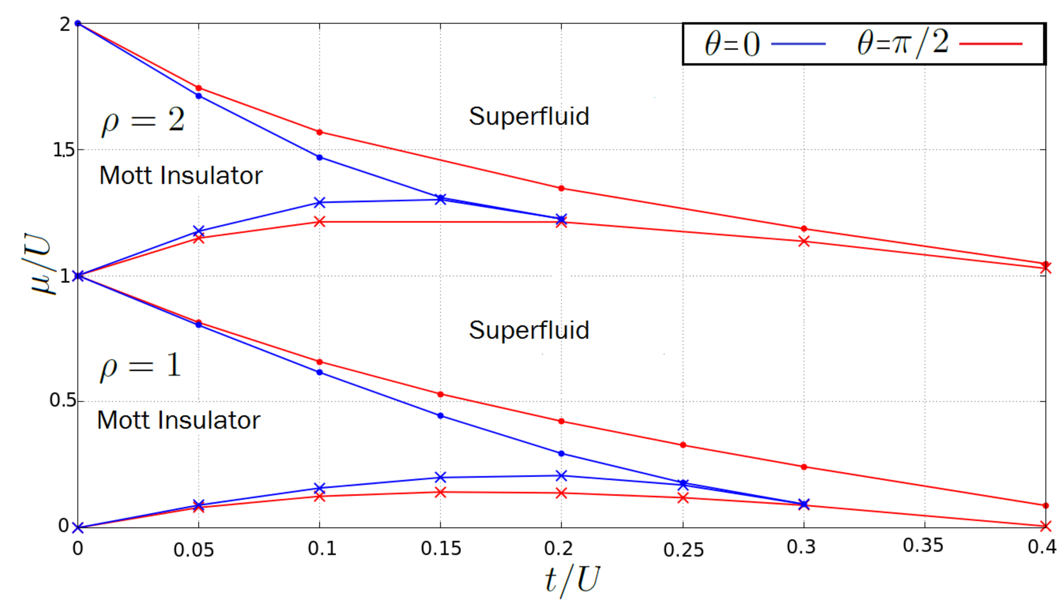

Fig 2: Comparison with bosonic case.

We can observe (Fig. 2) a superfluid phase surrounding an Mott insulator phase also we presented the comparison with the bosonic case ($\theta = 0$) for two densities ($\rho = 1$ and $\rho = 2$), the position of the critical point increasing with the angle for constant density and decreases with increasing of the density for the same angle.

In the (Fig. 3) we present the phase diagram for three densities and the inset we showed the evolution of critical points with the density.

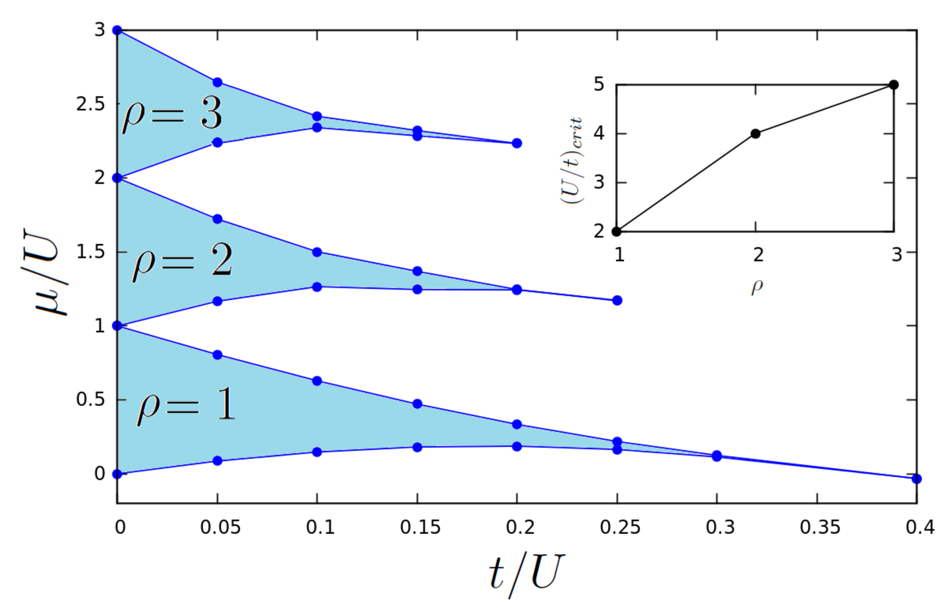

Fig 3: Phase diagram with $\theta = \pi/4$. Inset: Critical point dependence.

Results

The behavior of the von Neumann entropy (block entropy) $S_L(l)$ as a function of l

$$S_L(l) = \begin{cases} \frac{c}{3\eta} ln\left[\frac{\eta L}{\pi} sen(\pi l/L)\right] + \theta & critical, \\ \frac{c}{3\eta} ln\left[\zeta_L\right] + \theta' & noncritical, \end{cases}$$

Lauchli and Kollath proposed the estimator [3]

$$\Delta S_{LK} = S_L(L/2) - S_{L/2}(L/4) \tag{4}$$

$$\Delta S_{LK}(L) = \begin{cases} \frac{c}{3\eta} ln(2) & t \geq t_c \\ 0 & t < t_c \end{cases}$$

