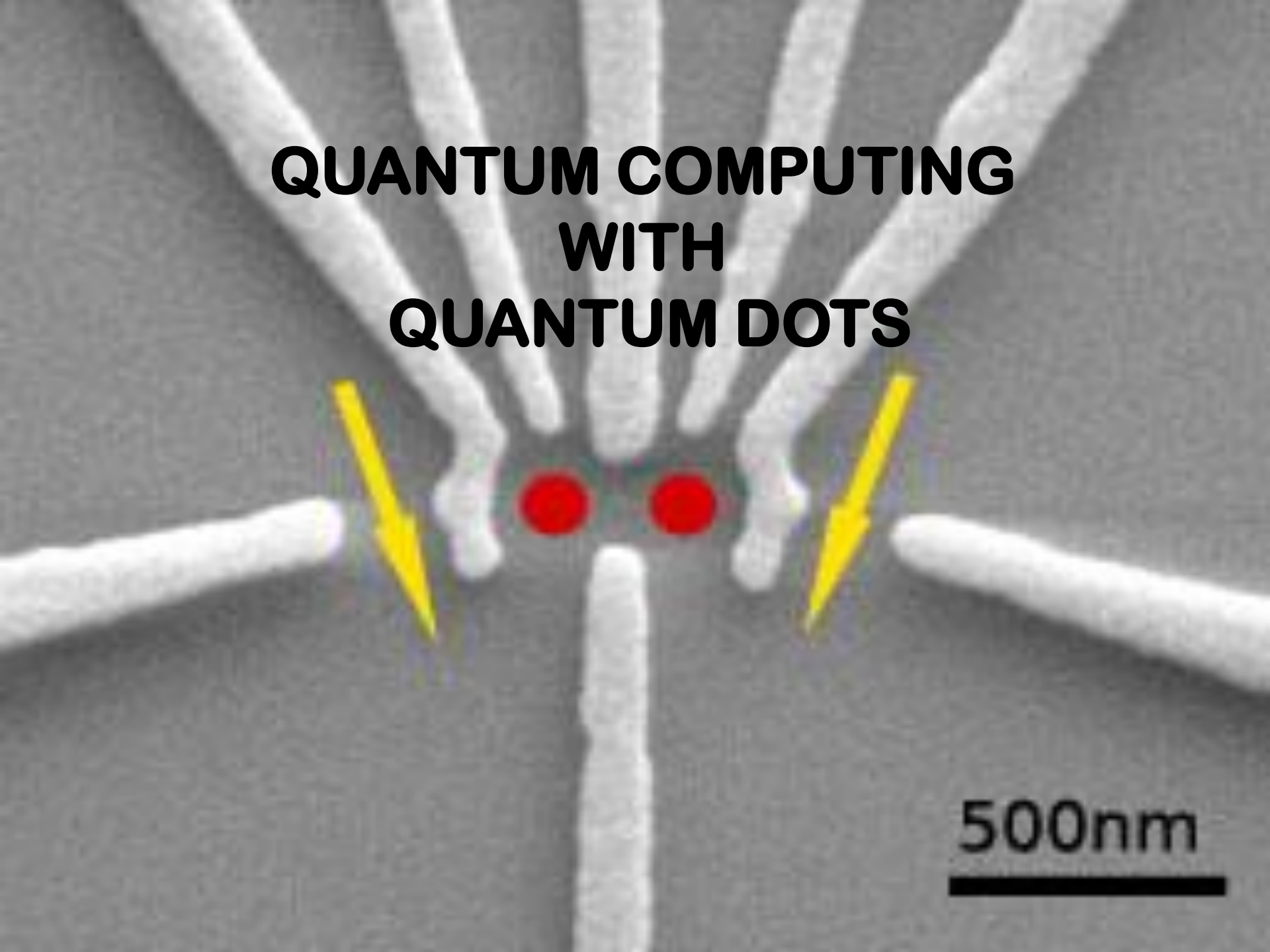
TWO LATERALLY-COUPLED QUANTUM DOTS

RESEARCH PROBLEM

CALCULATE IN AN ANALLITICAL FASHIN THE EXCHANGE ENERGY FOR A SYSTEM OF TWO SIDEWAYS COUPLED QUANTUM DOTS, EACH WITH AN ELECTRON, USING THE HEITLER-LONDON APPROACH

QUANTUM COMPUTING WITH QUANTUM DOTS

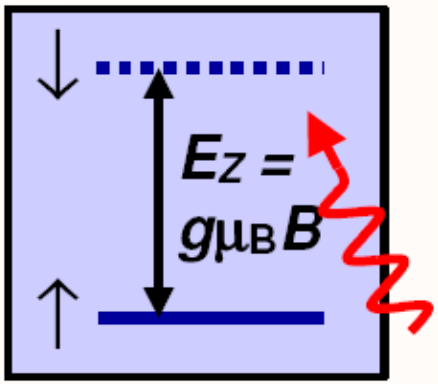


4.1. LOSS AND DIVINCENZO ARCHITECTURE

QUBIT : Electron spin in a quantum dot

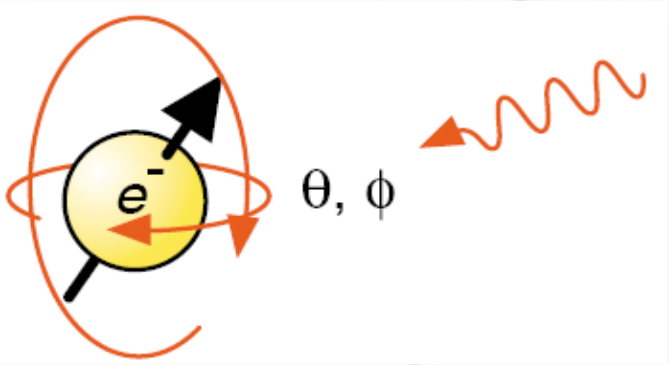
QUBIT : Energy level separation due to the Zeeman effect

Evolution for 1-qubit:



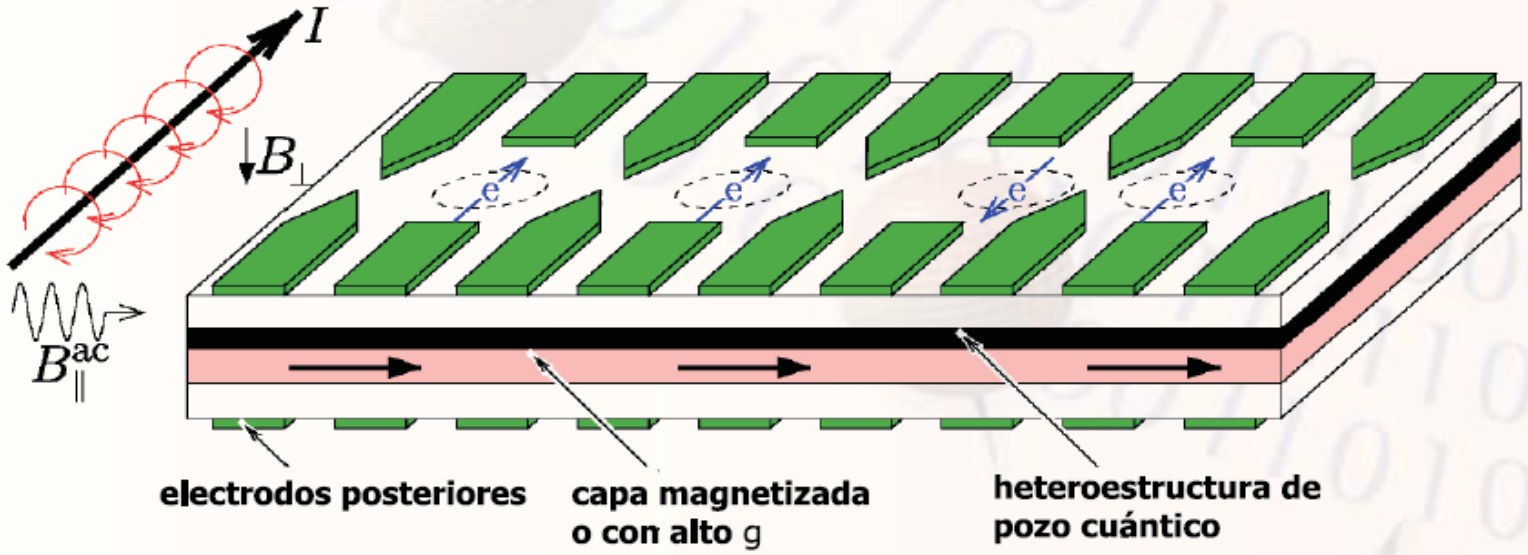
$$\hat{H}_s = \frac{g_s q_e \hbar}{4\mu} \hat{\sigma} \cdot \mathbf{B}$$

