Decoherence and relaxation of quantum systems

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R. Chitra & S. Camalet

Thesis

- Part I: Decoherence and relaxation in an open system
  - Bath of non-interacting electrons — *Restrepo, Camalet, Chitra, Dupont Phys Rev. B(2011)*
  - Bath with a long range order — *Restrepo, Camalet, Chitra (to be submitted)*
- Part II: Thermalization in a closed system — *Restrepo, Camalet New J. Phys (2010)*

Thesis

- Part I: Decoherence and relaxation in an open system
  - Bath of non-interacting electrons — *Restrepo, Camalet, Chitra, Dupont Accepted Phys Rev. B*
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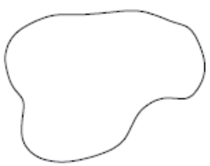
Decoherence and relaxation of a qubit coupled to a bath with long range order

Qubit + bath

- Qubit $|\psi\rangle = \alpha\,|\uparrow\rangle + \beta\,|\downarrow\rangle$

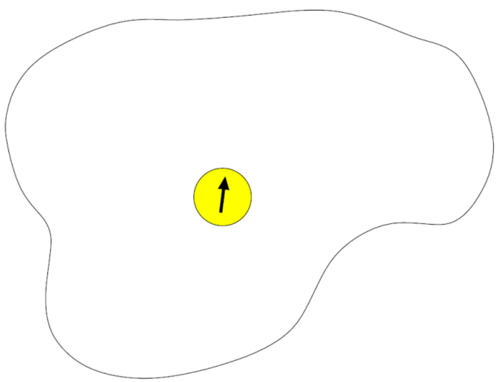
$$\rho_s = |\psi\rangle\,\langle\psi| = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{bmatrix}$$

Density matrix



- Bath ρ_B

Decoherence and relaxation



- Qubit
- Bath

Interaction

$$\rho_s = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{bmatrix}$$

Time evolution

- Decoherence
- Relaxation

Why study decoherence and relaxation?

- For quantum computation it is important to have coherent qubits.

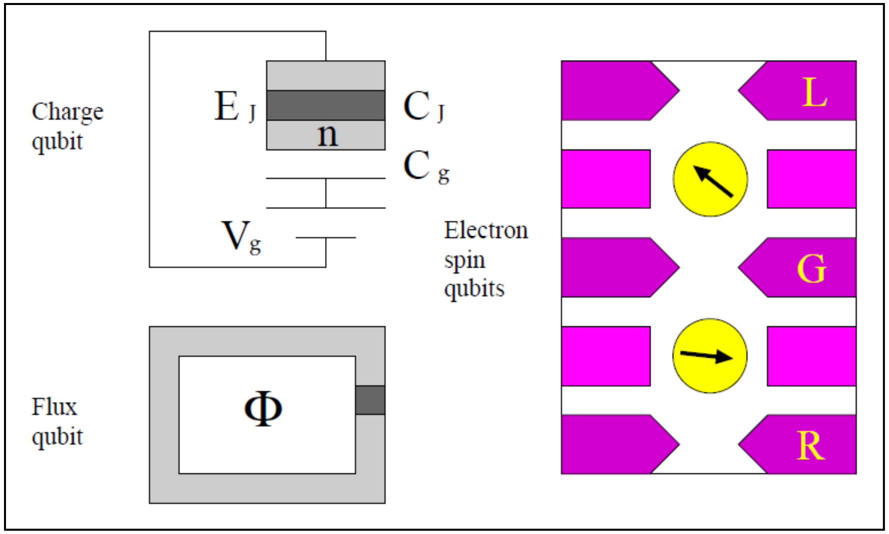
- Qubits can be used to probe the environment.

Physical realizations of qubits

Solid state qubits → Size advantage

Examples

- Josephson junctions
- Spin in quantum dots



Makhlin
Rev Mod Phys (2001)

DiVincenzo, Loss
Phys Rev A (1998)

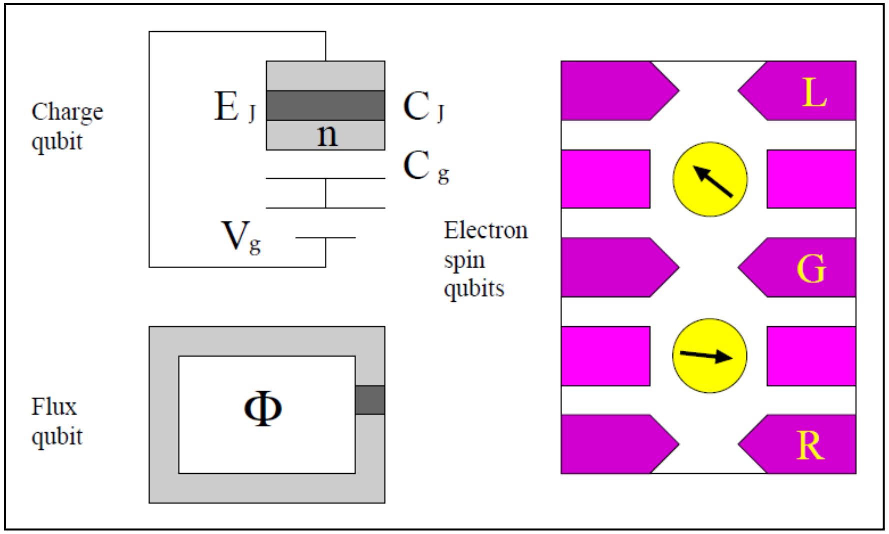
Physical realizations of qubits

Solid state qubits → Size advantage

Examples

- Josephson junctions
- Spin in quantum dots

→ Interacting nuclear spins and electronic baths



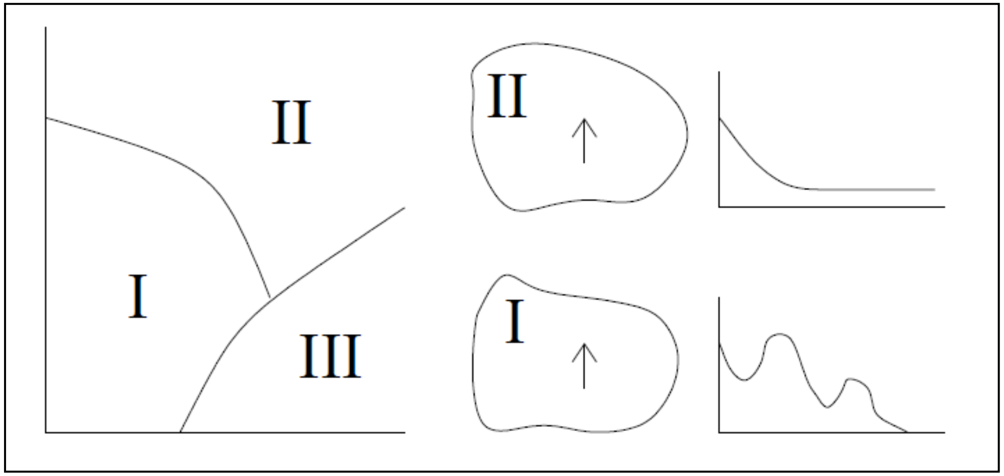
Coish, Loss
Phys Rev B (2008)