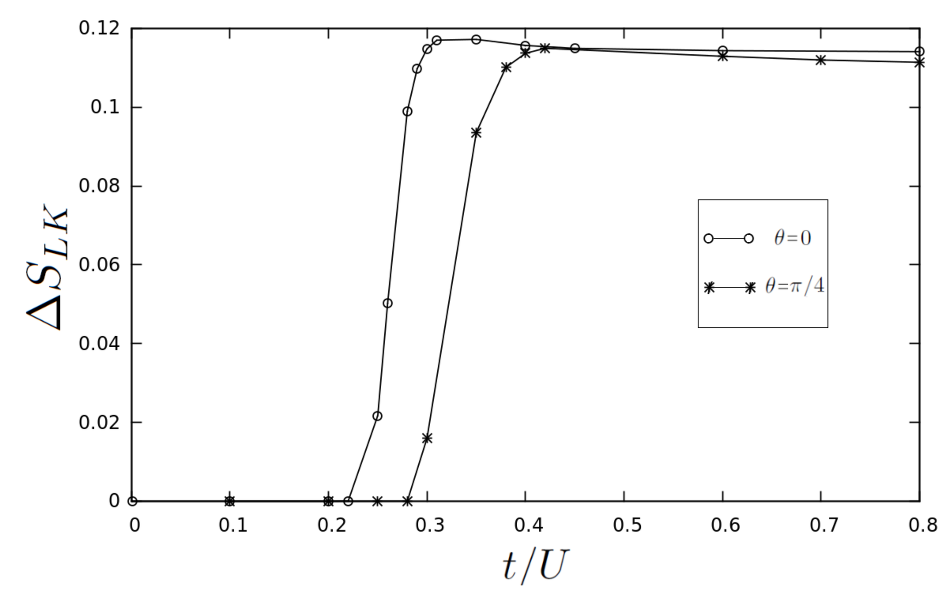

Fig 4: Determination of critical points. ΔS_{LK} as a function t/U.

The (Fig. 5) present the evolution of the critical point as a function of the density, calculate with the Lauchli and Kollath estimator

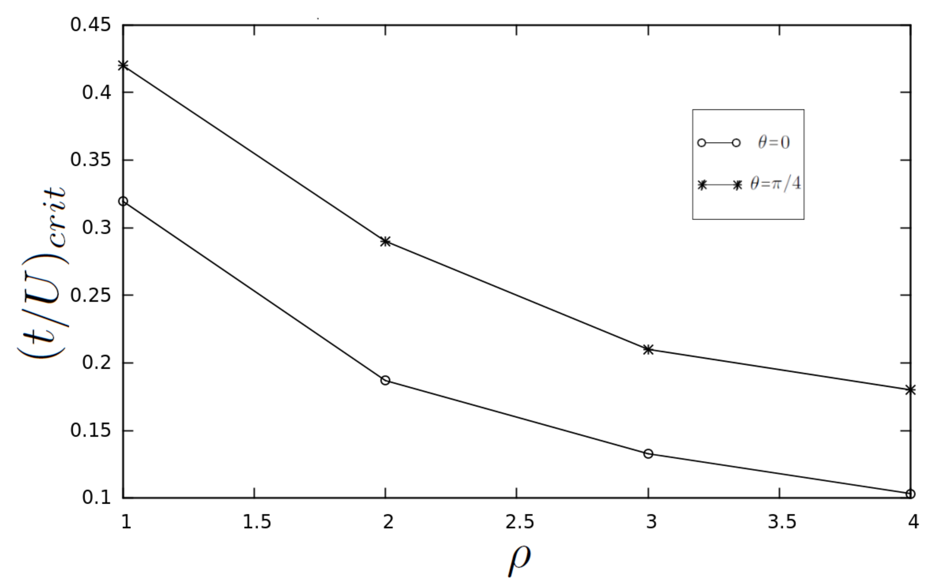

Fig 5: Evolution of the critical point.

We use the function for explain the type of the anyons transition [4]

$$\Delta\mu = Aexp\left[-b/\sqrt{t_c - t}\right] \tag{5}$$

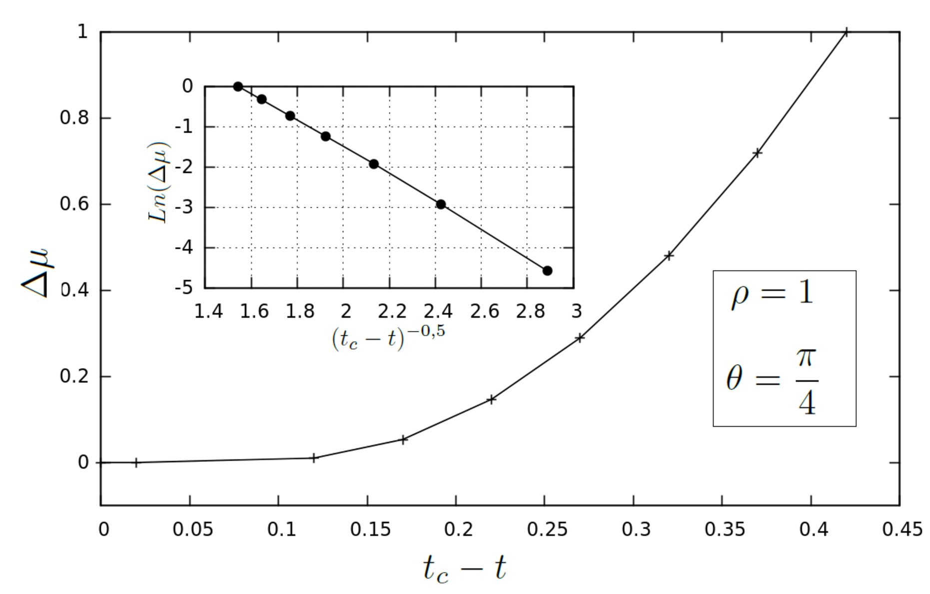

Fig 6: Adjustments to the Kosterlitz-Thouless transition.

which indicates that the Kosterlitz-Thouless behavior is suitable for describing the closing of the gap.

References

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