Spin rotation



4.1. LOSS AND DIVINCENZO ARCHITECTURE

Evolution for 2-qubit:



$$H_S(t) = J(t)\boldsymbol{S}_1 \cdot \boldsymbol{S}_2$$

System Hamiltonian

4.2. LOSS AND DIVINCENZO ARCHITECTURE

Evolution Operator

$$\hat{U}(t,t_0) = exp(-\tfrac{i}{\hbar}\int_{t_0}^{t} J(t)(\hat{S}_1 \cdot \hat{S}_2\ dt)$$

$$\hat{U}(t,t_0) = e^{-i\tilde{J}\hat{S}_1\cdot\hat{S}_2}$$

where

$$\tilde{J} = \tfrac{1}{\hbar}\int_{t_0}^{t} J(t)dt.$$

4.2. LOSS AND DIVINCENZO ARCHITECTURE

Possible cases

1. **Exchange gate**

$$\tilde{J} = \pi$$

$$\hat{U}_{SW}(i,j) = e^{-i\pi/4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

2. **Exchange gate**

$$\tilde{J} = \frac{\pi}{2}$$

$$\hat{S}(i,j) = \sqrt{\hat{U}_{SW}(i,j)} = e^{-i\pi/8} \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & e^{i\pi/4} & e^{-i\pi/4} & 0 \\ 0 & e^{-i\pi/4} & e^{i\pi/4} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix}.$$

4.2. LOSS AND DIVINCENZO ARCHITECTURE

Possible cases

3. Controlled gates:

$$\hat{U}_{CZ(1,2)} = e^{i\frac{3\pi}{2}\hat{S}_{1z}} e^{i\frac{\pi}{2}\hat{S}_{2z}} \hat{S}(1,2) e^{-i\pi\hat{S}_{1z}} \hat{S}(1,2) = e^{i\pi/4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\hat{U}_{XOR} = e^{-i\frac{\pi}{2}\hat{S}_{1y}} \hat{U}_{CZ(1,2)} e^{i\frac{\pi}{2}\hat{S}_{2y}} = e^{i\pi/4} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -0 \end{pmatrix}$$

4.2. DIVINCENZO CRITERIA

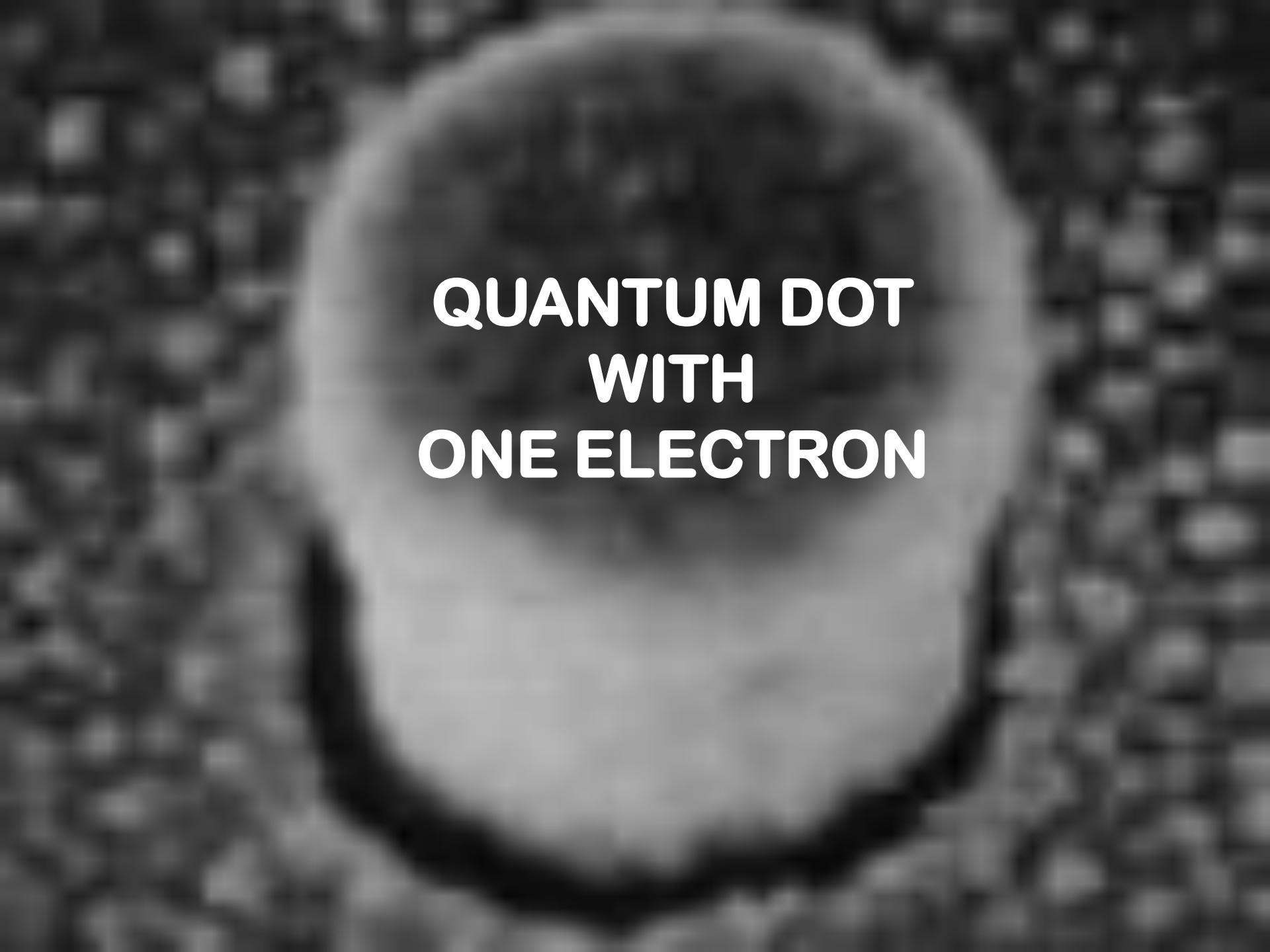
1. A physical system is scalable when the qubits are clearly defined

2. Ability to initialize the state of qubits in an initial single state

3. Decoherence time should be much longer than the operating time of the gate

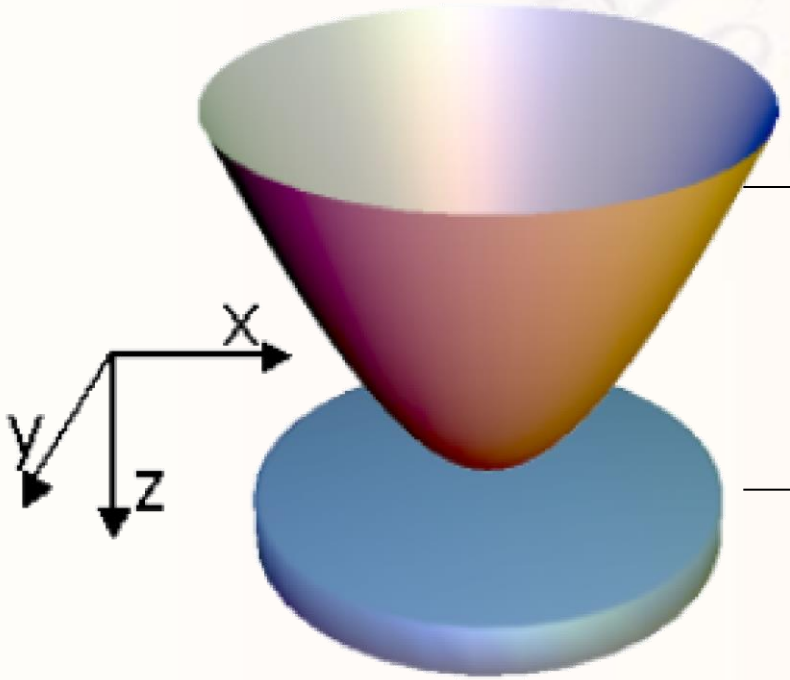
4. A set of universal quantum gates

5. Having the ability to measure a specific qubit


QUANTUM DOT
WITH
ONE ELECTRON

5.1 PHYSICAL SYSTEM FEATURES

Quantum dot of GaAs / AlGaAs with one electron confined inside with a magnetic field B being applied parallel to the z axis



$$\frac{\mu\omega_0^2}{2}\left(\hat{x}^2+\hat{y}^2\right)$$

$$u(\hat{z})$$

Spin-magnetic field B $\longrightarrow$ $\frac{1}{2}g_s\mu\hat{\sigma}\cdot\mathbf{B}$

5.1 PHYSICAL SYSTEM FEATURES

1. Electron free of interactions
2. Harmonic oscillator type confinement potential in the xy plane
3. Infinite potential box type confinement Potential in the z direction
4. The system temperature is T = 0K
5. There is a field perpendicular to the point plane
6. The electrostatic interactions of the atomic nucleus on the electron quantum dot are modeled under the effective mass approximation

5.2 PHYSICAL SYSTEM HAMILTONIAN

$$\hat{H} = \underbrace{\frac{1}{2\mu}\left[\hat{p} + \frac{q_e}{c}\mathbf{A}(\hat{r})\right]^2 + \frac{\mu\omega_0^2}{2}\left(\hat{x}^2 + \hat{y}^2\right) + u(\hat{z})}_{\text{Hamiltonian without spin effect}} - \underbrace{\frac{1}{2}g_s\mu\hat{\sigma}\cdot\mathbf{B}}_{\text{Hamiltonian with spin effect}}$$

