

*Quinta Escuela de Física-Matemática 2013*

*The Mathematics of  
Entanglement*

*Departamento de Matemáticas – Departamento de Física  
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# *Morning Lectures*

- **Fernando Brandão** (University College London, Reino Unido)  
*Entanglement measures and convex optimisation*
- **Matthias Christandl** (ETH Zürich, Suiza)  
*Multipartite entanglement and the quantum marginal problem*
- **Aram Harrow** (MIT, US)  
*Monogamy of entanglement and the de Finetti theorem*

In 1935, Einstein, Podolsky and Rosen discovered that quantum systems admit correlations that are stronger than any classical correlation. In addition to their fundamental importance for the build up of matter, these correlations – known as entanglement – are responsible for the advantages that quantum computation and quantum cryptography have to offer over their classical counterparts. Consequently, the study of entanglement has captured the fascination of many researchers and recent years have seen the development of a theory of entanglement. It is the goal of this school to give a mathematically sound introduction to the subject, which is guided by three active research topics:

- Entanglement measures and convex optimisation
- Multipartite entanglement and the quantum marginal problem
- Monogamy of entanglement and the de Finetti theorem

# *Short Communications*

**Tristram Bogart**

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## **NP-intermediate problems and quantum algorithms**

Quantum computers are not believed to be able to solve NP-hard problems in polynomial time. Thus, following Shor's discovery of a quantum polynomial-time factoring algorithm, attention in quantum computing has focused on a small set of other problems that do not have a known polynomial-time classical algorithm and yet are not known to be NP-hard. In combinatorics, the most famous such problem is Graph Isomorphism and its variants such as Graph Automorphism and Polytope Isomorphism. I will explain these problems and a little of what is known about both classical and quantum algorithms for solving them.

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**Vladimir Ballesteros Ballesteros**

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## **Homothety of entanglement in Grover's quantum algorithm**

Entanglement represents an important factor in the speed-up of quantum computational processes [1, 2, 3, 4]. It has been of recent interest to analyze the function of multipartite entanglement in quantum algorithms. On the other hand, it has been proved that multipartite entangled states are used in the first phase of the Grover's quantum search algorithm [1]. In this work, we describe the behavior of entanglement in this algorithm, computing it through GME (Geometric Measure of Entanglement). Then, we demonstrate that behavior of any type of entanglement is independent of the quantity  $q$  of qubits for large  $q$ , thus exhibiting a homothetic trans-

formation. Finally, we compare this outcome with the one obtained by a fixed-point quantum algorithm and we propose some guidelines for efficient simulation in the context of this quantum algorithm.

*Joint with José Alfonso Leyva Rojas, Pontificia Universidad Javeriana..*

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**Andrés Felipe Ducuara Garcia**

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## **On the superactivation of quantum nonlocality and the $B$ -nonlocality**

Some striking properties arise in quantum theory that are very ‘strange’ from a fundamental point of view, yet very useful in a practical way for the development of the so-called quantum technology. In this talk, we review two very recent results concerning entanglement and its relation to nonlocality, the latter understood in the sense of Bell’s theorem [1]. It is well known that all separable states are local states. As proved by Werner, there also exist entangled states that are local [6]. The first result, through the concept of superactivation of quantum nonlocality [5, 4], makes it possible to start from an entangled local Werner state  $W$  and, by taking  $k$  times its own



direct product ( $\rho_W^{\otimes k}$ ), obtain a new entangled state that is nonlocal. This very powerful and counterintuitive result suggests that the states  $W$  contain a kind of ‘hidden nonlocality’ that can be activated through tensor products. We review and discuss the proof of such superactivation via the use of quantum game theory, within the framework of the Khot-Vishnoi game [2]. The second result is that all entangled states contain a kind of nonlocality [3], the Buscemi ( $B$ )-nonlocality [3], which in turn has been proven useful in the so-called semi-quantum games. The relationship between the ‘standard’ nonlocality and this new ( $B$ )-nonlocality is an open problem; we give an outlook and discussion about this subject.

