The Mathematics of Entanglement
2013 School in Mathematical Physics
Universidad de los Andes Bogotá - Colombia May 27th-31st

RELATIONSHIPS BETWEEN QUANTUM CORRELATIONS AND PHASE TRANSITIONS IN AN EXCITON-POLARITON BEC CONFINED IN A NANOCAVITY

GUILLERMO ALONSO GUIRALES [a]- BORIS ANGHELO RODRÍGUEZ[a]-HERBERT VINCK POSADA[b]
[a] GRUPO DE FÍSICA ATÓMICA Y MOLECULAR, UNIVERSIDAD DE ANTIOQUIA, MEDELLÍN
[b] DEPARTAMANETO DE FÍSICA, UNIVERSIDAD NACIONAL DE COLOMBIA,BOGOTÁ
UNIVERSIDAD DE ANTIOQUIA
1 8 0 3

ABSTRACT

Recently, the two-dimensional nanocavity exciton-polariton system confined in semiconductor nanocavities has emerged as a promising alternative of Bose-Einstein Condensate (BEC). The strong quantum correlations presents in this system, make it an unique candidate for studying entanglement and quantum phase transitions —qualitative changes in the ground state properties—. In this work, we review previous evidence, obtained by us, in the phase transitions of a finite exciton-polariton system. First, by using a BCS wave function to compute the ground state energy of N excitons without the photonic field, a crossover from the high-density electron-hole phase to the BCS excitonic phase is found, at a density which is roughly four times the close-packing density of excitons. Second, by means of a self-consistent procedure with a trial function composed of a coherent photon field and a BCS function for the electron-hole pairs, we obtain the scaling of the critical temperature with the number of polaritons. Using the method proposed originally by Olliver and Zurek, and subsequently developed by Bo Li and Zhi-Xi Wang , we calculate the discord for a generalized density operator, proposed by Blaizot

THEORETICAL MODEL

N interacting excitons confined in a parabolic quantum dot and interacting with a quantized field mode in a nanocavity are described by the Hamiltonian [1] $H = H_x + H_\gamma + H_{int}$, where:

$$\begin{aligned} H_x &= \sum_n (E_n^{(e)} e_n^\dagger e_n + E_{\bar n}^{(h)} h_{\bar n}^\dagger h_{\bar n}) + \sum_{nm} (t_{nm}^{(e)} e_n^\dagger e_m + t_{\bar n \bar m}^{(h)} h_{\bar n}^\dagger h_{\bar m}) + \\ & \frac{\beta}{2} (\sum_{nmlk} (C_{lk}^{nm} e_n^\dagger e_m^\dagger e_k e_l + C_{lk}^{nm} h_{\bar n}^\dagger h_{\bar m}^\dagger h_{\bar k} h_{\bar l})) - \\ & \beta \sum_{nmlk} (C_{lk}^{n\bar m} e_n^\dagger h_{\bar m}^\dagger h_{\bar k} e_l); \\ H_\gamma &= \hbar\omega a^\dagger a; \qquad H_{int} = \hbar g \sum_n (e_n h_{\bar n} a^\dagger + e_n^\dagger h_{\bar n}^\dagger a). \end{aligned}$$

Describing the ground state of the system by a BCS wave function for the matter and a coherent state for the photonic field

$$|\alpha, \nu\rangle = |\alpha\rangle_\gamma \otimes |BCS\rangle_x = \exp(\alpha a^\dagger + \alpha^* a)|0\rangle_\gamma \otimes \prod_n (u_n + \nu_n e_n^\dagger h_{\bar n}^\dagger)|0\rangle_x,$$

we calculated the ground state energy $\tilde E = E - \mu N_p$, (by using the Wick theorem):

$$\begin{aligned} E &= \langle H \rangle = \sum_n E_n^{(x)} \nu_n^2 - \\ & \beta \sum_{nm} C_{mn}^{nm} (2\nu_n^2 \nu_m^2 + u_n \nu_n u_m \nu_m) - \hbar g (\alpha + \alpha^*) \sum_n u_n \nu_n + \hbar\omega |\alpha|^2 \end{aligned}$$