$\hat{H}_0$

$\hat{H}_s$

$$\hat{H} = \hat{H}_0 + \hat{H}_s$$

$$\mathbf{A}(\hat{r}) = \frac{B}{2}(-\hat{y}, \hat{x}, 0)$$

Vector Potentials

$$\nabla \cdot \mathbf{A}(\hat{r}) = 0$$

5.3 HAMILTONIAN WITHOUT SPIN EFFECT

$$\hat{H}_0 = \underbrace{\frac{1}{2\mu}\left[\hat{p} + \frac{q_e}{c}\mathbf{A}(\hat{r})\right]^2 + \frac{\mu\omega_0^2}{2}\left(\hat{x}^2 + \hat{y}^2\right)}_{\text{Hamiltonian in the xy plane}} + \underbrace{u(\hat{z})}_{\text{Hamiltonian in z}}$$

$$\hat{H}_\perp \qquad \hat{H}_{||}$$

$$\hat{H}_0 = \hat{H}_\perp + \hat{H}_{||}$$

5.3.1 Hamiltonian in the xy plane

$$\hat{H}_{\perp} = \frac{1}{2\mu}\left[\hat{p} + \frac{q_e}{c}\mathbf{A}(\hat{r})\right]^2 + \frac{\mu\omega_0^2}{2}\left(\hat{x}^2 + \hat{y}^2\right)$$

Considering

$$[\hat{x}, \hat{p_y}] = [\hat{y}, \hat{p_x}] = 0$$

$$\Omega^2 = \omega_0^2 + \frac{1}{4}\omega_c^2$$

$$\hat{L_z} = \hat{x}\hat{p_y} - \hat{y}\hat{p_x} \longrightarrow \text{Angunlar Momentum}$$

$$\omega_c = \frac{q_e B}{c\mu} \longrightarrow \text{Ciclotrinic frecuency}$$

5.3.1 Hamiltonian in the xy plane

$$\hat{H}_{\perp} = \frac{\hat{p_x}^2}{2\mu} + \frac{\hat{p_y}^2}{2\mu} + \frac{1}{2}\mu\Omega^2(\hat{x}^2 + \hat{y}^2) + \frac{1}{2}\omega_c\hat{L_z}$$

To determine the eigenvalues of the Hamiltonian and their associated wave function, we make use of the creation and destruction operators defined

$$\hat{a}^+ = \frac{1}{2}\left[\beta\hat{x} - \frac{i\hat{p_x}}{\beta\hbar}\right] \qquad \hat{b}^+ = \frac{1}{2}\left[\beta\hat{y} - \frac{i\hat{p_y}}{\beta\hbar}\right]$$

$$\hat{a} = \frac{1}{2}\left[\beta\hat{x} + \frac{i\hat{p_x}}{\beta\hbar}\right] \qquad \hat{b} = \frac{1}{2}\left[\beta\hat{y} + \frac{i\hat{p_y}}{\beta\hbar}\right]$$

5.3.1 Hamiltonian in the xy plane

$$\hat{H}_{\perp} = \hbar\omega\left(\hat{a}^{+}\hat{a} + \hat{b}^{+}\hat{b} + 1\right) + \frac{i\omega_c\hbar}{2}\left(\hat{a}\hat{b}^{+} - \hat{a}^{+}\hat{b}\right)$$

To further reduce the system ,two new pairs of operators, which are defined:

$$\hat{B}^{+} = \frac{1}{\sqrt{2}}\left(\hat{a}^{+} - i\hat{b}^{+}\right) \qquad \hat{A}^{+} = \frac{1}{\sqrt{2}}\left(\hat{a}^{+} + i\hat{b}^{+}\right)$$

$$\hat{B} = \frac{1}{\sqrt{2}}\left(\hat{a}^{+} + i\hat{b}^{+}\right) \qquad \hat{A} = \frac{1}{\sqrt{2}}\left(\hat{a}^{+} - i\hat{b}^{+}\right)$$

5.3.1 Hamiltonian in the xy plane

$$\hat{H}_\perp = \hbar\Omega\left(\hat{A}^+\hat{A} + \hat{B}^+\hat{B} + 1\right) + \frac{\omega_c\hbar}{2}\left(\hat{A}^+\hat{A} - \hat{B}^+\hat{B}\right)$$

Noting

$$\hat{A}^+\hat{A} = \hat{N}_A \qquad \hat{B}^+\hat{B} = \hat{N}_B$$

Whose eigenvalues are: n_A, n_B

$$\hat{H}_\perp = \hbar\Omega\left(\hat{N}_A + \hat{N}_B + 1\right) + \frac{\omega_c\hbar}{2}\left(\hat{N}_A - \hat{N}_B\right)$$

5.3.1 Hamiltonian in the xy plane

Taking:

$$\omega_A = \Omega + \tfrac{1}{2}\omega_c \qquad \omega_B = \Omega - \tfrac{1}{2}\omega_c$$

The energy of the system in this plane is:

$$E_\perp = \hbar\omega_A\left(n_A + \tfrac{1}{2}\right) + \hbar\omega_B\left(n_B + \tfrac{1}{2}\right)$$

Conssidering:

$$n = n_A + n_B \qquad m = n_A - n_B$$

That the principal and azimuthal quantum number we have:

$$E_\perp = \hbar\Omega\left(n + 1\right) + \tfrac{1}{2}\omega_c\hbar m$$

5.3.2 Wave function in the xy plane

Similar to the problem for a two-dimensional harmonic oscillator we have:

$$|\varphi_{n_A,n_B}\rangle = \frac{1}{\sqrt{(n_A)!(n_B)!}}(\hat{A}^+)^{n_A}(\hat{B}^+)^{n_B}\,|\varphi_{00}\rangle$$

The fundamental state is dettermined by the relationships:

$$\left(\frac{\beta}{2}z^* + \frac{1}{\beta}\partial_z\right)\psi_{00}(z,z^*) = 0$$

$$\left(\frac{\beta}{2}z + \frac{1}{\beta}\partial_{z^*}\right)\psi_{00}(z,z^*) = 0$$

We have then:

$$|\psi_{00}\rangle = \sqrt{\frac{\beta^2}{\pi}}e^{-\frac{\beta^2}{2}(x^2+y^2)}$$

5.3.3 Hamiltonian in z

This Hamiltonian is considered frequently in introductory courses in quantum mechanics

$$\hat{H}_{||} = \frac{\hat{p_z}^2}{2\mu} + u(\hat{z})$$

Its solutions are

$$|\varphi_{n_z}\rangle = \begin{cases} \sqrt{\frac{2}{L}}\cos(k_n z) k_{n_z} = \frac{n_z \pi}{L}; n_z = 1, 3, 5, ... \\ \sqrt{\frac{2}{L}}\sin(k_n z) k_{n_z} = \frac{n_z \pi}{L}; n_z = 2, 4, 6, ... \end{cases}$$

Whos energies are:

$$\boxed{E_{||} = \frac{n_z^2 \pi^2 \hbar^2}{2\mu L^2}}$$

5.4 HAMILTONIAN WITH SPIN EFFECT

This term represents the external magnetic field interaction with the electron spin

$$\hat{H}_s = -\hat{\mu}_s \cdot \mathbf{B}$$

With:

$$\hat{\mu}_s = -\frac{g_s \mu_B \mathbf{S}}{\hbar}, \mu_B = \frac{q_e \hbar}{2\mu}, \hat{S} = \frac{\hbar}{2}\hat{\sigma}$$

where specifically

$$\sigma_z \left|\varphi_{m_z}\right\rangle = \sigma \left|\pm\right\rangle = \pm \left|\pm\right\rangle = \pm \left|\varphi_{m_z}\right\rangle$$

