Maximazing information flow between coupled qubits under dissipation

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Outline

- Quantum and classical correlations
  - Quantum discord
  - Entanglement of formation
- Koashi-Winter monogamic relations
- Model of the system
- Information flow: correlations dynamics
- Conclusion

Quantum and classical correlations

Classical correlations

Quantum Mutual Information: All types of correlations

$$\mathcal{I}_{AB} = S_A + S_B - S_{AB}.$$

Hamieh, Kobes, & Zaraket, Phys. Rev. A 70, 052325 (2004)

The locally accessible information are the classical correlations

$$\mathcal{J}_{AB}^{\leftarrow} = \max_{\Pi_k} \left[S_A - \sum_k S_{A|k} \right],$$

Henderson & Vedral, J. Phys. A 34, 6899 (2001)

where $S_{A|k} \equiv S(\rho_{A|k})$, with

$$\rho_{A|k} = \frac{1}{p_k} \mathrm{Tr}_B \left(\Pi_k \rho_{AB} \Pi_k \right) \qquad \text{Reduced state A}$$

$$\text{After the outcome } k$$

$$p_k = \mathrm{Tr}_{AB}(\Pi_k \rho_{AB} \Pi_k) \qquad \{\Pi_k\} \ \text{(POVM)}$$

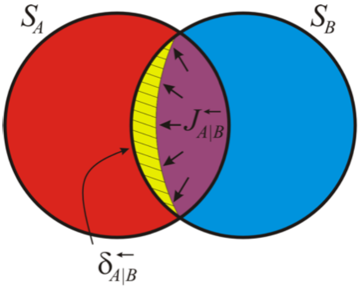
Quantum discord

A measure of the quantumness of correlations

$$\delta_{AB}^{\leftarrow} = \mathcal{I}_{AB} - \mathcal{J}_{AB}^{\leftarrow},$$

Ollivier & Zurek, Phys. Rev. Lett. 88, 017901 (2002)

Mutual information that is not locally accessible.



Fanchini, Castelano, Cornelio & de Oliveira, New J. Phys. 14, 013027 (2012)

Entanglement of formation

$$E_{AB} = \min_{\{p_i, |\psi_i\rangle\}} \sum_i p_i S(\mathrm{Tr}_B(|\psi_i\rangle \langle\psi_i|)).$$

The min is over the $\{p_i, |\psi_i\rangle\}$ satisfying $\sum_i p_i |\psi_i\rangle \langle\psi_i| = \rho_{AB}$

An analytical formula for any arbitrary two-qubit system

$$E_{AB} = h\left(\frac{1+\sqrt{1-(C_{AB})^2}}{2}\right),$$

$h(x) = -x\log_2 x - (1-x)\log_2(1-x)$

Concurrence: $C_{AB} = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$

λ_i's square root of the eigenvalues of the non-Hermitian matrix

$$\rho_{AB}(\sigma_y \otimes \sigma_y)\bar{\rho}_{AB}(\sigma_y \otimes \sigma_y)$$

Wootters, Phys. Rev Lett. 80, 2245 (1998)

Koashi-Winter monogamic relations

The monogamic relations

For pure states

$$E_{AB} = \delta^{\leftarrow}_{AB}.$$

For mixed states?

The Koashi-Winter relation

Theorem: When ρ_{AE} is B-complement of ρ_{AB},

$$E_{AB} + \mathcal{J}^{\leftarrow}_{AE} = S_A$$
$$E_{AE} + \mathcal{J}^{\leftarrow}_{AB} = S_A$$

- ρ_{AE} is the B complement of ρ_{AB} when there exists a tripartite **pure** state ρ_{ABE} such that $\mathrm{Tr}_B(\rho_{ABE}) = \rho_{AE}$ and $\mathrm{Tr}_E(\rho_{ABE}) = \rho_{AB}$.
- The state B complement of ρ_{AB} is unique up to local unitary operations on system E: $\rho_{AE} = (\mathbb{I}_A \otimes U_E)\rho'_{AE}(\mathbb{I}_A \otimes U_E^{\dagger})$.

Koashi & Winter, Phys. Rev. A 69, 022309 (2004)

Proof of the K-W relations

Let $\{p_i, |\psi_i\rangle\}$ be the ensemble achieving: $E_{AB} = \min \sum_i p_i S(\mathrm{Tr}_B(|\psi_i\rangle\langle\psi_i|))$. Since $\sum_i p_i |\psi_i\rangle\langle\psi_i| = \rho_{AB}$, there exists $\{\tilde{M}_i\}$ on E such that, $\{\tilde{M}_i\}$ on ρ_{ABE}, gives the outcome i with p_i, leaving the state of A and B in $|\psi_i\rangle$. If we neglect system B, this implies that $\{\tilde{M}_i\}$ on ρ_{AE} leaves the state of A in $\mathrm{Tr}_\mathrm{B}(|\psi_i\rangle\langle\psi_i|)$. From the definition of $\mathcal{J}^{\leftarrow}$:

$$\begin{aligned}\mathcal{J}^{\leftarrow}_{AE} &\geq S_A - \textstyle\sum_i p_i S(\mathrm{Tr}_B(|\psi_i\rangle\langle\psi_i|)) \\ &= S_A - E_{AB}\end{aligned}$$