With a self-consistent method [1, 2] is possible to find numerically all parameters. The most important are gaps, Hartree-Fock energies, and occupations:

$$\Delta_k = 2\beta \sum_n C_{nk}^{kn} u_n \nu_n + \hbar g \alpha \qquad E_k^{HF} = E_k^{(x)} - 2\beta \sum_n C_{nk}^{kn} \nu_n^2$$
$$\nu_k^2 = \frac{1}{2}\left(1 - \frac{E_k^{HF} - \mu}{\sqrt{\Delta_k^2 + (E_k^{HF} - \mu)^2}}\right)$$

DENSITY AND GENERALIZED DENSITY OPERATORS

The density operator has been used to measure correlations between states but it is more complicated in systems with a big number of particles. However, in a BCS state is possible to calculate the density operator by pairs. Also, it is possible to introduce a the density generalized operator of a particle R, where [3]:

$$\widetilde{R} = \vec{\alpha}^\dagger \otimes \vec{\alpha},$$

with $\vec\alpha$ a column vector $\vec\alpha = \begin{pmatrix} \vec a \\ \vec a^\dagger \end{pmatrix}$ and $\vec a$ a vector of destruction operators in second quantization. Hence, we define $\vec a = \begin{pmatrix} \vec e \\ \vec h \end{pmatrix}$ with $\vec e$ a n dimension vector of electron destruction operator and $\vec h$ a n dimension vector of hole destruction. operator

Discord is defined like $\mathcal{Q}(\rho) = \mathcal{I}(\rho) - \mathcal{C}(\rho)$ or $\mathcal{Q}(\rho) = S(\rho^B) + \sum_i \lambda_i \log_2(\lambda_i) + \min_{\{B_i\}} S(\rho|\{B_i\})$, where $S(\rho)$ is the von Neumann entropy, λ_i are the eigenvalues and $S(\rho|\{B_i\})$ is the conditional entropy [4,5]

Density operator	State	Negativity	Discord
$\begin{pmatrix} u^2 & 0 & 0 & uv \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ uv & 0 & 0 & v^2 \end{pmatrix}$	Pure	uv	$1 - \left(u^2 \log_2(u^2) + v^2 \log_2(v^2)\right)$
$\begin{pmatrix} \{v_i^2 \delta_{ij}\} & 0 & 0 & \{u_i v_i \delta ij\} \\ 0 & \{v_i^2 \delta_{ij}\} & \{-u_j v_j \delta_{ij}\} & 0 \\ 0 & \{-u_i v_i \delta_{ij}\} & \{u_i^2 \delta_{ij}\} & 0 \\ \{u_j v_j \delta ij\} & 0 & 0 & \{u_i^2 \delta_{ij}\} \end{pmatrix}$	Mixed	0	$1 - \left(u^2 \log_2(u^2) + v^2 \log_2(v^2)\right)$

SELF-CONSISTENT ALGORITHM

1. Put seed for Δ_k^0 , $E_k^{HF(0)}$
2. Calculate $\mu^{(0)}$ and $\Delta^{(0)}$, with:

$$(\hbar\omega - \mu)\alpha = \hbar g \sum_n \frac{\Delta_k^2}{\sqrt{\Delta_k^2 + (E_k^{HF} - \mu)^2}}$$

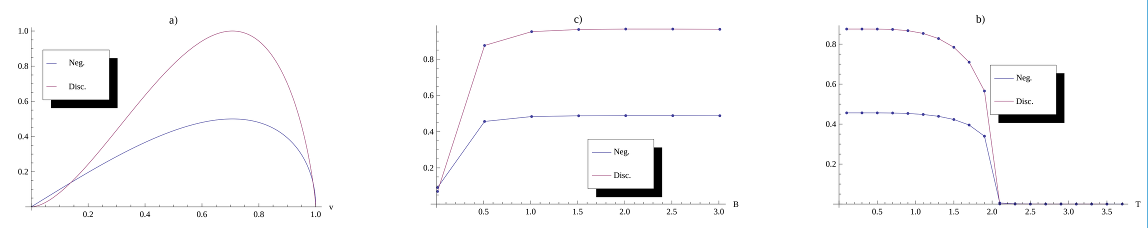
$$N_P = \alpha^2 + \sum_k 1 - \frac{E_k^{HF} - \mu}{\sqrt{\Delta_k^2 + (E_k^{HF} - \mu)^2}}$$