**Keywords:** Nonlocality, Superactivation, Entanglement, Quantum games, Semiquantum games.

*Joint with John H. Reina, Universidad del Valle, Colombia.*

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## Relativistic Dynamical Quantum Non-Localities

In nonrelativistic quantum mechanics, quantum correlations are largely thought to be absolute. However, when they are studied in the framework of relativistic quantum mechanics they could depend on the reference frame [1]. In particular, two particles could be entangled in one reference frame but unentangled in another one, thus quantum non-locality depends upon the reference frame. Here, the non-locality of quantum dynamics was tracked, by working to the Weyl’s representation of quantum mechanics, to the superposition principle. This is a kind of single particle non-locality, of different nature as the discussed above [2]. We extend this work to the relativistic framework of quantum mechanics. To do so, we review the basics of the relativistic Weyl’s formalism and discuss the construction of the path-integral representation of the Wigner function, as well as the influence of the reference frame on this dynamical quantum non-locality.

**Keywords:** Dynamical non-locality, path integrals, Wigner function

*Joint with Leonardo Pachón, Universidad de Antioquia, Colombia.*

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**Ivonne Guevara**

GRIFFITH UNIVERSITY, AUSTRALIA,  
UNIVERSIDAD NACIONAL, BOGOTÁ, COLOMBIA

## **Entanglement Dynamics in Open Systems: A quantum trajectory approach**

We use quantum diffusive trajectories to analyze the effects of decoherence in the entanglement dynamics of open quantum systems and test this approach from various perspectives. We study the dynamics of concurrence for  $2 \times 2$  systems coupled in different ways to thermal, dephasing and depolarizing channels. In specific cases we prove that the time evolution of bipartite entanglement under spontaneous emission can be fully characterized by an optimal continuous monitoring of the system. We analytically determine this optimal unraveling, and derive a deterministic evolution equation for the system's concurrence. Furthermore, we propose an experiment to monitor the entanglement dynamics, and to determine the disentanglement time from a single trajectory. We use entanglement of formation and concurrence of assistance to also explore the possibilities this approach has in the preparation and control of ensembles with small variance or of states with high values of entanglement. We find specific cases with time independent unravellings that protect entanglement, specially for maximally entangled states, even under noisy environments.

**Keywords:** quantum entanglement, open quantum systems, decoherence, quantum trajectories, quantum computation and information.

A partir del enfoque de trayectorias cuánticas difusivas se analizan los efectos de la decoherencia en la dinámica del entrelazamiento de sistemas abiertos probando este enfoque desde varias perspectivas. Se estudia la dinámica de concurrencia de sistemas  $2 \times 2$  acoplados a diferentes entornos térmicos, desfazantes y despolarizantes. En casos específicos se demuestra que la evolución temporal del entrelazamiento bipartito bajo emisión espontánea puede ser completamente caracterizado por un monitoreo óptimo o continuo del sistema. Se determina analíticamente este deshilamiento óptimo o y se deriva una ecuación de evolución determinística para la concurrencia del sistema. Además, se aprovechan las medidas de entrelazamiento de formación y concurrencia de asistencia para explorar las posibilidades que el enfoque de trayectorias cuánticas tiene en la preparación y control de ensambles con mínima varianza o máximo valor promedio de entrelazamiento. Se encuentran casos específicos con independencia temporal que de hecho protegen

esta propiedad, en particular para estados máximamente entrelazados, aún bajo el efecto de ambientes ruidosos.

**Palabras claves:** Entrelazamiento cuántico, sistemas cuánticos abiertos, decoherencia, trayectorias cuánticas, computación e información cuántica.

*Joint with Carlos Viviescas, Universidad Nacional, Bogotá..*

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## **Servio Tullio Pérez Merchancano**

DEPARTAMENTO DE FÍSICA

FACULTAD DE CIENCIAS NATURALES EXACTAS Y DE LA EDUCACIÓN

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### **Coupled Quantum Dots and Other Solid State Physical Models for Quantum Computing**

In this work, the exchange of energy  $J$  for a system of two laterally-coupled quantum dots, each one with an electron, is calculated analytically and in a detailed form, considering them as hydrogen-like atoms, under the Heitler-London approach. The atomic orbitals, associated to each quantum dot, are obtained from translation relations, as functions of the Fock-Darwin status. Our results agree with the ones reported by Burkard, Loss and DiVincenzo in their model of quantum gates based on quantum dots, as well as with the recent experimental reports.

