Escuela de Física-Matemática 2012

Quantum Integrable Systems

Departamento de Matemáticas
Departamento de Física
Universidad de los Andes

28 Mayo – 1 Junio 2012

Lunes, 28 de Mayo (Salón: SD-805)	
8:00 – 9:00	*Registration*
9:00 – 9:15	*Opening*
9:15 – 10:15	Vladimir Korepin: *Quantum non-linear Schrödinger equation*
10:15 – 11:15	Ed Corrigan: *An introduction to integrable classical and quantum field theory in two dimensions*
11:15 – 11:45	Break
11:45 – 12:45	Andreas Klümper: *Thermodynamics of Spin Chains*
12:45 – 14:00	Lunch Break
Salón B-202	
14:00 – 14:35	Andrés Ángel: *Quantum cobordism and moduli spaces of curves*
14:35 – 15:10	Gabriel Téllez: *Expanded Vandermonde powers: applications to the two-dimensional one-component plasma*
15:10 – 15:45	Alexander Cardona: *Integrable Systems, Dirac structures and quantization*

Martes, 29 de Mayo (Salón: SD-805)	
9:00 – 10:00	Vladimir Korepin: *Quantum non-linear Schrödinger equation*
10:00 – 11:00	Ed Corrigan: *An introduction to integrable classical and quantum field theory in two dimensions*
11:00 – 11:30	Break
11:30 – 12:30	Andreas Klümper: *Thermodynamics of Spin Chains*
12:30 – 14:00	Lunch Break
Salón B-202	
14:00 – 14:35	Servio Pérez: *Density Matrix Renormalization Group (DMRG)*
14:35 – 15:10	Juliana Restrepo: *Decoherence and relaxation in quantum systems*
15:10 – 15:45	Otto Rendón: *Two - particle scattering by a finite interacting region: In the T- matrix approach*

Miercoles, 30 de Mayo (Salón: SD-805)

9:00 – 10:00	Vladimir Korepin: *Quantum non-linear Schrödinger equation*
10:00 – 11:00	Ed Corrigan: *An introduction to integrable classical and quantum field theory in two dimensions*
11:00 – 11:30	Break
11:30 – 12:30	Andreas Klümper: *Thermodynamics of Spin Chains*

Jueves, 31 de Mayo (Salón: SD-805)	
9:00 – 10:00	Vladimir Korepin: *Quantum non-linear Schrödinger equation*
10:00 – 11:00	Ed Corrigan: *An introduction to integrable classical and quantum field theory in two dimensions*
11:00 – 11:30	Break
11:30 – 12:30	Andreas Klümper: *Thermodynamics of Spin Chains*
12:30 – 14:00	Lunch Break
Salón B-202	
14:00 – 14:35	Primitivo Acosta-Humánez: *A Galoisian approach to SUSY QM and beyond*
14:35 – 15:10	David Blázquez: *An application of differential Galois theory to the computation of exact eigenvalues and eigenfunctions of some Sturm-Liouville problems*
15:10 – 15:45	Andrés Reyes: *Entanglement entropy, Particle Identity and the GNS Construction*

Viernes, 1 de Junio (Salón: SD-805)	
9:00 – 10:00	Vladimir Korepin: *Quantum non-linear Schrödinger equation*
10:00 – 11:00	Ed Corrigan: *An introduction to integrable classical and quantum field theory in two dimensions*
11:00 – 11:30	Break
11:30 – 12:30	Andreas Klümper: *Thermodynamics of Spin Chains*
12:30 – 14:00	Lunch Break
Salón B-202	
14:00 – 14:20	Miguel Rodríguez: *La métrica retrocausal de Gödel*
14:20 – 14:40	Otaivin Martínez: *Al menos un tercio de los zeros no triviales de la función zeta de Riemann tienen parte real 1/2*
14:40 – 15:00	Emilio Torres: *TBA*

Morning Lectures

Ed Corrigan (University of York, UK)

An introduction to integrable classical and quantum field theory in two dimensions

The main example will be the sine-Gordon model, which will be used to illustrate features that are key to the study of many other models. For example, the classical and quantum spectrum, conservation laws, scattering (including a brief look at the Yang-Baxter equation) and bound states. If time allows there will be a brief exploration of other topics especially the influence of boundaries.