3. Calculate $\nu^{(1)}$ and $\mu^{(1)}$, using:

$$\nu_k^2 = \frac{1}{2}\left(1 - \frac{E_k^{HF} - \mu}{\sqrt{\Delta_k^2 + (E_k^{HF} - \mu)^2}}\right)$$

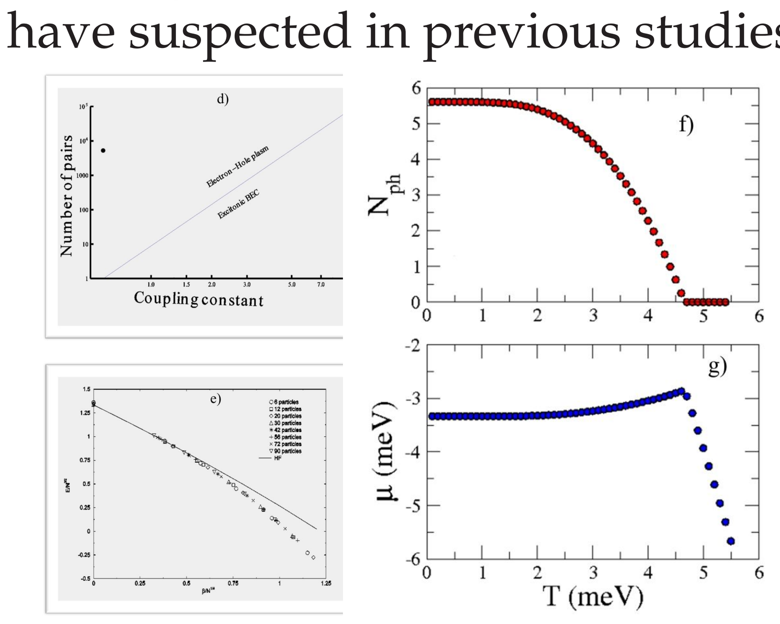
4. Restart the algorithm with $\Delta_k^{(1)}$, $E_k^{HF(2)}$ and stop when the convergence is achieved.

PARTIAL RESULTS



In a BCS system the negativity and the discord of a pair show the same behavior; it may means that the entanglement is the most relevant quantity among all correlations in the system, as it is show in the figure (a).

On the other hand, the figures (b) and (c) show a clear signature of a phase of a phase transition in the quantum correlations of the system, as we have suspected in previous studies [1,2]



d) Crossover between the electron-hole plasma and the BCS excitonic phase. The line correspond to the critical coupling constant which is a change of phase. e) Scaling of the ground state energy as a function of the coupling constant. Note that while the number of particles is increased, there is a departure from the Hartree-Fock energy. f) and g) Number of photons and chemical potential as a function of the temperature, it can be seen that there is a critical temperature for the phase transition [1].

REFERENCES

[1] H. Vinck-Posada, B. A. Rodriguez, A. Gonzalez, Superlatt. and Microstruc. **43**, 500 (2008).
[2] B. A. Rodriguez, A. Gonzalez, *et al.* Int. J Mod. Phys. B, **14**, 71 (2000).
[3] J.-P. Blaizot and G. Ripka, Quantum Theory of Finite Systems, The MIT Press, 1985
[4] H. Ollivier and W. H. Zurek, Phys. Rev. Lett.**88**, 017901(2001).
[5] B. Li, Z. Wang and S. Fei, Phys. Rev. A. **83**, 022321 (2011).