On the other hand we have studied other models of quantum tunneling effects that include spin effects that are sources of the entanglement and that, based in solid state physics, contribute to the understanding and potential production of quantum mechanisms.

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## **Andrés Schlieff**

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### **Entanglement spectrum in the fractional quantum Hall effect**

The study of the Fractional Quantum Hall Effect (FQHE) discovered in 1982 by Tsui, Stormer and Gossard [5] has been enlightening in various fields of

physics. Particularly, in the past two decades this effect has been of major interest for the study of highly correlated systems entanglement. During this short communication I will present the concept of *Entanglement Spectrum* and its application to the FQHE [2]. In particular, I will present numerical results for Laughlin states in three different geometries and different Hilbert space partitions [2, 3, 4]. A particular emphasis will be given in the interpretation of the entanglement spectra as a clear sign of the so-called *Topological Order* [1]. In order to do this, the entanglement spectra will be used as numerical evidence of Haldane and Li’s conjecture: The low-lying entanglement spectrum corresponds to the energy spectrum of the theory’s edge modes described by a particular CFT [2, 3, 4].

**Keywords:** Entanglement, Topological Order, Entanglement Spectrum

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**Cristian E. Susa**

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### **Maximizing information flow between coupled qubits under dissipation**

Quantum entanglement and, in general, quantum properties of physical systems have been studied for many years as important resources for quantum processing tasks and quantum information protocols. In this work, we study the information flow (as quantified by discord and entanglement of formation) in an optically-driven bipartite qubit system (quantum emitters) coupled to a common dissipative environment. While the correlations between qubit systems have been widely studied in the literature, our interest is to gain information about how each emitter is correlated to the environment. To this end, we use a monogamic relation which allows us to calculate the emitter-environment correlations without any prior knowledge about the state of the environment at any time. We analyze the behavior of these correlations in terms of the inner properties of the emitters and the control parameters of the laser field. We show that a broader study involving the information flow between the qubits and the environment gives new insights into the dynamics of the interqubit quantum correlations. These aspects increase our knowledge of the usefulness of the dissipative environment, and lead us to identify how to maximize the distribution of correlations between the qubits.

**Keywords:** Quantum entanglement, discord, information, decoherence.

*Joint with John H. Reina, Universidad del Valle y Felipe Fanchini UNESP, Bauru, Brasil.*

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# Posters

**Juan David Botero**

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## **On the Dynamics of Spin-1/2 particles: A phase-space Path integral Approach**

The two-level quantum system is the most fundamental element in quantum-information-processing theory (QIPT) and one of its more natural physical implementations comprises a spin-1/2-system. Entangling these systems and their subsequent manipulation, based on the non-local character of quantum correlations, are the most fundamental protocols in QIPT. The non-locality that is exploited in those protocols is a non-locality between quantum systems; however, in order to get a complete picture of the quantum correlations, one has to analyze the influence of the non-local character of the quantum dynamics itself (dynamical non-locality). We use the proposal given by Björk et al [1] to construct the Wigner function in a discrete phase space, then with the aim to analyze the dynamics of the spin-1/2 particles, we develop a formula for the discrete Wigner propagator and calculate it by means of a direct method based on the path integral formalism for discrete systems [2]. Having already the explicit form for the Wigner propagator, we can see explicitly the non-local behavior of the quantum dynamics for the discrete systems.