The energy eigenvalue is

$$\boxed{E_s = \pm \frac{q_e \hbar}{2\mu} B}$$

5.5 RESULTS

The total energy of the system with all interactions is

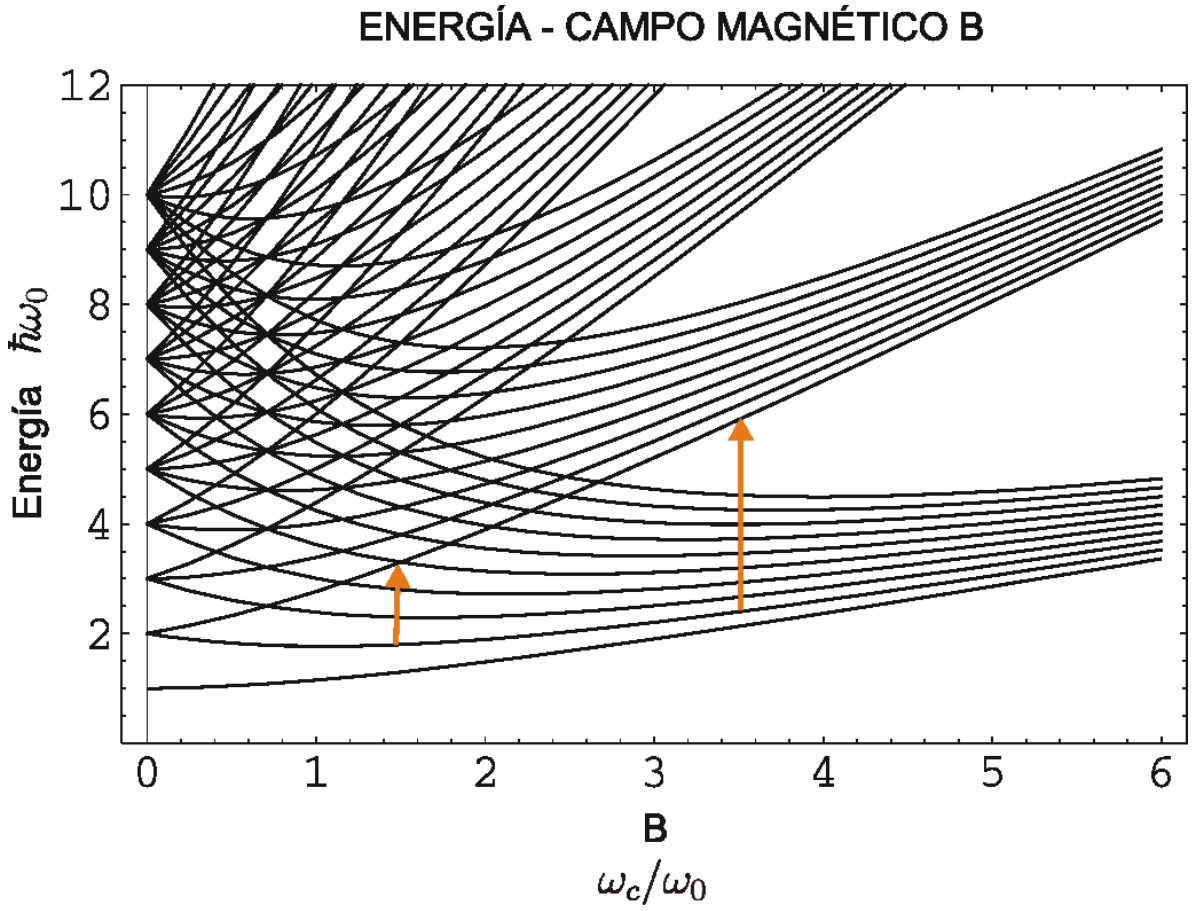
$$E = \hbar\Omega\,(n+1) + \frac{\hbar\omega_c}{2}\,(m) + \frac{n_z^2\pi^2\hbar^2}{2L^2\mu} \pm \frac{q_e\hbar}{2\mu}B.$$

Factorizing the varible the expression ω_0 is obtained in terms of the variable ω_c/ω_0 , which depends exclusively on the magnetic field, therefore the energy in the xy plane is

$$E_\perp = \hbar\omega_0\left(\left[1+\frac{1}{4}\left(\frac{\omega_c}{\omega_0}\right)^2\right]^{\frac{1}{2}}(n+1) + \left(\frac{\omega_c}{\omega_0}\right)\left(\frac{m}{2}\right)\right).$$

Takingω_0 constant we have

5.5 RESULTS

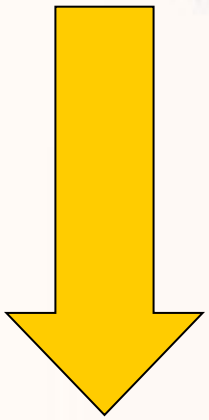


Energy behavior of a constant radius quantum dot under the action of external magnetic field parallel to the z axis in terms of the variable ω_c/ω_0

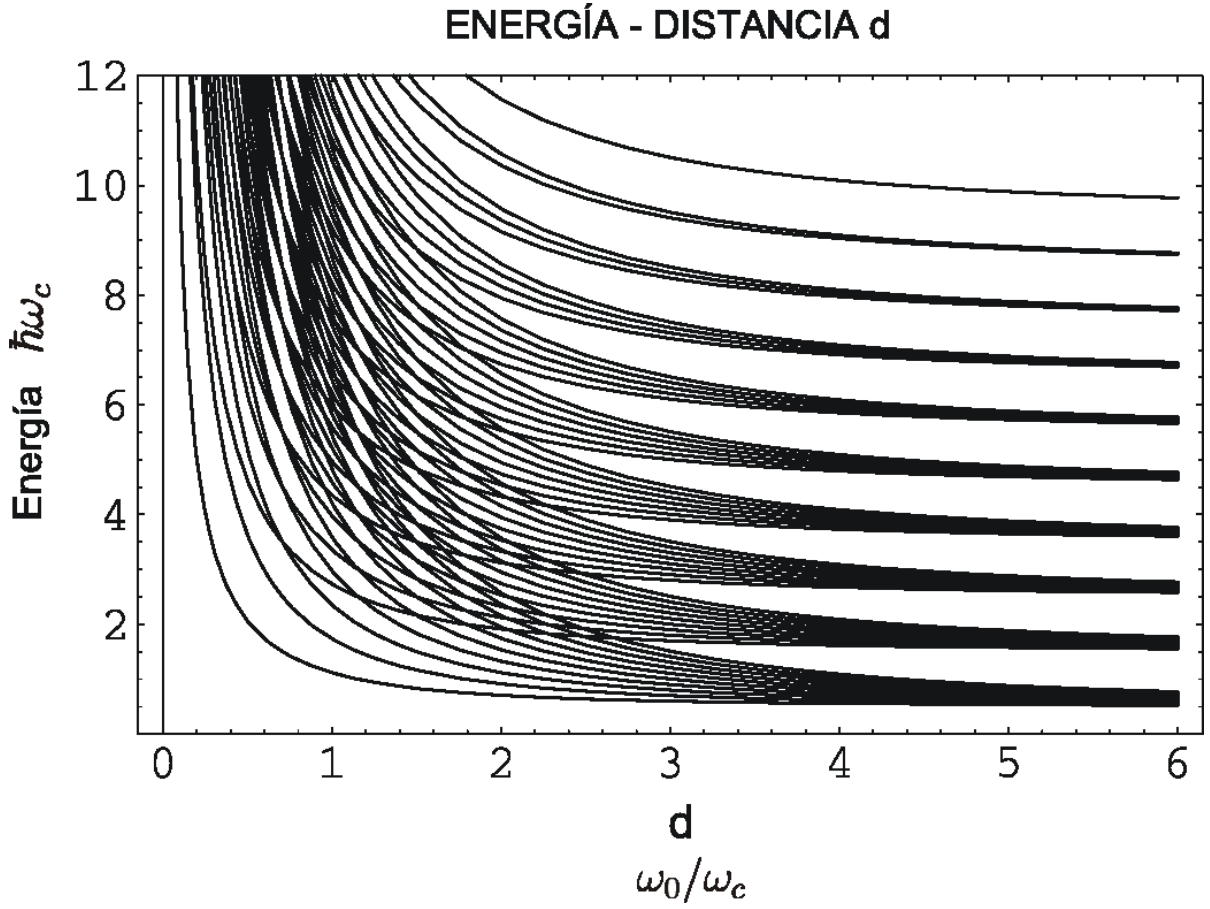
5.5 RESULTS

Factoring now the variable ω_c we have that ω_0/ω_c depends solely on the quantum dot radius, therefore the energy of the system for its xy plane confinement is represented by the equation

$$E_\perp = \hbar\omega_c \left[\left[\left(\frac{\omega_0}{\omega_c} \right)^2 + \frac{1}{4} \right]^{\frac{1}{2}} [n+1] + \frac{m}{2} \right].$$