Koashi & Winter, Phys. Rev. A 69, 022309 (2004)

$\{M_i\}$ on E that achieves the maximum for $\mathcal{J}^{\leftarrow}$: $\mathcal{J}^{\leftarrow}_{AE} = S_A - \sum p_i S_i$. A decomposition $M_i = \sum_j M_{ij}$ into rank-1 nonnegative operators, gives a new measurement $\{M_{ij}\}$ that satisfies: $p_i = \sum_j p_{ij}$ and $p_i\rho_i = \sum_j p_{ij}\rho_{ij}$. From the concavity of the entropy: $S_A - \sum_{ij} p_{ij} S_{ij} \geq S_A - \sum_i p_i S_i = \mathcal{J}^{\leftarrow}_{AE}$, but the definition of $\mathcal{J}^{\leftarrow}$ leads to the opposite inequality, so we have the equality. For the outcome ij: system AB becomes in a pure state $|\phi_{ij}\rangle$ (since M_{ij} is rank-1). Neglecting B: $\{M_{ij}\}$ on ρ_{AE} leaves A in $\mathrm{Tr}_\mathrm{B}(|\phi_{ij}\rangle\langle\phi_{ij}|) = \rho_{ij}$. Therefore:

$$\begin{aligned}E_{AB} &\leq \textstyle\sum_{ij} p_{ij} S(\mathrm{Tr}_A(|\phi_{ij}\rangle\langle\phi_{ij}|)) \\ &= \textstyle\sum_{ij} p_{ij} S_{ij} = S_A - \mathcal{J}^{\leftarrow}_{AE}\end{aligned}$$

Combining these two results, the Koashi-Winter relation is proved

The monogamic relation between E and $\delta^{\leftarrow}$

Given the Koashi-Winter relation

$$E_{AB} + \mathcal{J}^{\leftarrow}_{AE} = S_A,$$

And by applying the definition of the quantum discord for the system AE

$$E_{AB} - \delta^{\leftarrow}_{AE} = S_{A|E},$$

where $S_{A|E} = S_{AE} - S_E$ is the conditional entropy prior any measure.

Because $S_{A|E} = -S_{A|B}$ for the pure state ρ_{ABE},

$$\delta^{\leftarrow}_{AE} = E_{AB} + S_{A|B}.$$

Fanchini *et al.* Phys. Rev. A 84, 012313, (2011); New J. Phys. 14, 013027 (2012)

In conclusion:

For measures on subsystem B

$$E_{AE} = \delta^{\leftarrow}_{AB} + S_{A|B},$$
$$\delta^{\leftarrow}_{AE} = E_{AB} + S_{A|B}.$$

For measures on subsystem A

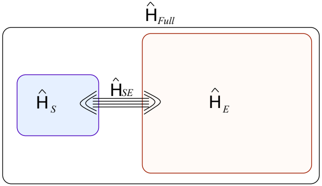
$$E_{BE} = \delta^{\leftarrow}_{BA} + S_{B|A}$$
$$\delta^{\leftarrow}_{BE} = E_{BA} + S_{B|A}$$

Information on the quantum correlation between each subsystem A or B and the environment, without any knowledge of state of E.

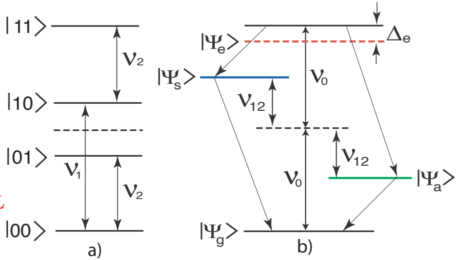
Model of the system

Model for two interacting qubits

- Computational basis $\{|0\rangle, |1\rangle\}$ for two qubits $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$



- Bare energies $\hat{H}_0 = -\frac{1}{2}\nu_1\sigma_1^z - \frac{1}{2}\nu_2\sigma_2^z$. In general, $\nu_1 \neq \nu_2$
- Dipolar coupling V: $\hat{H}_{12} = \frac{V}{2}\left(\sigma_1^x \otimes \sigma_2^x + \sigma_1^y \otimes \sigma_2^y\right)$
- Light-matter interaction: $\hat{H}_L = \ell^{(i)}(\sigma_i^- e^{i\omega_L t} + \sigma_i^+ e^{-i\omega_L t})$
  $\ell_i = -\boldsymbol{\mu}_i \cdot \boldsymbol{E}_i \qquad \Delta_+/2 \equiv \frac{\nu_1+\nu_2}{2} - \nu_L$
- σ^x, σ^y and σ^z are the Pauli matrices.