*Joint with Leonardo A. Pachón Universidad de Antioquia, Colombia.*

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**Leonardo Calderón**

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**Interaction between localized photons and electrons in a Young’s double-slit experiment**

The model of photon is usually expressed for an electromagnetic field confined inside a cavity. Thus, each photon has a uniform spatial distribution within the cavity, and therefore the single-mode state of photons are delocalized. However, the typical sources of quantum-optical experiments produce photons described by a spatial wavepacket, i.e., with some degree of localization. We study the non-local effects in the interaction between localized photons and electrons in the double-slit experiment, where it is known that the interference pattern is related to degree of coherence of the electron wave function, i.e., the correlations between the different components of the wave function.

**Keywords:** Localized photons, Double slit experiment, Coherence.

*Joint with Rafael Torres, Universidad de Santander.*

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**Santiago Echeverri**

UNIVERSIDAD DEL QUINDÍO, ARMENIA, QUINDÍO, COLOMBIA

**Entanglement and PL spectra like signatures in cQED via phonon-assisted cavity feeding**

We propose a theoretical model for a dissipative quantum dot-microcavity system interacting via cavity phonon-assisted (CPA). The Lindblad master equation formalism is considered to study the full dynamics for both weak



and strong coupling regime. In particular, this model reproduce the intermediate regime that had reported experimentally as anomalous effect in the photoluminescence spectra. The entanglement sudden death of the system is calculated, we found that revivals and sudden death are evidenced.

*Joint with Edgar Gómez, Universidad del Quindío; Herbert Vinck-Posada, Universidad Nacional de Colombia .*

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**Andrés F. Estrada**

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MEDELLÍN, COLOMBIA

### **Non-Markovian effects in the dynamics of entanglement in the high temperature limit**

In the past years, some quantum phenomena have been observed at macroscopic scales. In particular, superconductivity, coherent superpositions of Bose-Einstein condensates and interference patterns in fullerenes have been detected. This fact has made that the border between the quantum and classical realms become more diffuse and intricate, although, more interesting, than before. However, in order to observe these quantum features, one needs to reach the low temperature regime,  $\hbar\omega/k_{\text{B}}T \gg 1$ , where  $\hbar\omega$  denotes a characteristic system energy-scale and  $k_{\text{B}}T$  the thermal energy. Therefore, some delicate and elaborate cooling processes have been developed. Our work aims to show that, even in the high temperature regime, some quantum features such entanglement can be present, if the system is placed out from equilibrium. In particular, we study the non-Markovian dynamic of two different harmonic oscillators coupled to different baths at different temperatures and with different coupling-to-the-bath-strengths. We found that, despite the absence of symmetries in the parameters space, entanglement between the oscillators can be created and maintained in the long-time regime. We also discuss the implementation of our setup for studying the influence of the non-Markovian dynamics in the optimal sideband cooling of nano-mechanical resonators.

**Keywords:** quantum information, computation, decoherence, and entanglement

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**Guillermo Guirales**

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MEDELLÍN, COLOMBIA

### **Relationships between entanglement and phase transitions in an exciton-polariton BEC confined in a nanocavity**

Recently, the two-dimensional nanocavity exciton-polariton system confined in semiconductor nanocavities has emerged as a promising alternative of Bose-Einstein Condensation (BEC). The strong quantum correlations present in this system, makes it a unique candidate for studying entanglement and quantum phase transitions qualitative changes in the ground state properties. In this work, we review previous evidence, obtained by us, in the phase transitions of a finite exciton-polariton system. First, by using a BCS wave function to compute the ground state energy of  $N$  excitons without the photonic field, a crossover from the high-density electron-hole phase to the BCS excitonic phase is found, at a density which is roughly four times the close-packing density of excitons. Second, by means of a self-consistent procedure with a trial function composed of a coherent photon field and a BCS function for the electron-hole pairs, we obtain the scaling of the critical temperature with the number of polaritons. Using the method proposed by Comte and Nozieres, and generalizing it, we will expect to find an interpolating function between the distinct phases and provide clues about the relationship between the phase diagram and the entanglement in this system.

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**Paul Daniel Marin**

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### **Puertas lógicas en computación cuántica topológica**

La computación cuántica basada en aniones es un novedoso tipo de computación que se caracteriza por ser muy robusta respecto a perturbaciones debidas al entorno. La propuesta se basa en la existencia de estados topológicos de la materia cuyas cuasipartículas excitadas no son ni bosones ni fermiones, sino partículas conocidas como aniones.