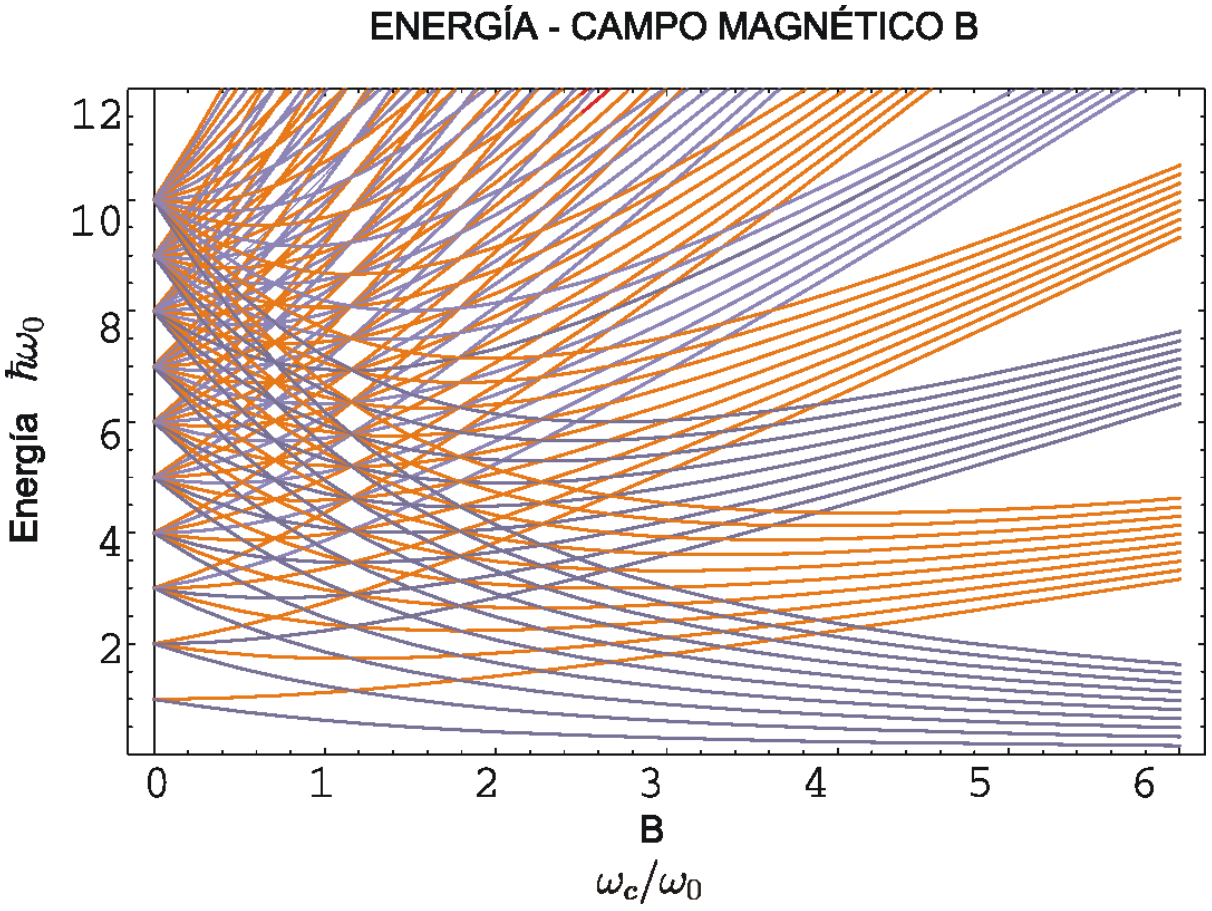


5.5 RESULTS



Energy spectrum for a variable radius quantum dot affected by a constant magnetic field in the z direction

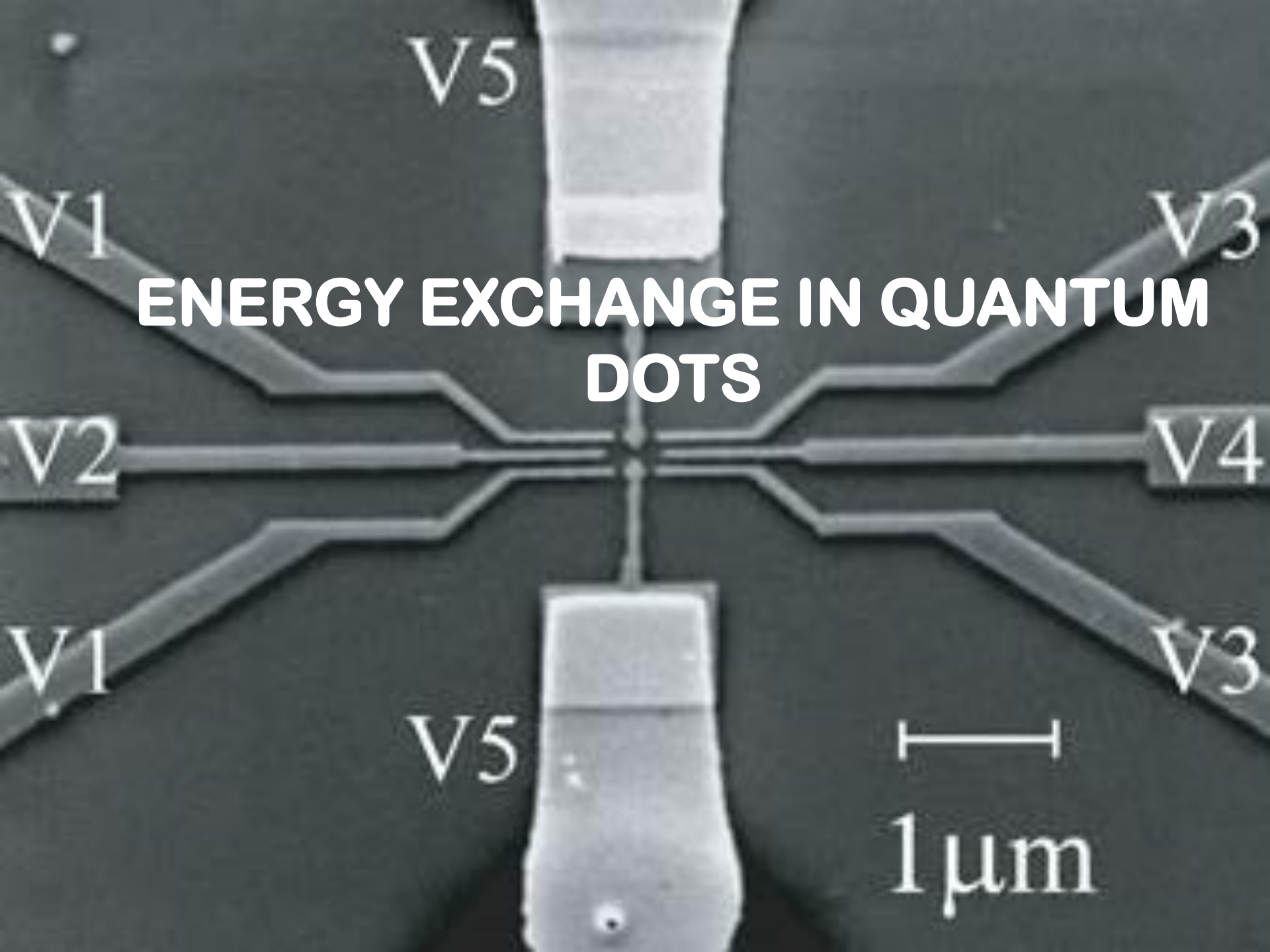
5.5 RESULTS



Fock-Darwin spectrum for a given quantum dot spin effects.

5.6 CONCLUSIONS

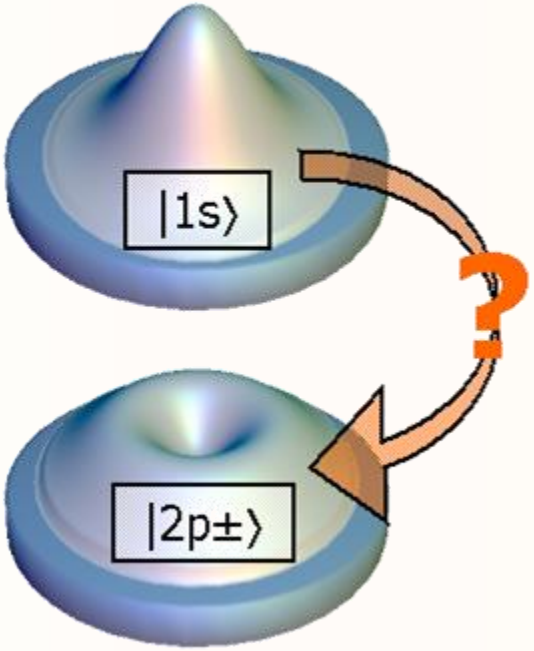
1. Based on the formalism of creation and destruction operators we were able to determine the energy eigenvalues and the general form of the wave functions associated with a GaAs quantum dot
2. The energy of this system without considering the effects of electron spin describes an interesting behavior, the fact that some of the energy levels cross each. This phenomenon, known as Fock-Darwin spectrum presents an important feature that increases with the magnetic field strength $ \ B $, the degeneration and the quantization of energy levels that becomes more representative due to the energy this interaction makes and clearly shows the increase in the kinetic energy of the system Hamiltonian.

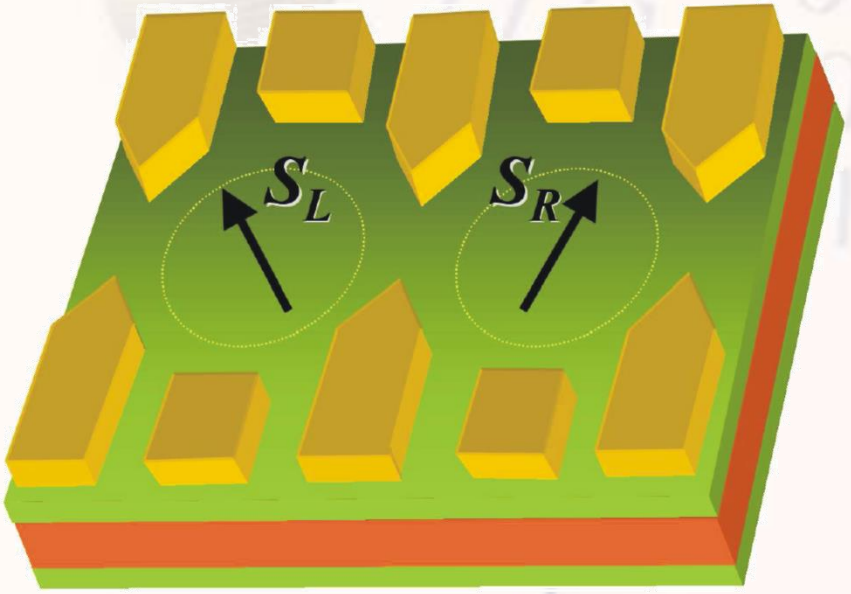


ENERGY EXCHANGE IN QUANTUM DOTS

7.1 PHYSICAL MODEL

The physical system under study is constituted by two GaAs quantum dots coupled laterally, obtained through the technique of 2DEG. Each of these points has an electron confined inside and the spacing between these heterostructures is 2a