Vladimir Korepin (State University of New York at Stony Brook, USA)

Quantum non-linear Schrödinger equation

The course will be about the Lieb-Liniger model [it can also be called one dimensional Bose gas with delta interaction or quantum non-linear Schrödinger equation]. It is based on my book Quantum inverse scattering method and correlation functions. The last part of the lecture series will deal with the Lieb-Liniger model of 1D anyons, based on the papers Large-Distance Asymptotic Behavior of the Correlation Functions of 1D Impenetrable Anyons at Finite Temperatures (arXiv:0811.2419) and Non-conformal asymptotic behavior of the time-dependent field-field correlators of 1D anyons (arxiv.org/abs/0906.0431).

Slides and a video recording of lectures about Lieb-Liniger model of 1d anyons by Professor Korepin given at the Kavli Institute for Theoretical Physics at the University of California in 2011 can be found at `http://online.kitp.ucsb.edu/online/integral11/korepin/`.

Andreas Klümper (Bergische Universität Wuppertal, Alemania)

Thermodynamics of Spin Chains

1. Thermodynamical Bethe Ansatz (TBA)
2. Lattice Path Integral Formulation and Quantum Transfer Matrix (QTM)
3. Fusion Algebra: T- and Y-Systems
4. Non-Linear Integral Equations
5. Higher Rank Models and Continuum Systems

The main topic of my lectures will be the finite temperature physics of integrable 1d quantum systems. The discussion will be rather comprehensive and detailed for the case of the spin-1/2 Heisenberg chain, but not restricted to this. In the first lecture I will discuss the combinatorial TBA method introduced by Yang and Yang for the single component Bose gas and generalized by Gaudin and Takahashi to the Heisenberg chain. Lectures 2 and 3 are devoted to an algebraic approach to the thermodynamical properties of integrable quantum chains. The finite temperature systems are mapped to classical models on 2d lattices. The partition function is obtained from just the largest eigenvalue of the column-to-column transfer matrix, also called the 'quantum transfer matrix' which acts in an infinite dimensional space. A hierarchy of transfer matrices is derived by the fusion method, and algebraic relations of various type are established. Lecture 4: By use of the so-called T- and Y-systems (i) the TBA equations are derived rigorously, (ii) alternative, especially finite sets of non-linear integral equations are derived, (iii) correlation lengths/mass gaps are calculated. In Lecture 5 generalizations to higher rank models and continuum limits like Bose gases are discussed.

Short Communications

Primitivo Acosta-Humánez
(Universidad del Norte, Colombia)

A Galoisian approach to SUSY QM and beyond

This talk will be concerning to a Differential Galois (Picard-Vessiot) Theory point of view of the Supersymmetric Quantum Mechanics. The main object is the non-relativistic stationary Schrödinger equation, where are introduced the concepts of Algebraic Spectrum and Hamiltonian Algebrization. Using the Kovacic's Algorithm and the Hamiltonian Algebrization are analyzed Darboux transformations, Crum iterations and supersymmetric quantum mechanics, including their Algebrized Versions from a Galoisian approach. In particular are obtained the ground state, eigenvalues, eigenfunctions, the differential Galois groups and eigenrings of some Schrödinger equations with potentials such as exactly solvable, quasi-exactly solvable and shape invariant potentials. Finally is introduced one methodology to find Algebraically Solvable and Algebraically Quasi-Solvable Potentials. It consists in to apply the Hamiltonian Algebrization, as inverse process, over families of second order linear differential equations integrables in the Picard-Vessiot sense for a set of parameters, in particular, involving orthogonal polynomials and special functions.