En la computación cuántica topológica, la información cuántica se almacena en las trayectorias bidimensionales de las cuasipartículas, las cuales se pueden describir en un espacio-tiempo 2+1 dimensional. Las líneas de mundo de las cuasipartículas se cruzan unas sobre otras para formar los trenzados (braiding) en ese espacio tridimensional. Las puertas lógicas cuánticas dependen solo de la topología del trenzado. En el presente trabajo, de tipo pedagógico, se muestra como hallar trenzados que dan lugar a un conjunto universal de puertas cuánticas para bits cuánticos codificados en una clase de cuasipartículas que, según la literatura consultada, dará lugar a implementaciones promisorias de la computación cuántica. Ciertos experimentos con efecto Hall cuántico fraccionario indican que estos elementos pueden ser creados en el mundo real usando semiconductores hechos de arseniuro de galio cerca al cero absoluto y sujetos a campos magnéticos fuertes. Herramientas primordiales de dicha propuesta son los grupos de trenzados aplicados a los estados de aniones.

*Joint with C. Estrada, J. Mahecha, Universidad de Antioquia, Colombia.*

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## **Dynamical Quantum Non-locality**

Non-locality of correlations between systems is one of the hallmarks of quantum mechanics. Non-locality is responsible for unexpected features of system dynamics at the quantum regime such as tunnelling and entanglement; it has also recently been extensively exploited as a resource for quantum information [2]. The origin of non-locality of quantum measurements and its relations to the fundamental postulates of quantum mechanics, such as the uncertainty principle, have been only recently elucidated [3]. However, quantum interference problems involve two kind of non-localities: a non-locality expressed in terms of the Bell-inequalities (of kinematic nature, the one discussed in [3]) and the non-locality of the quantum equation of motion of a physical observable (of dynamic nature) [1, 4]. The latter has been barely discussed, explored or understood [4]. We trace here the origin of dynamical non-locality to the superposition principle mediated by the presence of non-linear interactions between systems and discuss the disappearance of non-locality in the classical realm. This relation adds to the more fundamental understanding of nature's quantum dynamics and allows us to establish and identify how the uncertainty principle and the superposition determine the non-local character of the outcome of quantum measurements. As a consequence, dynamical quantum non-locality emerges, naturally, as the responsible for the suppression of chaos, understood in the classical sense, in the quantum dynamics.

**Keywords:** Dynamical non-locality, path integrals, Wigner function.

*Joint with Leonardo A. Pachón, Instituto de Física, Universidad de Antioquia, Colombia.*

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## Relationships between entanglement and phase transitions in an exciton-polariton BEC confined in a nanocavity

Los estados de puntos cuánticos han sido propuestos como los estados base de un qubit. Hay numerosas propuestas acerca de cuales estados usar: estados de carga (charge qubit), estados de espín (spin states), estados singlete y triplete de dos electrones atrapados en puntos cuánticos dobles, etc. Las dificultades para calcular las propiedades espectrales y el entrelazamiento de estos estados son diversas. Hay pocos ejemplos de cálculo ab-initio del entrelazamiento en estados de dos electrones ligados a un punto cuántico. Una de las configuraciones experimentales más interesantes para la cual no hay un cálculo del entrelazamiento de este tipo es en la transición singlete-triplete, la cual es producida por la aplicación de un campo magnético externo. En este poster presentaremos algunos de los detalles del cálculo de la energía de transición y del entrelazamiento en un punto cuántico simple con dos electrones.