7.1 PHYSICAL MODEL

The Hamiltonian describing this system is

$$\hat{H} = \hat{H}_{orb} + \hat{H}_z$$

Because the dynamics of interaction between the electrons confined in each of the quantum dots is performed in the xy plane, this study is limited to consider the Hamiltonian orbital, which is

$$\hat{H}_{orb} = \hat{H}_1(\hat{p}_1, \hat{r}_1) + \hat{H}_2(\hat{p}_2, \hat{r}_2) + C_{12}(\hat{r}_1, \hat{r}_2) + W(\hat{r}_1, \hat{r}_2)$$

donde

$$\hat{H}_1(\hat{p}_1, \hat{r}_1) = \frac{1}{2\mu}\left[\hat{p}_1 - \frac{e}{c}\mathbf{A}(\hat{r_1})\right]^2 + eE\hat{x}_1 + \frac{m\omega_0^2}{2}\left[(\hat{x}_1 + a)^2 + \hat{y}_1^2\right]$$

$$\hat{H}_2(\hat{p}_2, \hat{r}_2) = \frac{1}{2\mu}\left[\hat{p}_2 - \frac{e}{c}\mathbf{A}(\hat{r_2})\right]^2 + eE\hat{x}_2 + \frac{m\omega_0^2}{2}\left[(\hat{x}_2 - a)^2 + \hat{y}_2^2\right]$$

7.1 PHYSICAL MODEL

$$W(\hat{r}_1, \hat{r}_2) = W_1(\hat{r}_1) + W(\hat{r}_2)$$

$$W_j = \frac{\mu\omega_0^2}{2}\left[\frac{1}{4a^2}\left(\hat{x}_j^2 - a^2\right)^2 - \left(\hat{x}_j \pm a\right)^2\right], \qquad j = 1\text{“}+\text{”} \text{ y } \; j = 2\text{“}-\text{”}$$

The two Hamiltonians $\hat{H}_1(\hat{p}_1, \hat{r}_1), \hat{H}_1(\hat{p}_1, \hat{r}_1)$ are basically Fock-Darwin Hamiltonians, for which a translation operators act. Therefore, the Hamiltonian can be expressed as

$$\hat{H}_1 = \hat{H}_{FD1} - \left[q_e E a + \frac{q_e^2 E^2}{2\mu\omega_0^2}\right] \qquad \hat{H}_2 = \hat{H}_{FD2} - \left[\frac{q_e^2 E^2}{2\mu\omega_0^2} - q_e E a\right],$$

$$\hat{\tilde{H}}_i = \frac{1}{2\mu}\left[\hat{p}_i - \frac{q_e}{c}\mathbf{A}(\hat{x}_{\pm i}, \hat{y}_i, 0)\right]^2 + \frac{\mu\omega_0^2}{2}\left[\hat{x}_{\pm i}^2 + \hat{y}_i^2\right]$$

7.1 PHYSICAL MODEL

The wave functions are represented by

$$\phi_0^{(1)}(x_1, y_1) = e^{-\frac{i}{\hbar}\left(\frac{q_e^2 BE}{2\mu\omega_0^2 c} + \frac{q_e Ba}{2c}\right) y_1} \sqrt{\frac{\mu\Omega}{\pi\hbar}}\, e^{-\frac{\mu\Omega}{2\hbar}\left(x_{-1}^2 + y_1^2\right)}$$

$$\phi_0^{(2)}(x_2, y_2) = e^{-\frac{i}{\hbar}\left(\frac{q_e^2 BE}{2\mu\omega_0^2 c} - \frac{q_e Ba}{2c}\right) y_2} \sqrt{\frac{\mu\Omega}{\pi\hbar}}\, e^{-\frac{\mu\Omega}{2\hbar}\left(x_{+2}^2 + y_1^2\right)}$$

These wave functions represent the molecular orbitals of the quantum dot system.

7.2 HEITLER-LONDON METHOD

Technique developed in 1927 by Heitler and London that is used to determine the variation energy when two atoms, each one with an electron (as in the case of hydrogen atoms) are close and form a molecule

Heitler and London questioned the location of the electrons in each of their respective orbitals when the two atoms approach each other, thus constructing two test wave functions, a skew-symmetric and other, this allows the variation energy to be analytically determined with the associated the singlet and triplet states represented by the energy exchange J

7.2 HEITLER-LONDON METHOD

Atomic orbitals have the form

$$A(j) = \phi_0^{(1)}(x_j, y_j) \qquad B(j) = \phi_0^{(2)}(x_j, y_j)$$

The wave function proposed by Heitler and London is of the form

$$\Psi_{\pm} = \frac{|A(1)B(2)\rangle \pm |A(2)B(1)\rangle}{\sqrt{2(1 \pm S^2)}})$$

where

$$S = \left\langle \phi_0^{(2)}(x, y) | \phi_0^{(1)}(x, y) \right\rangle$$

Is the overlap integral system

7.2 HEITLER-LONDON METHOD

With these elements, you can define the exchange energy J as

$$J = \epsilon_t - \epsilon_S = \langle \Psi_- | \hat{H}_{orb} | \Psi_- \rangle - \langle \Psi_+ | \hat{H}_{orb} | \Psi_+ \rangle$$

Where J represents the variation of the expected values of energy between the singlet and triplet states

CALCULATE THIS TERM FOR TWO QUANTUM DOTS IS OUR GOAL!

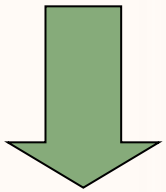
7.3 ENERGY EXCHANGE

The overlap integral is of the form

$$S = \left\langle \phi_0^{(2)}(x,y) | \phi_0^{(1)}(x,y) \right\rangle = e^{-\frac{\mu\Omega a^2}{\hbar} - \frac{q_e^2 B^2 a^2}{4\mu\hbar c^2 \Omega}}$$

and energy exchange is

$$J = \frac{S^2}{1+S^4}\left[\Upsilon_1 - \frac{\Upsilon_2}{S^2} + \Upsilon_3 - \frac{\Upsilon_4}{S^2} + \Upsilon_5\right]$$



7.3 ENERGY EXCHANGE

$$\Upsilon_1 = \langle A(1)|\, H_1\, |A(1)\rangle \langle B(2)|B(2)\rangle + \langle B(2)|\, H_2\, |B(2)\rangle \langle A(1)|A(1)\rangle + \langle B(1)|\, H_1\, |B(1)\rangle \langle A(2)|A(2)\rangle \langle A(2)|\, H_2\, |A(2)\rangle \langle B(1)|B(1)\rangle$$

$$\Upsilon_2 = \langle A(1)|\, H_1\, |B(1)\rangle \langle B(2)|A(2)\rangle + \langle B(2)|\, H_2\, |A(2)\rangle \langle A(1)|B(1)\rangle + \langle B(1)|\, H_1\, |A(1)\rangle \langle A(2)|B(2)\rangle \langle A(2)|\, H_2\, |B(2)\rangle \langle B(1)|A(1)\rangle\,,$$

$$\Upsilon_3 = \langle A(1)B(2)|\, V_{12}\, |A(1)B(2)\rangle + \langle A(2)B(1)|\, V_{12}\, |A(2)B(1)\rangle\,,$$

$$\Upsilon_4 = \langle A(1)B(2)|\, V_{12}\, |A(2)B(1)\rangle + \langle A(2)B(1)|\, V_{12}\, |A(1)B(2)\rangle\,,$$

$$\Upsilon_5 = \langle A(1)B(2)|\, W_1 + W_2\, |A(1)B(2)\rangle + \langle A(2)B(1)|\, W_1 + W_2\, |A(2)B(1)\rangle - \tfrac{1}{S^2}\left[\langle A(2)B(1)|\, W_1 + W_2\, |A(1)B(2)\rangle + \langle A(1)B(2)|\, W_1 + W_2\, |A(2)B(1)\rangle\right]$$