Glazman, Loss
Phys Rev Lett (2002)

Schiller et al
Phys Rev B (2006)

Yamada et al
Conf Proc (2007)

Qubits as probes



Phase diagram Decoherence

- Theoretical

Chitra, Camalet Phys Rev Lett (2007)

Winograd , Chitra, Rozenberg Phys Rev B (2010)

Vernier / Jiang Phys Rev A (2011)

- Experimental

Buttiker et al Phys Rev B (2010)

Vandersypen et al Rev Mod Phys (2005)

State of art

Baths

- Nuclear spins — *Glazman, Loss, Phys Rev Lett (2002)*; *Coish, Loss, Phys Rev B (2008)*
- Electrons — *Schiller et al, Phys Rev B (2006)*; *Yamada et al, Conf Proc (2007)*

Effect of

- Bath Interactions — *Paganelli et al, Phys Rev A (2002)*; *Tessieri et al, Jour Phys A (2003)*
- Phase transition — *Yuan et al, EPL (2005)*; *Chitra, Camalet, Phys Rev Lett (2007)*; *Wang et al, Phys Lett A (2008)*

What is the effect of order in the bath on decoherence and relaxation of the qubit?

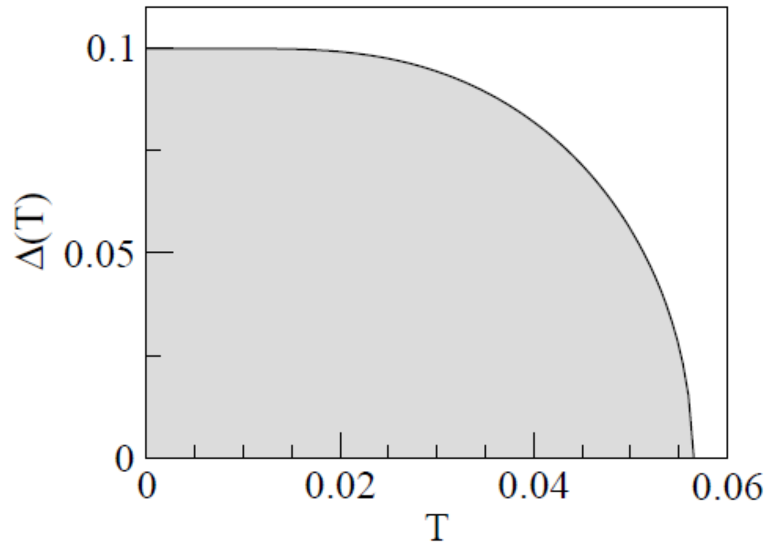
Model

$$H = H_S + H_B + H_I$$

- The qubit has no intrinsic dynamics

$$H_S = 0$$

- The bath is described by BCS Hamiltonian
  - T < Tc superconductor
  - T > Tc metal



- Interaction

$$H_I = \sum_{\beta=0,x,y,z} V_\beta \, \sigma_\beta^c$$

Interaction

- Charge $$H_I = \lambda\, \sigma_z^c \sum_{kp} \left(c_{k\uparrow}^\dagger c_{p\uparrow} + c_{k\downarrow}^\dagger c_{p\downarrow} \right)$$
- Kondo $$H_I = \lambda \boldsymbol{\sigma}^c \cdot \sum_{k,p,\alpha\beta} c_{k\alpha}^\dagger \, \boldsymbol{\sigma}_{\alpha\beta} \, c_{p\beta}$$
- Order $$H_I = \lambda \sigma_-^c \sum_{kp} c_{k\uparrow}^\dagger c_{p\downarrow}^\dagger - \lambda \sigma_+^c \sum_{kp} c_{k\uparrow} c_{p\downarrow}$$

Reduced dynamics of the qubit

- Initial state

$$\rho(0) = \rho_s(0) \otimes \rho_B(0)$$

$$\rho_s(0) = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \alpha^*\beta & |\beta|^2 \end{bmatrix}$$

$$\rho_B(0) = \frac{e^{-H_\beta/T}}{Z_B}$$

Reduced dynamics of the qubit

- Initial state $$\rho(0) = \rho_s(0) \otimes \rho_B(0)$$
- Unitary evolution $$\frac{d}{dt}\rho = -i\ [H, \rho]$$
- Reduced density matrix $$\rho_s(t) = \mathrm{Tr}_B\,[\,\rho(t)\,]$$

Weak coupling techniques

Master equation $$\frac{d}{dt}\rho_s(t) = \int dt'\Sigma(t',t)\rho_s(t')$$

Born approximation λ^2

$$\frac{d}{dt}\rho_s(t) = \int dt'\Sigma^{(2)}(t',t)\rho_s(t')$$

Nakajima-Zwanzig (NZ)

Local approximation

$$\frac{d}{dt}\rho_s(t) = \int dt'\Sigma^{(2)}(t',t)\rho_s(t)$$

Time convolutionless (TCL)

Master equation in Born aproximation

Laplace transform $\rho_s(z) = \frac{1}{2} \sum_{\beta=0,x,y,z} M_\beta(z)\, \sigma_\beta^c$

$$zM_\beta(z) - \sum_\alpha h_{\beta\alpha} M_\alpha(z) - \sum_\alpha \Sigma_{\beta\alpha}(z) M_\alpha(z) = \langle \sigma_\beta^c \rangle_0$$

First order $\lambda \langle V_\gamma \rangle$

Second order $\lambda^2 \langle V_\gamma(t) V_\delta \rangle$

Self energies→ Time evolution

Kondo coupling

- The problem is spin isotropic

- Bath operators have no net moment $h = 0$

Only one self energy $\Sigma(z)$

Kondo coupling

- The problem is spin isotropic
- Bath operators have no net moment

↓

Relaxation and decoherence are the same

$$M_\beta(z) = \underbrace{[z - \Sigma(z)]^{-1}}_{M(z)} \langle \sigma^c_\beta \rangle_0$$