*Joint with Omar Osenda, Universidad Nacional de Córdoba, Argentina.*

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## **Estudio de la complejidad de los estados resonantes del átomo de Helio a través de cuantificadores de la teoría de la información y del entrelazamiento cuántico**

Las distribuciones de densidad de átomos y moléculas son en principio objetos físicos observables, si bien sólo contienen información espacial proyectada de la propia función de onda. Estas densidades pueden pensarse como funciones de distribución de probabilidad sujetas al escrutinio de los métodos de análisis de la teoría de la información, como medidas de entropías y cuantificadores de complejidad. Los estados resonantes en átomos y moléculas presentes en espectros de colisiones electrónicas o fotónicas son estados de scattering metaestables embebidos en el continuo electrónico, cuya densidad electrónica, aún siendo estados del continuo, muestra una fuerte localización en zonas cercanas al núcleo. Su caracterización en energía y anchura ha sido motivo de estudio intenso en las últimas décadas. Sin embargo, la clasificación de estas resonancias en términos de etiquetas o números cuánticos aproximados no ha estado exenta de controversias.

En este átomo de dos electrones las etiquetas de la forma configuracional  $(n_1 l_1, n_2, l_2)$  pueden ser buenas aproximaciones para los estados simplemente excitados  $(1s, nl)$ , pero no así para los estados doblemente excitados, debido a la fuerte correlación electrónica que hace que la función de onda para un estado resonante venga descrita por una mezcla compleja de configuraciones. Han surgido así varios métodos de clasificación de estados doblemente excitados o resonantes [1, 3], destacando la propuesta de Herrick y Sinagoglu que rotula estos estados en términos de números cuánticos aproximados  $K, T$  generando series del tipo  $n_1(K, T)_{n_2}^A$  dentro de la serie de Rydberg de resonancias.

En este trabajo se pretende hallar una justificación de estas clasificaciones a partir del análisis topológico de las distribuciones de densidad  $\rho(r)$  y  $\rho(r_1, r_2)$  de la parte localizada de las resonancias, con cuantificadores de entropía tanto globales (entropía de Shannon, de Tsallis, de Rényi) como locales (información de Fisher), medidas de distancia de similaridad entre ellas [4], así como la magnitud de entrelazamiento cuántico a través de entropías de von Neumann y entropía lineal calculadas con la matriz de densidad reducida [5, 2].

Como ilustración, en la figura 1 puede observarse que la información de Fisher aplicada a las distribuciones de densidad radial  $\rho(r)$  muestra un comportamiento diferente para estados ligados y estados resonantes. Adicionalmente, dentro de la serie de Rydberg de estados resonantes, la información de Fisher es capaz de discriminar notablemente el comportamiento de las tres diferentes series  $n_1(K, T)_{n_2}^A$  presentes en la simetría  $1P^o$  del Helio, antes de colapsar en el segundo umbral de ionización.

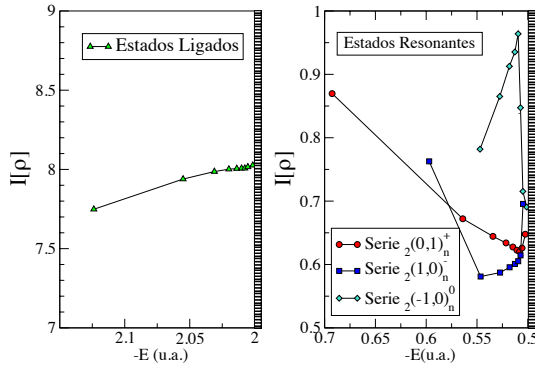


FIGURA 1. Entropía o información de Fisher  $I[\rho(r)]$  para las densidades de los estados ligados y resonantes de la simetría  $1P^o$  en el átomo de Helio, localizados energéticamente por debajo del primer umbral ( $E=-2.0$  a.u.) y del segundo umbral de ionización ( $E=-0.5$  a.u.), respectivamente.

Joint with José Luis Sanz-Vicario, Universidad de Antioquia, Colombia.