7.3 ENERGY EXCHANGE

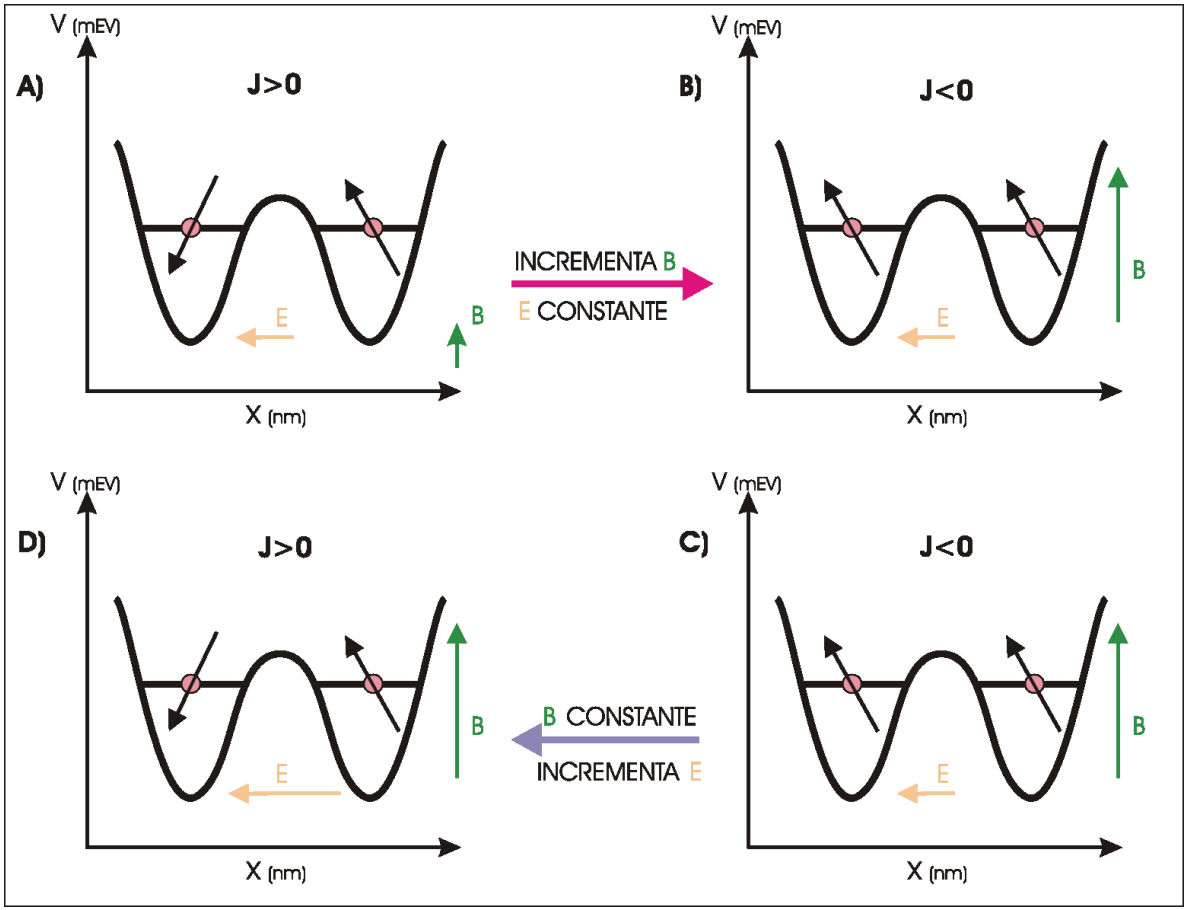
Finally we get an analytical energy exchange term

$$J = \frac{\hbar\omega_0}{\sinh\left[2d^2\left(2b-\frac{1}{b}\right)\right]}\left[C\sqrt{b}\left\{e^{-bd^2}\mathbf{I}_0\left(bd^2\right) - e^{d^2\left(b-\frac{1}{b}\right)}\mathbf{I}_0\left(d^2\left[b-\frac{1}{b}\right]\right)\right\}\right.$$
$$\left. + \frac{3}{4b}\left(1+bd^2\right) + \frac{3}{2}\frac{1}{d^2}\left(\frac{q_e E a}{\hbar\omega_0}\right)^2\right]$$

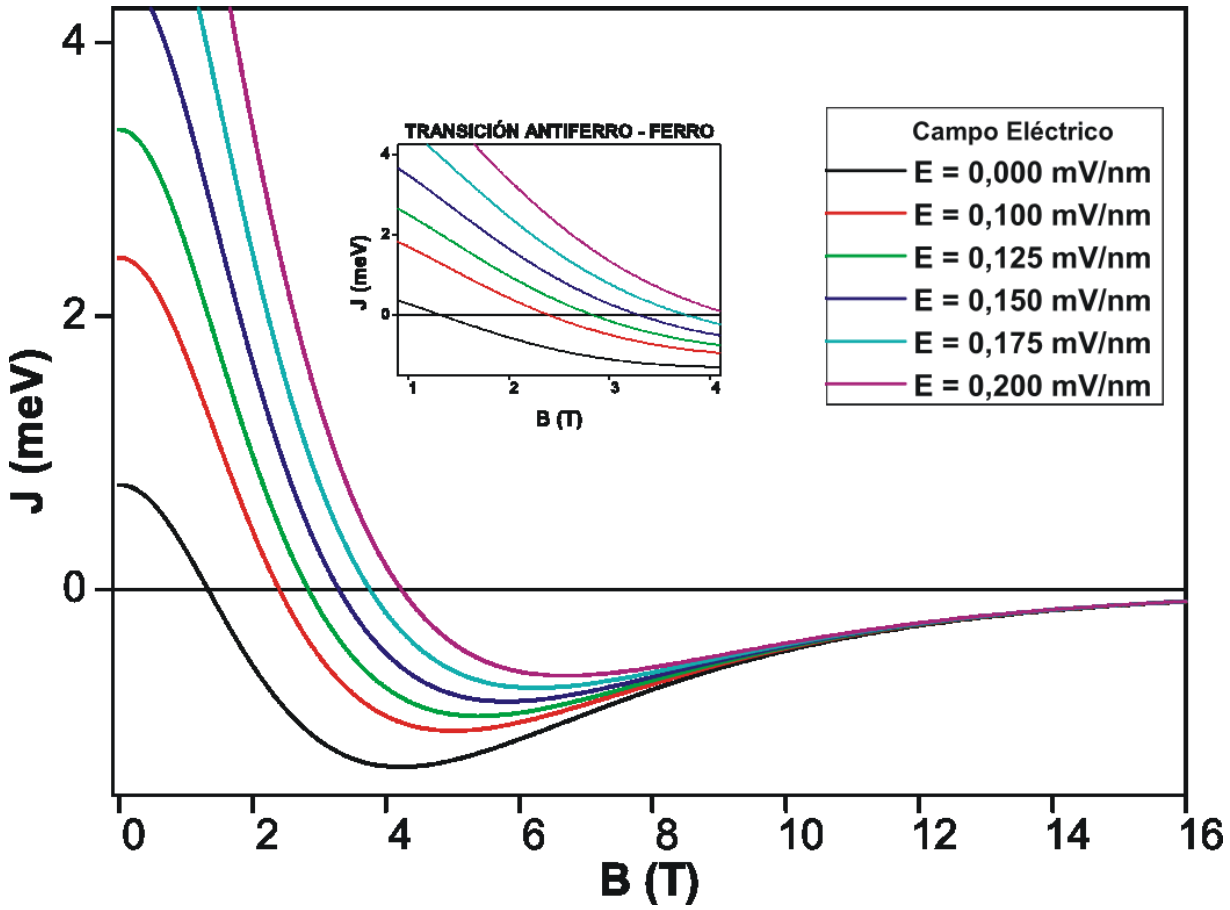
where

$$d = a\sqrt{\mu\omega_0/\hbar},\ b = \frac{\Omega}{\omega_0}, C = \frac{q_e^2}{\kappa}\frac{1}{\hbar\omega_0}\sqrt{\left(\frac{\pi\mu\omega_0}{2\hbar}\right)},$$