Dynamics in NZ and TCL

$$\Gamma(\omega) = -\Im m \lim_{\eta \to 0} \Sigma(\omega + i\eta)$$

- Time convolutionless

$$\ln M_{TCL}(t) = -\frac{2}{\pi} \int d\omega \frac{\sin^2 \omega t/2}{\omega^2} \Gamma(\omega)$$

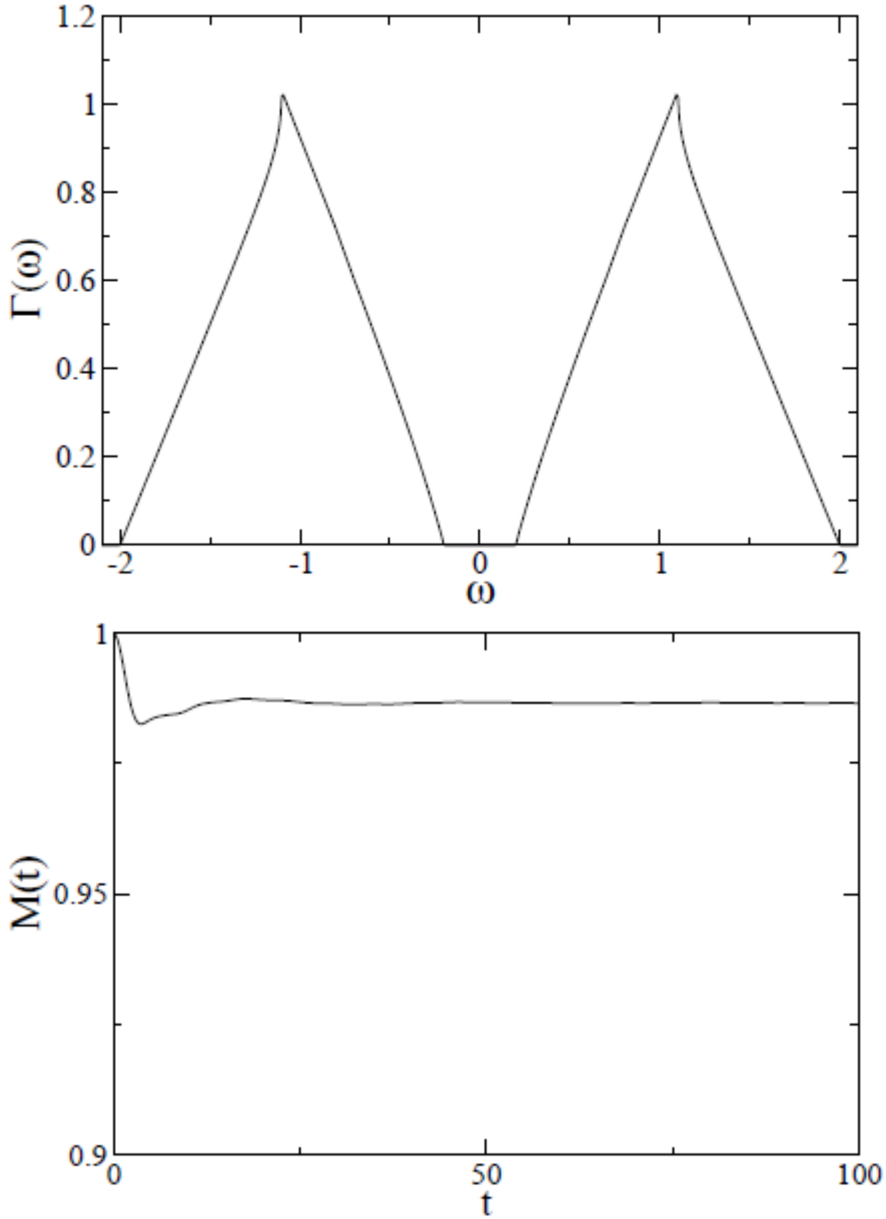
- Nakajima-Zwanzig

$$M_{NZ}(t) = \frac{1}{\pi} \int_0^\infty d\omega \cos(\omega\, t) \tilde{\Gamma}(\omega)$$

$$\Gamma(\omega)$$

Markovian asymptotic evolution $M(t) \simeq e^{-\Gamma(0)t}$

Asymptotic dynamics for T=0 (TCL, NZ)