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### **Principio de incertidumbre en las mediciones desde el experimento de Schrödinger**

La mecánica cuántica nace del hecho de observar ciertos eventos, y aun cuando se repitan bajo las mismas condiciones no se obtienen los mismos resultados como ocurriría desde la mecánica clásica, por ende ya no se puede hablar de situaciones y resultados concretos y/o ciertos sino de posibles resultados. Es así como se formula un simple experimento dado por el Físico Erwin Schrödinger que a partir de una situación simple logra explicar dicha discrepancia, metiendo un gato en una caja con una ampolla de veneno y cerrando la caja sin saber en qué momento el gato podría seguir vivo o muerto. De esta forma se puede hablar de un principio de simultaneidad, puesto que si no se conoce en qué estado se encuentra un sistema, este continuara afirmando tener al mismo tiempo todos los estados posibles, y de esta forma se logra dar explicaciones sobre el comportamiento de partículas extremadamente pequeñas.

**Keywords:** Experimentos, Simultaneidad, Estados.



The Quantum Mechanics is born from the fact of checking certain events, which repeated under similar conditions do not show similar results, as it would occur throughout classical mechanics. For that reason, it is not longer possible to talk about either concrete situations or certain results, but possible results. Thus, a simple experiment given by physicist Erwin Schrödinger can explain throughout a simple situation such a discrepancy; in Schrödingers case, a cat is put inside a box with a piece of poison, subsequently, the box is closed without knowing at what point the cat would be death or alive. In this way, we can talk about a simultaneity principle, since if we do not know what the stage of a system is; it will continue having at the same time all possible stages which, certainly, explains the behavior of extremely small particles.

**Keywords:** Experiments, simultaneity, states .

*Joint with Víctor Andrés Heredia Heredia, Universidad Pedagógica Nacional, Colombia.*

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## **Regime at thermal equilibrium**

The aim of constructing and designing machines working at the nanometre-length scale, such as atomic motors, photocells, gyrators or heat engines, has boosted the developing of a quantum version of thermodynamics. One of the foundational conundra in this emerging field is, to what extent nanomachines can display quantum features and how this quantum behaviour could be used to improve their efficiency. Intuitively, one can suggest that if the energy of the thermal fluctuations is much smaller than the typical energy scale of the nanosystem, then there is room for the nanosystem to reveal its quantum nature. However, as it has been discussed recently in almost all fields related to quantum mechanics (e.g quantum information science, quantum biophysics, nanotechnology, quantum chemistry or condensed matter physics), the border between the quantum/classical operating regime is far from being trivial. We predict here, at thermodynamical equilibrium, the existence of a regime where, e.g., nanoelectromechanical structures or optomechanical systems can be found in an entangled state at high temperature assisted by the non-Markovian interactions. Complementarily, we report the existence of a second regime, characterized by Markovian interactions at low temperature, where quantum nanodevices do not thermalize into the canonical Boltzmann distribution, and therefore all their thermodynamical properties are expected to deviate, even, from current quantum thermodynamics. Our findings not only provide a solid ground for understanding the presence of quantum features in most of current investigations in bio and handmade systems, but also points out the direction to follow in protecting and isolating of quantum systems.

**Keywords:** Non-Markovian, entangled state, thermodynamical equilibrium.

*Joint with Leonardo Pachón, Universidad de Antioquia, Colombia.*

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Lunes, 27 de Mayo (Salón: B-202)

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8:00 – 9:00 *Registration*

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9:00 – 9:15 *Opening*

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9:15 – 10:15 Fernando Brandão:  
*Entanglement measures and convex optimisation I*

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10:15 – 11:15 Matthias Christandl:  
*Multiparticle entanglement and the quantum marginal problem I*

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11:15 – 11:45 Break

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11:45 – 12:45 Aram Harrow:  
*Monogamy of entanglement and the de Finetti theorem I*

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12:45 – 14:00 Lunch Break

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14:00 – 15:00 Sesión de problemas

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15:00 – 15:30 Break

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15:30 – 16:10 Ivonne Guevara: *Entanglement dynamics in open systems: A quantum trajectory approach*

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16:10 – 16:50 Cristian Susa: *Maximizing information flow between coupled qubits under dissipation*

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Martes, 28 de Mayo (Salón: B-202)

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9:00 – 10:00 Fernando Brandão: *Entanglement measures and convex optimisation II*

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10:00 – 11:00 Matthias Christandl: *Multiparticle entanglement and the quantum marginal problem II*