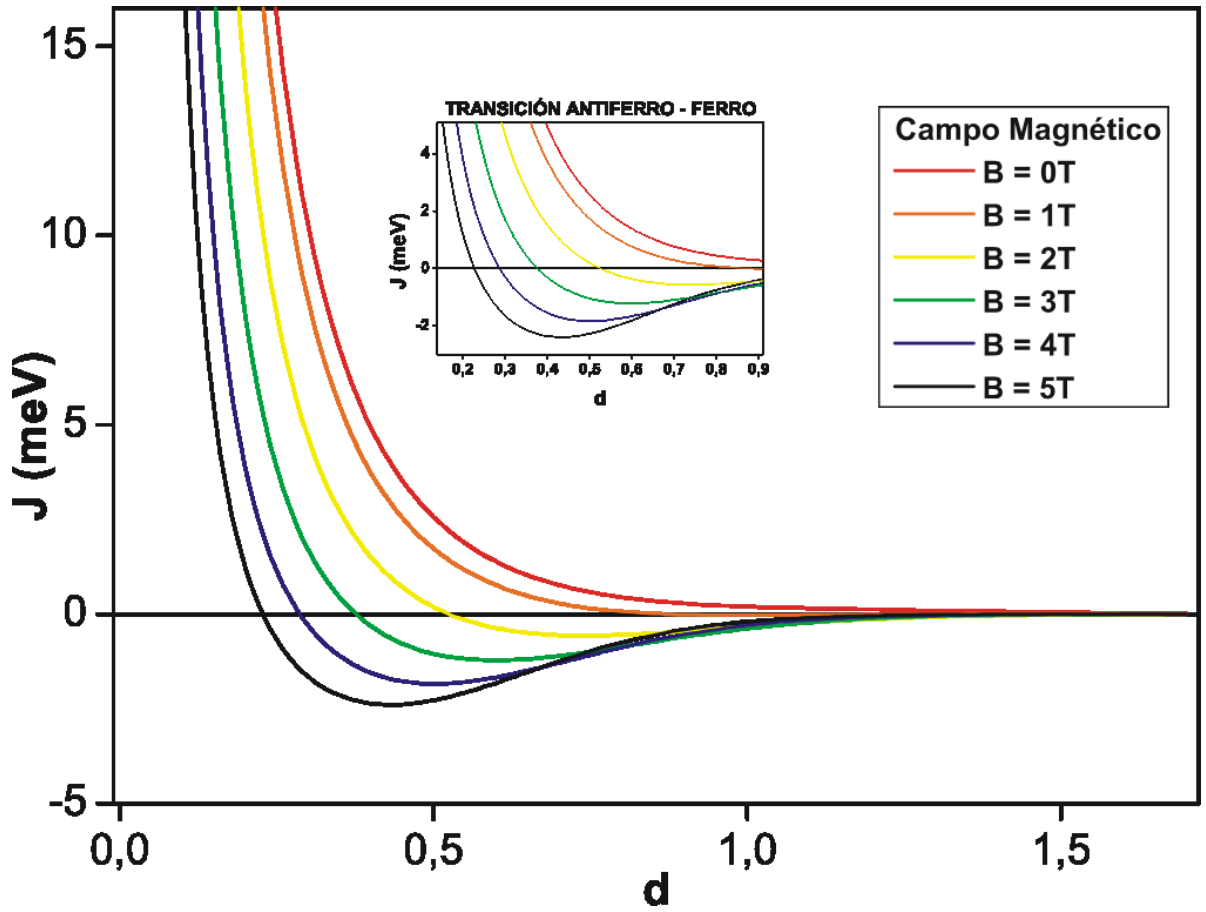
7.4 RESULTS



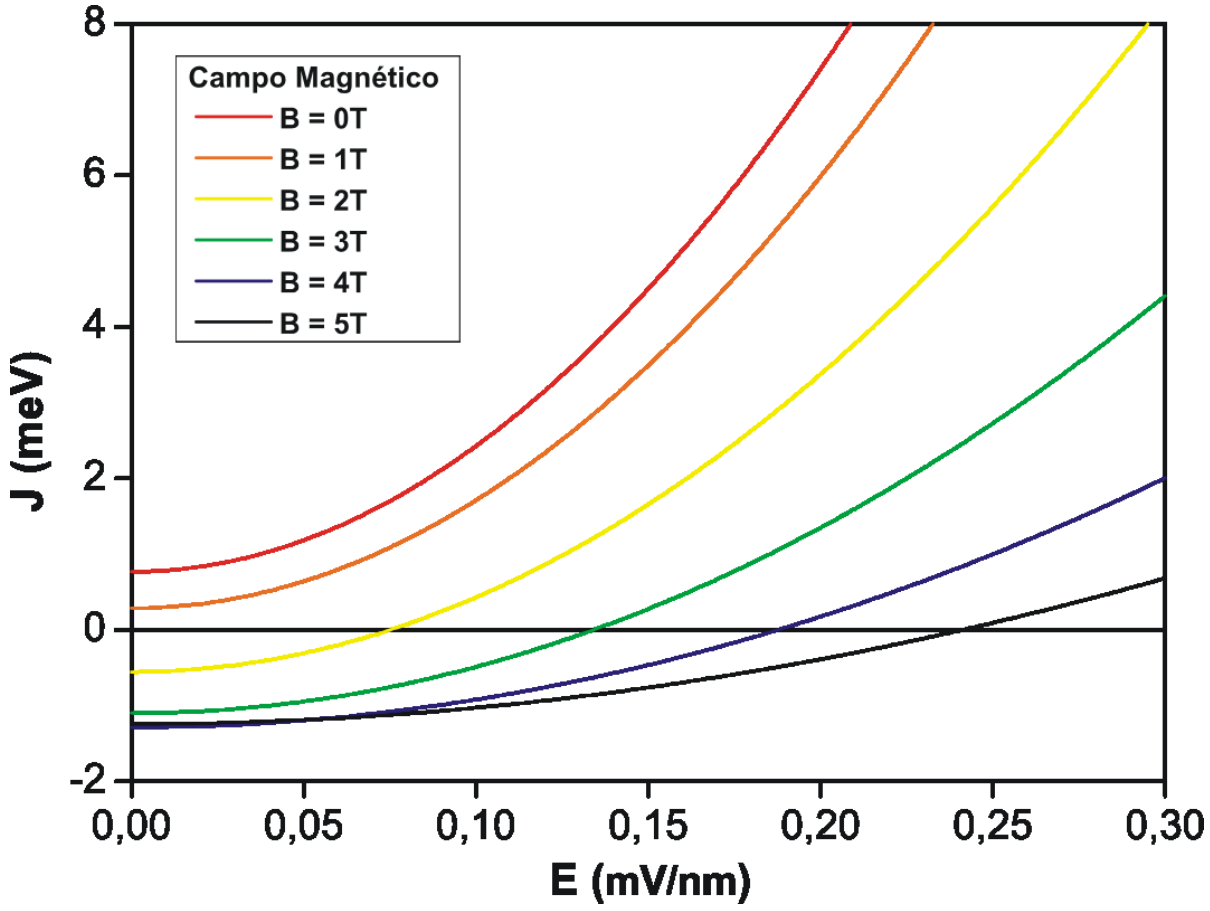
Switching scheme proposed in this work for a two laterally coupled quantum dots, operating as a quantum gate.



Exchange energy J(B) for various electric fields E



Exchange energy J(d) for various electric fields E



Transition energy exchange J (E) for various magnetic fields B

7.5 CONCLUSIONS

It is explicitly described the whole process to calculate the energy exchange J(B, E, A) between electron spins of a two laterally coupled quantum dots (each containing an electron) as a function of electric field and external magnetic applied to the system as well as the distance between points, using for this approximate Heitler-London.

It is shown that J(B, E, d) changes sign with increasing field before disappearing B exponentially.

A similar trend occurs when upon the system varies the electric field E, keeping constant B and d. In such a situation the transition energy exchange J(E) is opposite to that observed in J(B).

This particular feature, in which it is possible to produce a sign change in J (E) has not been reported so far.

8. GENERAL CONCLUSIONS

In this work, modeling is done of the physical behavior of a quantum dot with one electron under the influence of a magnetic field parallel to the z axis. The description of this system is carried out using the formalism of creation and destruction operators, which allows the a transparent, and quick calculation of theenergy levels as well as the general form of the wave functions for each of the possible states of system.

For the system of a quantum dot on an external magnetic field B, energy spectrum is obtained which, besides having a strong dependence of B, which results in the presence of Landau levels, is closely related to the dimensions of the point quantum, in particular to its radius. This strong dependence is a typical feature of quantum dots, in which the decrease in radius produces a quantization of energy.

8. GENERAL CONCLUSIONS

Due to the effect of an external magnetic field B on the quantum dot, appears a coupling between this factor and the magnetic dipole moment of the electron spin. Such an interaction, known as anomalous Zeeman effect, describes a system of two distinct energy states, which clearly can operate as a quantum gate 1-qubit rotation and which can be controlled externally. Thus experimental investigations in this field over the past six years have achieved a high technical and technological development, allowing for a glimpse of the near future that such gates are realized.

Is accomplished through perturbation theory to determinate spin orbit interaction for an electron quantum dot, under the action of an external magnetic field in the z direction, obtaining a term generally describes the behavior for any power level nondegenerate.

8. GENERAL CONCLUSIONS

The energetic effect the spin-orbit interaction on the system has on a quantum dot with an electron under the interaction of a magnetic field B, provides a contrast ratio of seven orders of magnitude with respect to the lower energy level, which physically means that the effect of this amount is very small. In the present study and the encouragement to consider a very simple model two laterally coupled quantum dots, each with an electron, the effect of spin-orbit coupling is not taken into account for the calculation of the energy exchange.

It described the physical and mathematical formalism that allows to obtain an analytical term representing energy exchange J for a two laterally coupled quantum dots of GaAs, each confining an electron.

8. GENERAL CONCLUSIONS

The J term which represents the energy exchange depends explicitly of three parameters that can be controlled externally, such as the magnetic field B in the z direction, the electric field E parallel to the x-axis separation distance between the two quantum dots.

The possibility of creating a change in the sign of the energy exchange through an electric field E, is an additional method to those described and reported in the literature, allowing explore a new work area so far not experimentally has exploded.

We present a scheme change in the parameter J, which allows a quantum gate based on quantum dots reach its initial state after carrying out an operation without the need to suppress the interactions of magnetic fields B and electric E on the system.

8. GENERAL CONCLUSIONS

This description of the J factor depends on variables that can be controlled experimentally found taking architecture DiVincenzo Loss and opens the possibility of using laterally coupled quantum dots and quantum gate devices.

Although the description of this scheme was carried out for two coupled GaAs quantum dots laterally, their study can be generalized to many-body systems as well as vertically coupled quantum dots, atoms coupled to a Bravais lattice, overlapping shallow donors in silicon between others, allowing you to explore other forms of implementation for a quantum mechanical computing device.