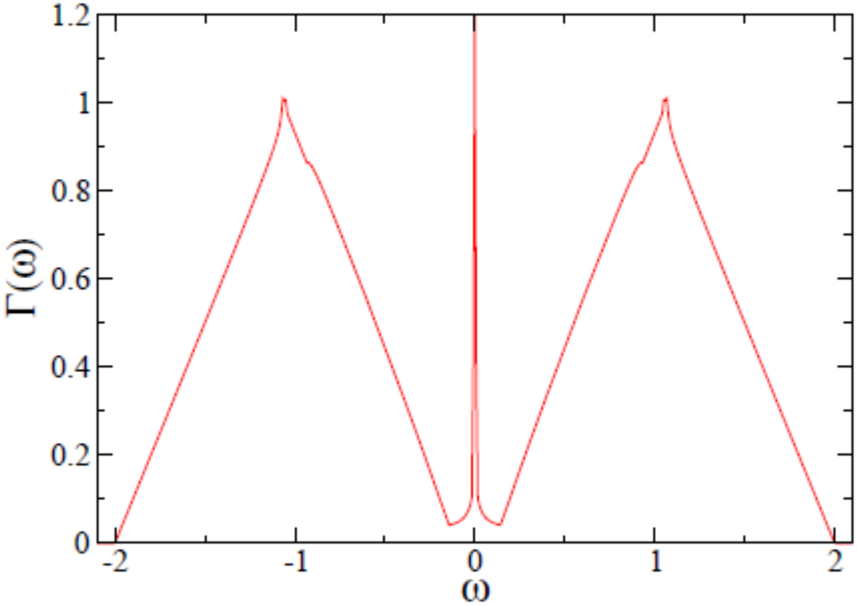


Gap at ω=0

Incomplete decoherence/ relaxation

$$M(t) \xrightarrow{t\to\infty} constant$$

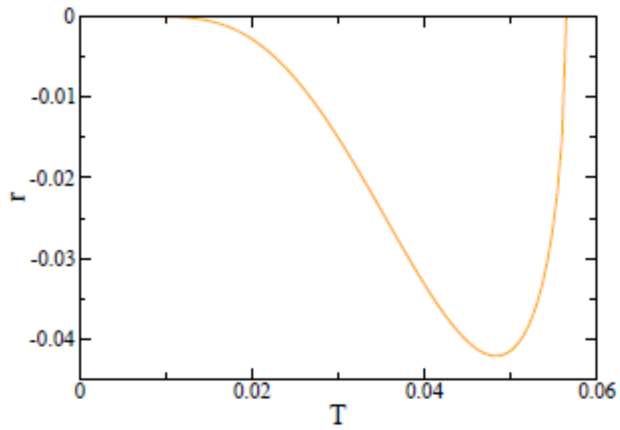
Self energy at finite temperature



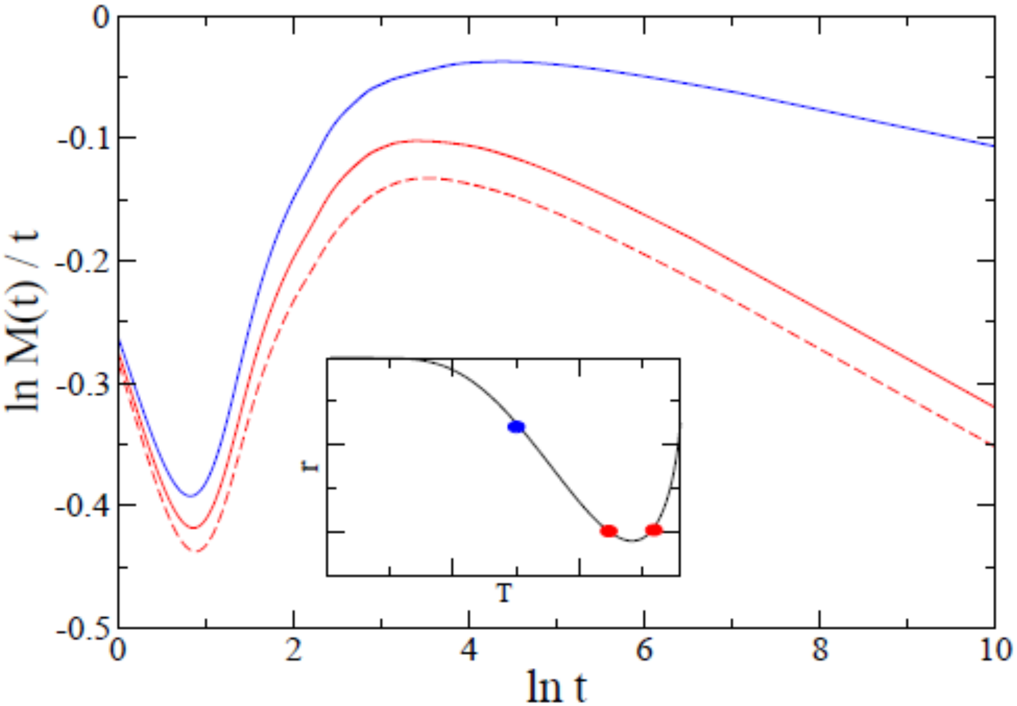
Logarithmic divergence at ω=0

$$\frac{\Gamma(\omega)}{2\pi\lambda^2} = r \ln \frac{\omega}{T}$$

r is non monotonic



Asymptotic dynamics for 0<T<Tc (TCL)



$$M_{TCL}(t) \simeq t^{2\pi\lambda^2 r(T)\, t}$$

Anomalous decoherence

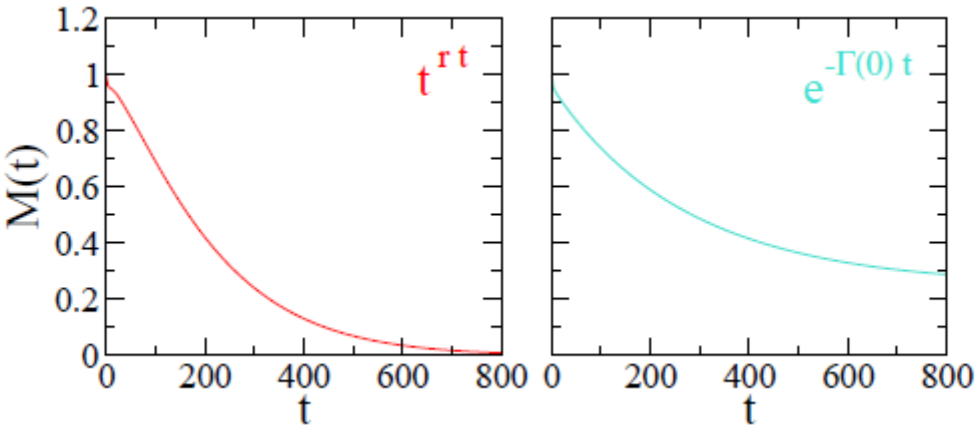
- Non Markovian
- Ultrafast
- ‘‘Reentrance’’

Scales

- Small T (t>> 1/T)
- Large T (t>> 1/Δ)

Asymptotic dynamics at 0<T<Tc (TCL)

$$M_{TCL}(t) \simeq t^{2\pi\lambda^2\, r(T)\, t}$$



Anomalous decoherence

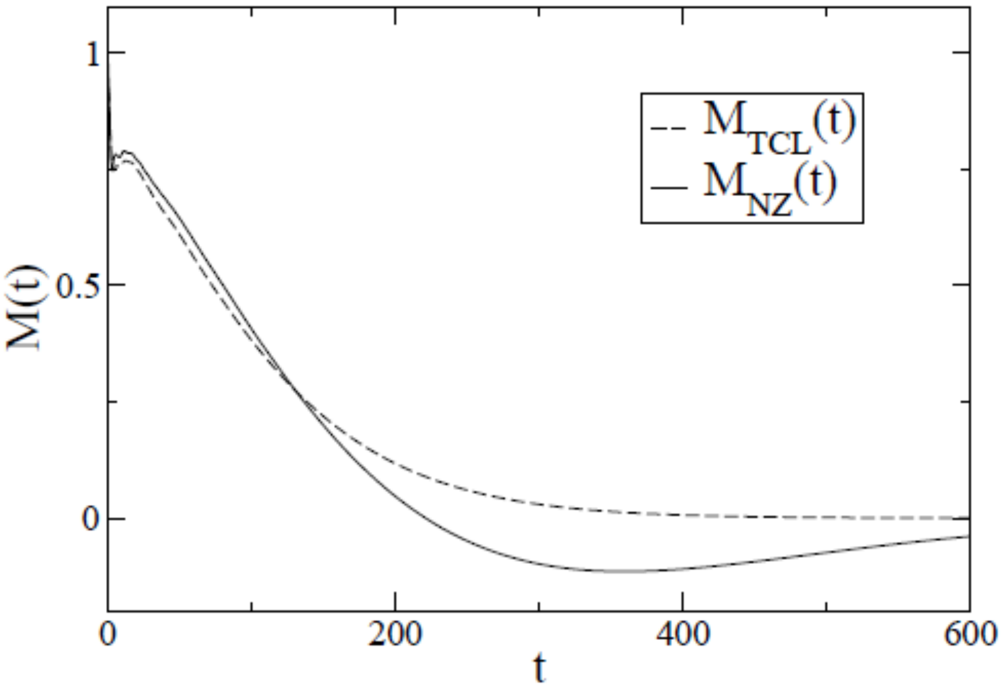
- Non Markovian
- Ultrafast
- Reentrance