Andrés Ángel
(Universidad de los Andes, Colombia)

Quantum cobordism and moduli spaces of curves

In this talk I will report on work of Givental and Coates interpreting intersection numbers of moduli spaces of stable curves as Gromov-Witten invariants with values in complex cobordisms. This interpretation allows us to see the problem as a quantum version of the Hirzebruch-Riemann-Roch theory corresponding to the Chern-Dold character from the cobordisms to cohomology.

David Blázquez
(Universidad Sergio Arboleda, Colombia)

An application of differential Galois theory to the computation of exact eigenvalues and eigenfunctions of some Sturm-Liouville problems

We study a Sturm-Liouville type eigenvalue problem for second-order differential equations on the infinite interval. Here the eigenfunctions are nonzero solutions exponentially decaying at infinity. We prove that at any discrete eigenvalue the differential equations are integrable in the setting of differential Galois theory under general assumptions. Our result is illustrated with three examples for a stationary Schrödinger equation having a generalized Hulthen potential; a linear stability equation for a traveling front in the Allen-Cahn equation; and an eigenvalue problem related to the Lame equation. This talk is based in a collaborative research with K. Yagasaki, from Niigata University.

Alexander Cardona
(Universidad de los Andes, Colombia)

Integrable systems, Dirac structures and quantization

In this talk we will describe classical integrable systems as particular cases of twisted Dirac structures on manifolds, and we will describe a geometric context to study the quantization of the Poisson algebra of admissible functions associated to it.

Otaivin Martínez
(Universidad Industrial de Santander, Colombia)

Al menos un tercio de los zeros no triviales de la función zeta de Riemann tienen parte real 1/2

La idea de esta charla es exponer, a modo de sketch, la demostración del teorema de Levinson, el cual asegura que al menos un tercio de los ceros no triviales de la función zeta de Riemann tiene parte real 1/2. Además de mostrar las mejoras que se han logrado para el estimativo, como lo es el teorema de Young: Al menos el 40% de los ceros no triviales tiene parte real 1/2.

Servio Pérez
(Universidad del Cauca, Colombia)

Density Matrix Renormalization Group (DMRG)

El método de matriz densidad y Grupo de renormalización (DMRG) se ha convertido en un poderoso método numérico que se puede aplicar a sistemas de baja dimensionalidad, fermiónicos y bosónicos fuertemente correlacionados. Este permite un cálculo muy preciso de las propiedades estáticas, dinámicas y termodinámicas. Su campo de aplicación se ha extendido más allá de la Materia Condensada, y es utilizado con éxito en mecánica estadística y también en física de altas Energías. En cada caso se revisan los aspectos principales del método y se comentan algunas de las aplicaciones más relevantes, a fin de dar una visión general sobre el alcance y las posibilidades de DMRG y mencionar

las extensiones más importantes del método; como el cálculo de las propiedades dinámicas, la aplicación a los sistemas clásicos, la inclusión de la temperatura, fonónes y desorden, teoría de campos, propiedades que dependen del tiempo y el cálculo ab initio de los estadoséelectrónicos en las moléculas.

Otto Rendón
(Universidad de Carabobo, Venezuela)

Two - particle scattering by a finite interacting region: In the T- matrix approach

We have studied scattering of two identical particles [1, 2]. The device consists

of one input lead and output leads attached to a quantum dot. The movement of two fermions is without interaction in the leads but exist repulsion U when the dot is doubly occupied. In this device, one may avoid both single and triple processes with putting certain values in the leads energy. Within this model We have done formal derivations for transition amplitudes, in terms of the T-matrix, to second orders in the coupling to the dot $V_{coupling}$ and consider a finite lead bandwidth, V. In fact, we put $V_{coupling} \ll V$. At $T = 0$K, the devices filters singlet entangled pairs if $U \neq 0$. However, here the postselection is not used. Moreover, resonance structure for the singlet transition amplitude is studied as function of energy difference between the input lead and the dot single-particle state. In $U = 0$, there isn't tunneling and the two-fermion scattering matrix tend to the obtained by custom scattering matrix theory for noninteracting electrons. The first part of this work has been done in collaboration with Gladys León and Ernesto Medina D.