---

11:00 – 11:30 Break

---

11:30 – 12:30 Aram Harrow: *Monogamy of entanglement and the de Finetti theorem II*

---

12:30 – 14:00 Lunch Break

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14:00 – 14:40 Servio Tulio Pérez Merchancano: *Coupled quantum dots and other solid state physical models for quantum computing*

---

14:40 – 15:20 Vladimir Ballesteros Ballesteros: *Homothety of entanglement in Grover's quantum algorithm*

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15:20 – 15:50 Break

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15:50 – 16:50 Sesión de problemas

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17:30 – Sesión de posters & refrigerio

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Miercoles, 29 de Mayo (Salón: B-202)

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9:00 – 10:00 Fernando Brandão: *Entanglement measures and convex optimisation III*

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10:00 – 11:00 Matthias Christandl: *Multiparticle entanglement and the quantum marginal problem III*

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11:00 – 11:30 Break

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11:30 – 12:30 Aram Harrow: *Monogamy of entanglement and the de Finetti theorem III*

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Jueves, 30 de Mayo (Salón: B-202)

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9:00 – 10:00 Fernando Brandão: *Entanglement measures and convex optimisation IV*

---

10:00 – 11:00 Matthias Christandl: *Multiparticle entanglement and the quantum marginal problem IV*

---

11:00 – 11:30 Break

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11:30 – 12:30 Aram Harrow: *Monogamy of entanglement and the de Finetti theorem IV*

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12:30 – 14:00 Lunch Break

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14:00 – 14:40 Andrés Felipe Ducuara Garcia: *On the superactivation of quantum nonlocality and the B-nonlocality*

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14:40 – 15:20 Sebastián Duque Mesa: *Relativistic dynamical quantum nonlocality*

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15:20 – 15:50 Break

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15:50 – 16:50 Sesión de problemas

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Viernes, 31 de Mayo (Salón: B-202)

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9:00 – 10:00 Fernando Brandão: *Entanglement measures and convex optimisation V*

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10:00 – 11:00 Matthias Christandl: *Multiparticle entanglement and the quantum marginal problem V*

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11:00 – 11:30 Break

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11:30 – 12:30 Aram Harrow: *Monogamy of entanglement and the de Finetti theorem V*

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12:30 – 14:00 Lunch Break

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14:00 – 15:00 Sesión de problemas

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15:00 – 15:30 Break

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15:30 – 16:10 Tristram Bogart: *NP-intermediate problems and quantum algorithms*

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16:10 – 16:50 Andrés Schlieff: *Entanglement spectrum in the fractional quantum Hall effect*

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	Lunes	Martes	Miercoles	Jueves	Viernes
8:00 – 9:00	Registration				
9:00 – 10:00	F. Brandão	F. Brandão	F. Brandão	F. Brandão	F. Brandão
10:00 – 11:00	M. Christandl	M. Christandl	M. Christandl	M. Christandl	M. Christandl
11:00 – 11:30	<i>Break</i>	<i>Break</i>	<i>Break</i>	<i>Break</i>	<i>Break</i>
11:30 – 12:30	A. Harrow	A. Harrow	A. Harrow	A. Harrow	A. Harrow
12:30 – 14:00	<i>Break</i>	<i>Break</i>		<i>Break</i>	<i>Break</i>
	14:00 – 15:00 Problem Session	14:00 – 14:40 S. Pérez		14:00 – 14:40 A. Ducuara	14:00 – 15:00 Problem Session
	15:00 – 15:30 <i>Break</i>	14:40-15:20 V. Ballester		14:40-15:20 S. Duque	15:00 – 15:30 <i>Break</i>
	15:30-16:10 I. Guevara	15:20 – 15:50 <i>Break</i>		15:20 – 15:50 <i>Break</i>	15:30-16:10 T. Bogart
	16:10-16:50 C. Susa	15:20 – 15:50 Problem Session		15:20 – 15:50 Problem Session	15:30-16:10 A.. Schlie
		17:30 Poster Session			