Scales

- Small T (t>> 1/T)
- Large T (t>> 1/Δ)

Asymptotic dynamics for 0<T<Tc (NZ)

$$M_{NZ}(t) \simeq -\frac{1}{2\pi^2\lambda^2\, r}\,\frac{1}{t\ln t}$$



- Non Markovian decoherence/relaxation
- Different from TCL

Summary for Kondo coupling

- Relaxation = Decoherence
- At T=0 incomplete decoherence (TCL and NZ)
- For 0<T<Tc non Markovian decoherence (TCL ≠ NZ)
- For 0<T<Tc (TCL) Ultrafast non Markovian
  → disastrous for qubits

signature of order?

Order coupling

$$H_I = \lambda\sigma^c_- \sum_{kp} c^\dagger_{k\uparrow} c^\dagger_{p\downarrow} - \lambda\sigma^c_+ \sum_{kp} c_{k\uparrow} c_{p\downarrow}$$

- The problem is not isotropic
- Bath operators have net moment $h_{yz} \propto \Delta$

↓

Self energy matrix

Relaxation ≠ Decoherence

Order coupling

- The problem is not isotropic
- Bath operators have net moment $h_{yz} \propto \Delta$

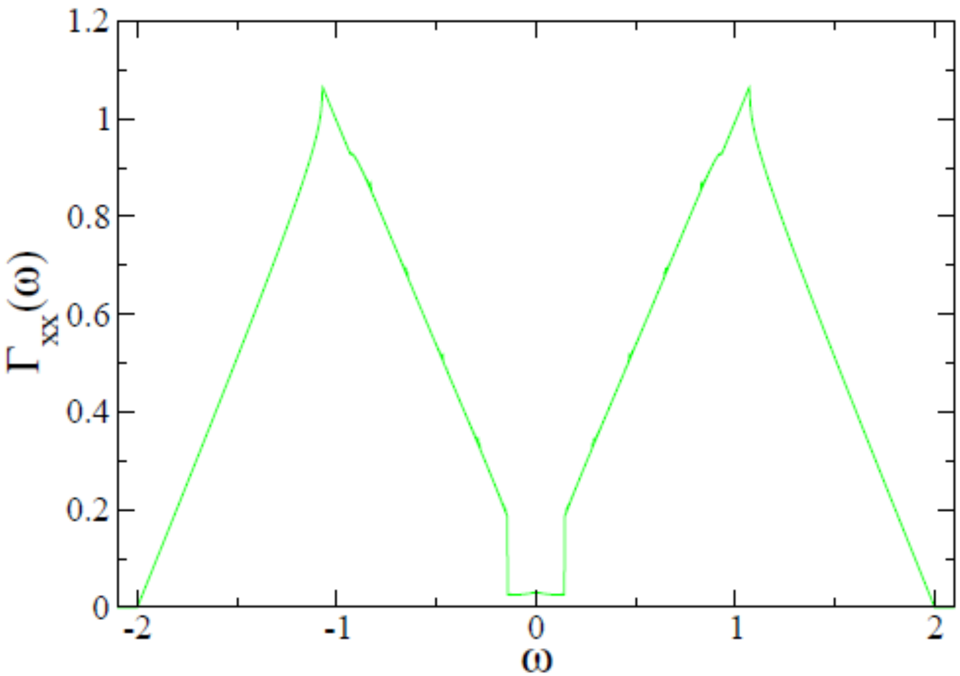
Self energy matrix

$$\begin{pmatrix} M_0(z) \\ M_x(z) \\ M_y(z) \\ M_z(z) \end{pmatrix} \begin{pmatrix} z & 0 & 0 & 0 \\ 0 & z-\Sigma_{xx} & 0 & 0 \\ 0 & 0 & z-\Sigma_{yy} & -h_{yz} \\ 0 & 0 & -h_{zy} & z-\Sigma_{zz} \end{pmatrix} = \begin{pmatrix} \langle\sigma_0^c\rangle_0 \\ \langle\sigma_x^c\rangle_0 \\ \langle\sigma_y^c\rangle_0 \\ \langle\sigma_z^c\rangle_0 \end{pmatrix}$$

The x component for 0<T<Tc (TCL)