References

[1] Oliver W. D., F. Yamaguchi, and Y. Yamamoto, Phys. Rev. Lett. vol. 63, 037901 (2002).
[2] G. León, O. Rendón, H. M. Pastawski, V. Mujica, and E. Medina, Europhys. Lett. vol. 66, 624 (2004).
[3] M. Bttiker, Y. Imry, R. Landauer, and S. Pinhas, Phys. Rev. B vol. 31, 6207(1985).

Juliana Restrepo
(Universidad Antonio Nariño, sede Medelin, Colombia)

Decoherence and relaxation in quantum systems

In this seminar, I will address a few fundamental questions related to the decoherence and relaxation of quantum systems. In particular I will present the study of the full evolution of a two level system or qubit coupled to a bath composed of non-interacting and interacting electrons and I will focus on the case where the bath exhibits long range order. The interest of this problem is related to quantum computation because in order to construct an efficient quantum computer one needs to understand the effect of the surroundings on the basic building block, i.e., a quantum bit or qubit. Alternatively, these problems are also interesting from the viewpoint of using the two level system as a probe

of the physics of the bath.

Andrés Reyes
(Universidad de los Andes, Colombia)

Entanglement entropy, Particle Identity and the GNS Construction

In this talk I will present an approach to entanglement which is based on the Gelfand-Naimark-Segal (GNS) construction. The conventional approach to the emergence of mixed from pure states based on taking partial traces is replaced by the more general notion of the restriction of a state to a subalgebra. For bipartite systems of nonidentical particles, this approach reproduces the standard results. But it also very naturally overcomes the limitations of the usual treatment of systems of identical particles. The content of this talk is based on joint work with A.P. Balchandran, T.R. Govindarajanand A.R de Queiroz.

Miguel Rodríguez
(Universidad Pedagógica Nacional)

La métrica retrocausal de Gödel

La métrica retrocausal de Gödel La charla se centrará en como la métrica de Gödel genera líneas de mundo cerradas, siendo estas las causantes de un universo retro causal teóricamente; siendo una solución exacta a la ecuación de campo de Einstein, además seria interesante el mostrar el comportamiento de la misma y que implicaciones físicas conllevaría un universo al cual solo seria aplicarle la propiedad de rotación a la materia.

Gabriel Téllez
(Universidad de los Andes, Colombia)

Expanded Vandermonde powers: applications to the two-dimensional one-component plasma

The two-dimensional one-component plasma (2dOCP) is a system of N mobile particles of the same charge q on a surface with a neutralising background. The Boltzmann factor of the 2dOCP at temperature T can be expressed as a Vandermonde determinant to the power = q2/(kBT). Recent advances in the theory of symmetric and anti-symmetric Jack polymonials provide an efficient way to expand this power of the Vandermonde in their monomial basis, allowing the computation of several thermodynamic and structural properties of the 2dOCP for N values up to 14 and equal to 4, 6 and 8. In this work, we present some applications of this formalism.

Emilio Torres
(Universidad de los Andes, Colombia)

TBA

	Lunes	Martes	Miercoles	Jueves	Viernes
8:00 – 9:00	Registration				
9:00 – 10:00	V. Korepin	V. Korepin	V. Korepin	V. Korepin	V. Korepin
10:00 – 11:00	E. Corrigan	E. Corrigan	E. Corrigan	E. Corrigan	E. Corrigan
11:00 – 11:30	*Break*	*Break*	*Break*	*Break*	*Break*
11:30 – 12:30	A. Klümper	A. Klümper	A. Klümper	A. Klümper	A. Klümper
12:30 – 14:00	*Break*	*Break*		*Break*	*Break*
14:00 – 14:35	A. Ángel	S. Pérez		P. Acosta	M. Rodríguez
14:35 – 15:10	G. Téllez	J. Restrepo		D. Blázquez	O. Martínez
15:10 – 15:34	A. Cardona	O. Rendón		A. Reyes	E. Torres