Susa & Reina, Phys. Rev. A 82, 042102 (2010); Phys. Rev. A 85, 022111

Dissipative Master Equation

- System: $\hat{H} = \hat{H}_0 + \hat{H}_{12} + \hat{H}_L$

$$\begin{aligned}\dot{\hat{\rho}} &= -\frac{i}{\hbar}\big[\hat{H}, \hat{\rho}\big] \\ &\quad - \sum_{i=1}^{2} \frac{\Gamma_i}{2} \left(\hat{\rho}\sigma_i^+\sigma_i^- + \sigma_i^+\sigma_i^-\hat{\rho} - 2\sigma_i^-\hat{\rho}\sigma_i^+\right) \\ &\quad - \sum_{i\neq j} \frac{\Gamma_{ij}}{2} \left(\hat{\rho}\sigma_i^+\sigma_j^- + \sigma_i^+\sigma_j^-\hat{\rho} - 2\sigma_i^-\hat{\rho}\sigma_j^+\right)\end{aligned}$$

- $\sigma_i^+ = |1_i\rangle\langle 0_i|$, and $\sigma_i^- = |0_i\rangle\langle 1_i|$.
- $\Gamma_{ij} = \Gamma_{ji} = \gamma$.

Interaction Strength and Incoherent Decay

$$V = \frac{3}{4}\Gamma\Big(- [\hat{\boldsymbol{\mu}}_1 \cdot \hat{\boldsymbol{\mu}}_2 - (\hat{\boldsymbol{\mu}}_1 \cdot \hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2 \cdot \hat{\mathbf{r}}_{12})]\frac{\cos z}{z} + [\hat{\boldsymbol{\mu}}_1 \cdot \hat{\boldsymbol{\mu}}_2 - 3(\hat{\boldsymbol{\mu}}_1 \cdot \hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2 \cdot \hat{\mathbf{r}}_{12})]\Big[\frac{\cos z}{z^3} + \frac{\sin z}{z^2}\Big]\Big),$$

$$\gamma = \frac{3}{2}\Gamma\Big([\hat{\boldsymbol{\mu}}_1 \cdot \hat{\boldsymbol{\mu}}_2 - (\hat{\boldsymbol{\mu}}_1 \cdot \hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2 \cdot \hat{\mathbf{r}}_{12})]\frac{\sin z}{z} + [\hat{\boldsymbol{\mu}}_1 \cdot \hat{\boldsymbol{\mu}}_2 - 3(\hat{\boldsymbol{\mu}}_1 \cdot \hat{\mathbf{r}}_{12})(\hat{\boldsymbol{\mu}}_2 \cdot \hat{\mathbf{r}}_{12})]\Big[\frac{\cos z}{z^2} - \frac{\sin z}{z^3}\Big]\Big),$$

$z = nk_0 r_{12}$, $k_0 = \frac{\omega_0}{c}$, $\omega_0 = \frac{\omega_1+\omega_2}{2}$

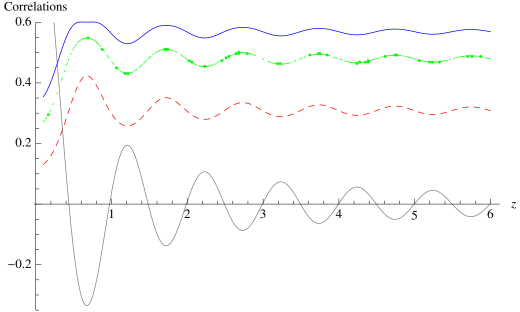
Information flow: correlations dynamics

Our interest of using the Koashi-Winter relations

- How does each subsystem get entangled/correlated with the environment?
- How does this depend on the properties of the qubits system (system AB)?
- How can this be controlled by means of an external field?

Critical distance for the maximum information flow

- Correlations between subsystem A and the environment, even without any knowledge about E
- Identical qubits. Dipoles parallel to each other and perpendicular to the inter-qubit axis.