$$\Gamma_{xx}(\omega) \Rightarrow M_x(z)$$

$$\langle \sigma_x^c(t) \rangle$$

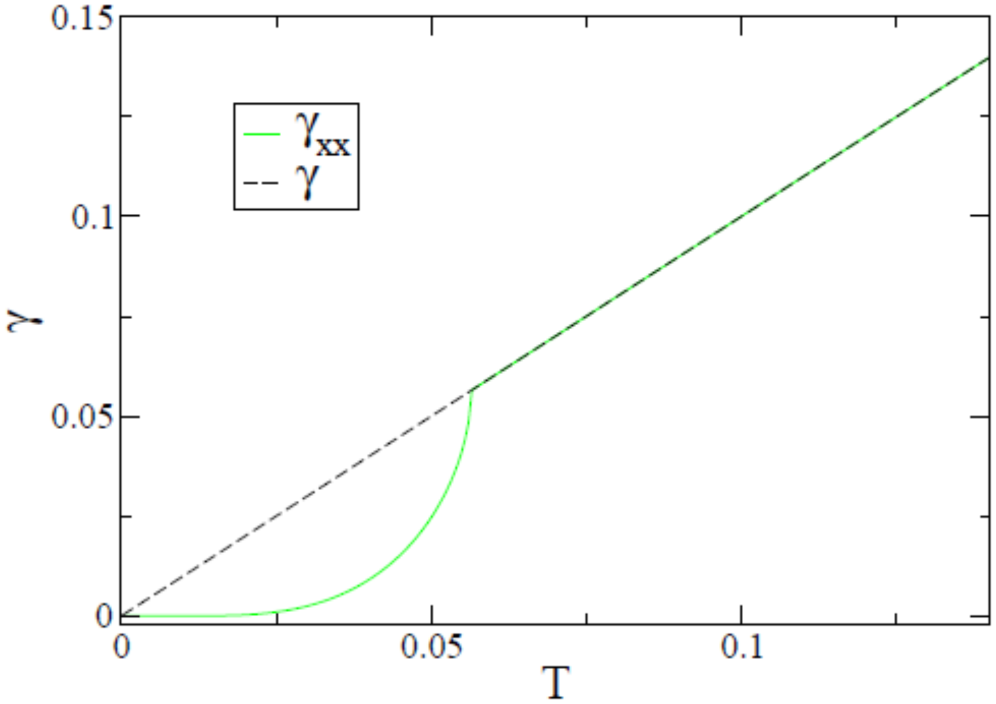


- Related to charge fluctuations
- Finite at ω=0 → Markovian evolution

The x component for 0<T<Tc (TCL)

$$\Gamma_{xx}(\omega) \Rightarrow M_x(z)$$

$$\langle \sigma_x^c(t) \rangle$$



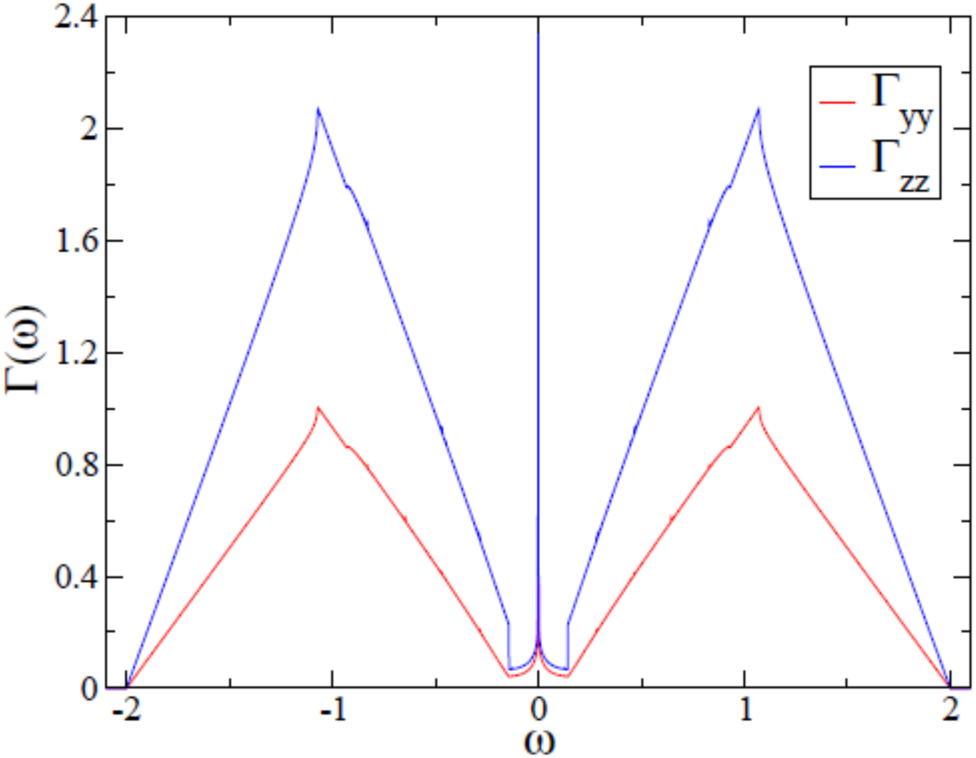
- Markovian
- Very slow

Black and Fulde
Jour Phys Lett (1979)

The y and z components for 0<T<Tc (TCL)

$$h_{yz} \quad \Gamma_{zz}(\omega) \quad \Gamma_{yy}(\omega) \quad \Rightarrow \quad M_y(z) \quad M_z(z)$$

$$\langle \sigma_y^c(t) \rangle \quad \langle \sigma_z^c(t) \rangle$$



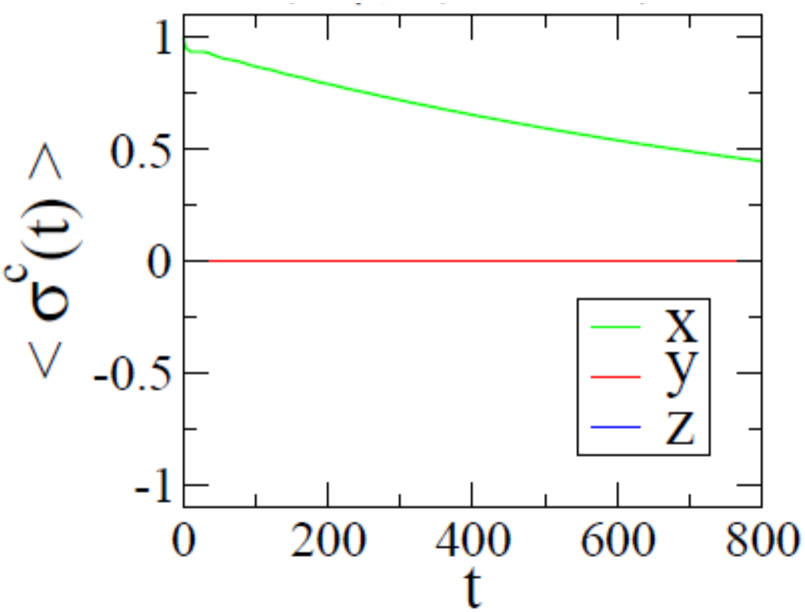
- Related to spin fluctuations
- Infrared divergence → non Markovian ultrafast evolution
- First order term → oscillations