Berrada, Fanchini & Abdel-Khalek, Phys. Rev. A 85, 052315 (2012)

- Quantum discord AB, quantum discord AE, entanglement AE, collective decay rate γ.
- Interqubit separation $k_0 r$, for $\Gamma t = 1$. Initial state $|\Psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$

Explaining the critical distance

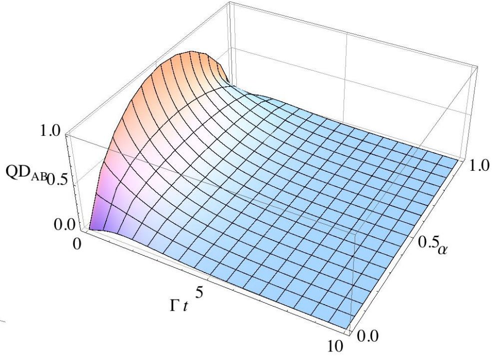
- The collective damping defines the distance R_c.
- The free evolution of the emitters is independent of the interqubit interaction V.
- For the initial states $|\Psi(\alpha)\rangle = \sqrt{\alpha}\,|01\rangle + \sqrt{1-\alpha}\,|10\rangle$:

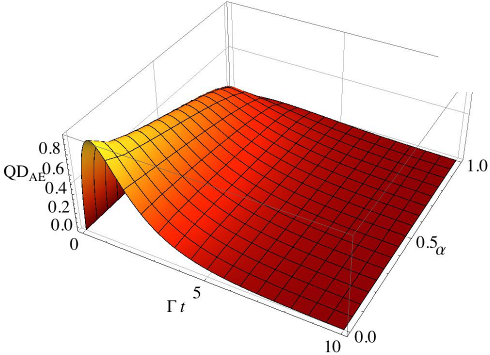
$$
\begin{aligned}
\rho_{00,00}(t) = &\quad 1 - \rho^{+}_{22}(t) - \rho^{-}_{33}(t), \\
\left\{ \begin{array}{c} \rho^{+}_{01,01}(t) \\ \rho^{-}_{10,10}(t) \end{array} \right\} = &\quad \frac{e^{-\Gamma t}}{2}\Big[\cosh(\gamma t) - 2\sqrt{\alpha(1-\alpha)}\sinh(\gamma t) \pm \\
&\quad (2\alpha-1)\cos(2Vt)\Big], \\
\rho_{01,10}(t) = \rho^{*}_{10,01}(t) = &\quad \frac{e^{-\Gamma t}}{2}\Big[2\sqrt{\alpha(1-\alpha)}\cosh(\gamma t) - \sinh(\gamma t) + \\
&\quad \mathrm{i}(2\alpha-1)\sin(2Vt)\Big],
\end{aligned}
$$

- For $\alpha = 1/2$ (past slide) the solution does not depend on V.

Full time evolution for correlations

- Quantum discord AB ($k_0 R_c \sim 0.674$)
- Initial state $|\Psi(\alpha)\rangle$, $0 \leq \alpha \leq 1$
- Asymptotic decay except for $\alpha \to 0$ and $\alpha \to 1$
- Symmetric respect to α

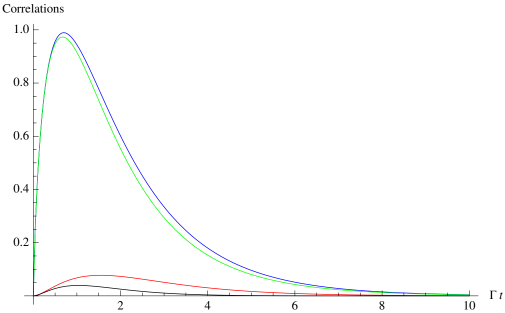




- Quantum discord AE
- Initially uncorrelated as expected
- Asymmetric respect to α
- Maximum reached at $t\Gamma = 1$

Comparing correlations of system AB with those of AE

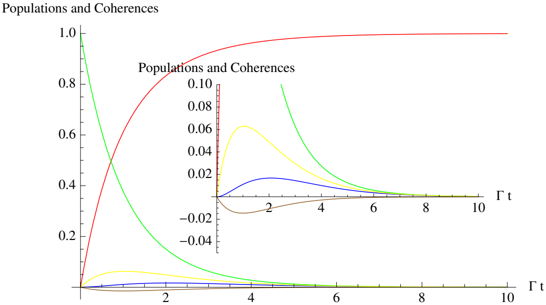
- Entanglement AB, quantum discord AB, quantum discord AE, entanglement AE. For $\alpha = 0$.



- The same behaviour for all the correlations
- System AE reached almost the maximum of correlations ($\to 1$)

Comparing correlations of system AB with those of AE

- $\rho_{00,00}$, $\rho_{10,10}$, $\rho_{01,01}$, $\text{Re}\left[\rho_{01,10}\right]$, $\text{Im}\left[\rho_{01,10}\right]$.



- Initial state $|10\rangle$
- Subsystem B always remains close to its ground state throughout the whole dynamics.
- Most of information about AE is not able to be extracted by measuring locally the system B.

Conclusion

Conclusion

- Correlations between a subsystem of a physical system and its environment may be explore without prior knowledge of the state of the environment.
- Koashi-Winter relations allow to understand the distribution of the entanglement of formation and the quantum discord.
- Quantum correlations of a multipartite system may be interpreted in terms of the locally inaccessible information.
- The information flow may be controlled by means of the properties of the physical system.

Thanks a lot