Sensitivity to initial conditions

$$\begin{pmatrix} M_0(z) \\ M_x(z) \\ M_y(z) \\ M_z(z) \end{pmatrix} \begin{pmatrix} z & 0 & 0 & 0 \\ 0 & z - \Sigma_{xx} & 0 & 0 \\ 0 & 0 & z - \Sigma_{yy} & -h_{yz} \\ 0 & 0 & -h_{zy} & z - \Sigma_{zz} \end{pmatrix} = \begin{pmatrix} \langle \sigma_0^c \rangle_0 \\ \langle \sigma_x^c \rangle_0 \\ \langle \sigma_y^c \rangle_0 \\ \langle \sigma_z^c \rangle_0 \end{pmatrix}$$

Sensitivity to initial conditions

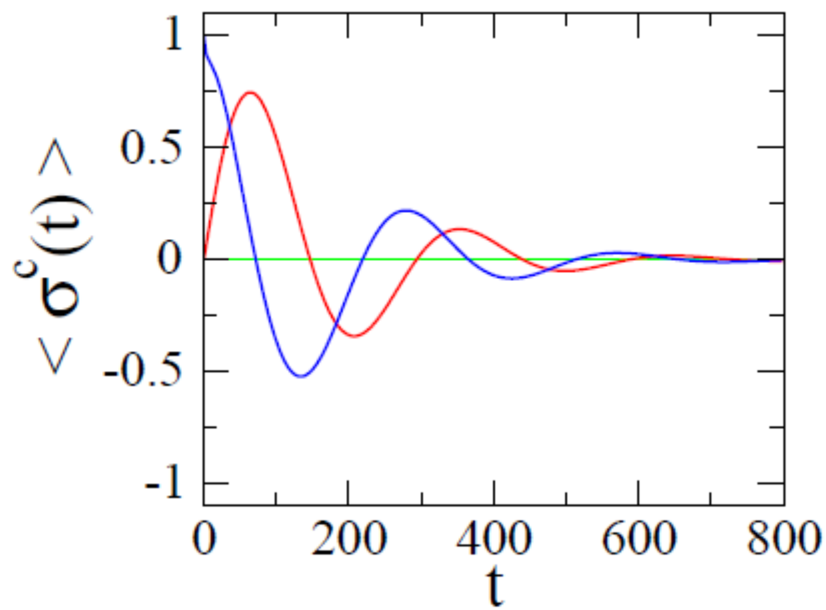
$$|\psi\rangle = \frac{1}{\sqrt{2}}|\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle$$



- No relaxation

- Markovian decoherence

$$|\psi\rangle = |\uparrow\rangle$$



- Non Markovian relaxation

- Non Markovian decoherence

Summary for Order coupling

- Relaxation ≠ Decoherence
- Sensitivity to initial conditions
- Difference between x, y, z
  - Charge channel: Markovian and slow
  - Spin channel: Like Kondo with oscillations

→ Important to couple to « good operators » and measure « good quantities »

→ disastrous or good for qubits

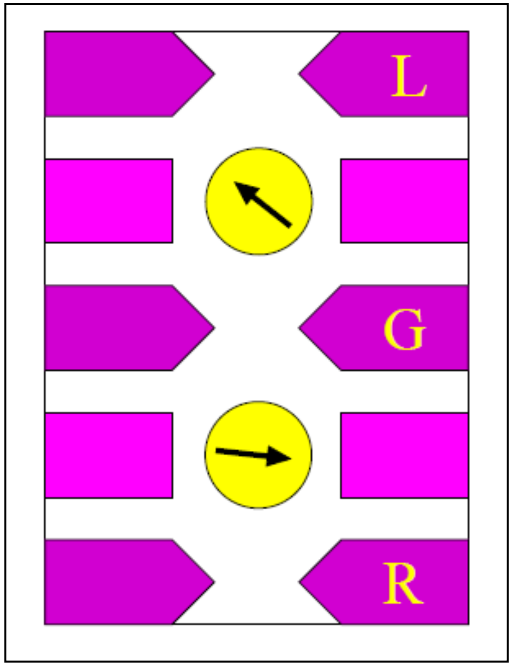
Perspectives

- $T \rightarrow T_c$ from metallic side
- Next order in TCL, NZ
- Other ordered baths
- Two qubits (entanglement and linear entropy)
- One qubit coupled to two baths:
  - Order + bosonic
  - Two ordered at different temperatures
- Strong coupling (Numerical approach)

Thank you, Gracias

P1: Dos qubits

¿Cuál es la dinámica de dos qubits cuando interactuan con un baño en fase ordenada?

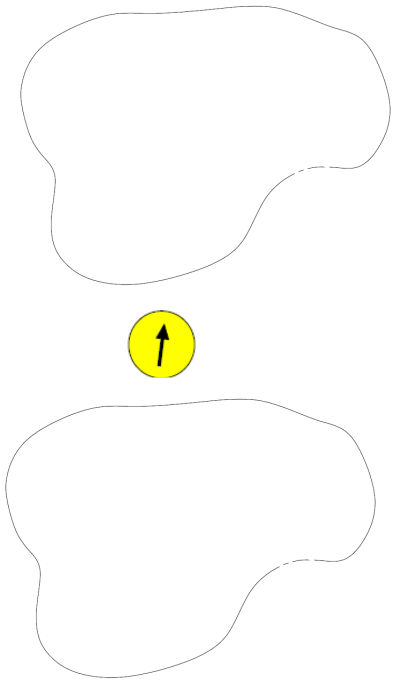


- Importante para aplicaciones, experimentos
- Entrelazamiento, entropía lineal
- Extensión « facil »

R. Chitra (ETH, Zurich) S. Camalet (LPTMC, Paris)

P2: Dos baños en equilibrio

¿Cuál es la decoherencia de un qubit cuando interactua con un baño de bosones y uno ordenado?

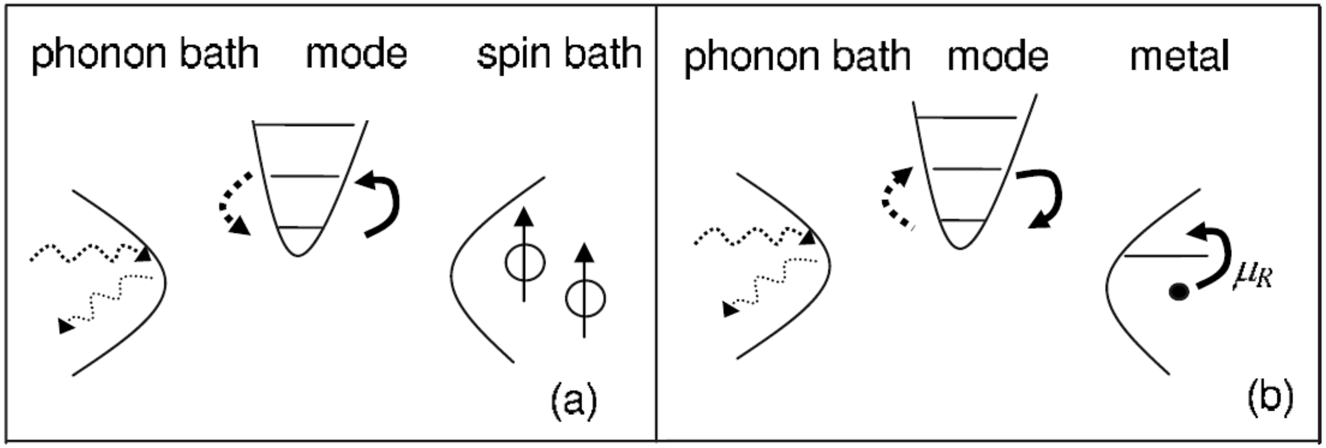


- Modelo más real
- Cada baño -> dinámicas diferentes
- ¿Quién gana?

R. Chitra (ETH, Zurich) S. Camalet (LPTMC, Paris)

P3: Dos baños a diferente temperatura

¿Cuál es la decoherencia de un qubit cuando interactua con dos baños a diferente temperatura?



Lian-Ao (PRE 2009)

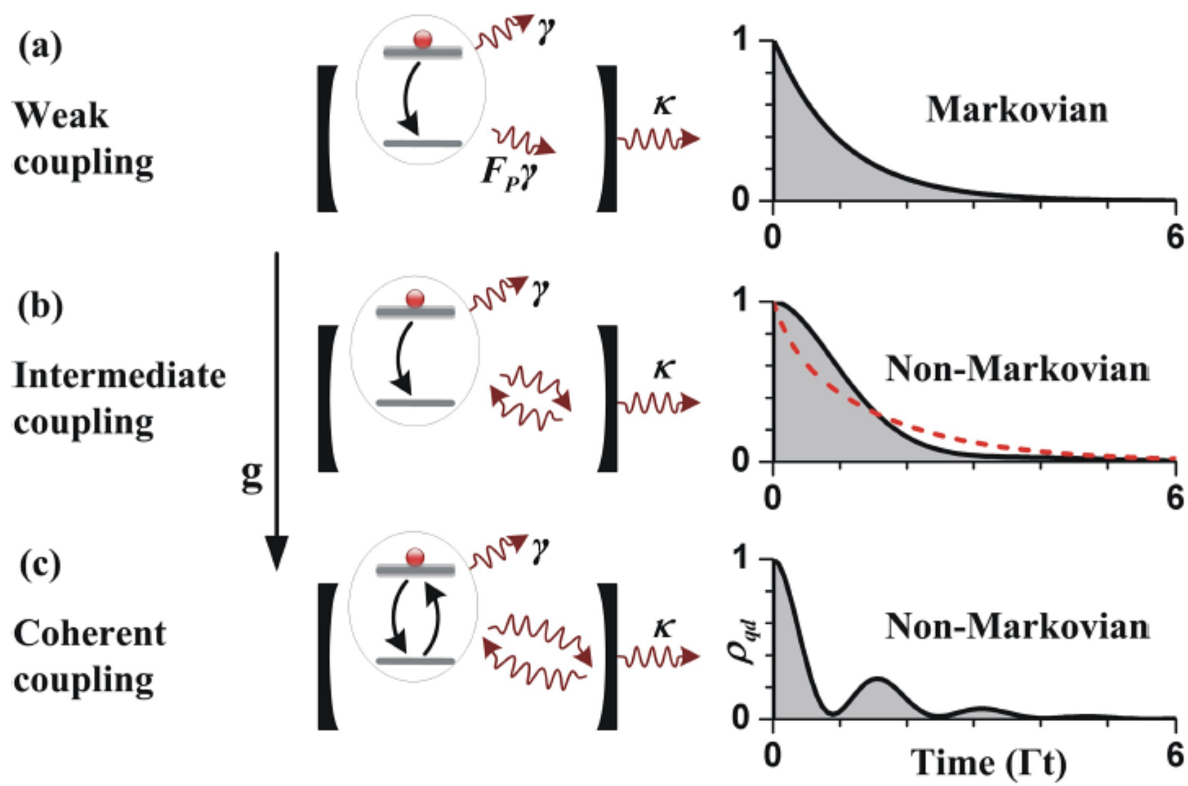
$$J = \frac{1}{2}\sum_{n,m} E_{m,n}|S_{m,n}|^2 P_n \times [k^L_{n\to m}(T_L) - k^R_{n\to m}(T_R)]$$

- Estudiar la transferencia de calor
- Resultados preliminares

P4: Excitones en un punto cuántico

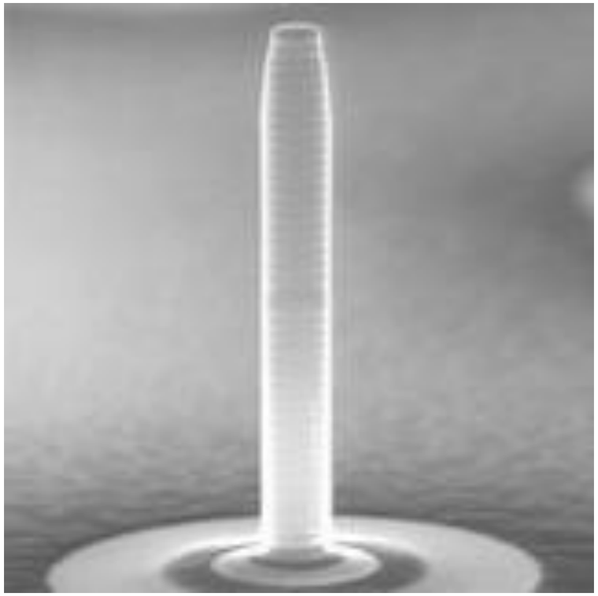
Caracterizar la dinámica en un sistema de excitones confinados en un punto cuántico

Madsen (PRL 2011)



P4: Excitones en un punto cuántico

Caracterizar la dinámica en un sistema de excitones confinados en un punto cuántico



- Cantidades dinámicas relacionadas con propiedades ópticas medibles

- No existe modelo teórico

B. Rodriguez (UDEA) L. Pachon (UDEA) M. Tarzia (LPTMC, Paris) L. Cugliandolo (LPTHE, Paris)