Two–particle scattering by a finite interacting region: In the T – Matrix approach.

Otto G. Rendón R.
Univerisdad de Carabobo, (UC).
Valencia - Venezuela.

May 29, 2012
h: 15:10-15:45.

Abstract.

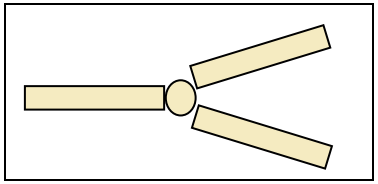
- We have studied scattering of two identical particles [1, 2]. The device consists of one input lead and output leads attached to a quantum dot. The movement of two fermions is without interaction in the leads but exist repulsion U when the dot is doubly occupied. In this device, one may avoid both single and triple processes with putting certain values in the leads energy. Within this model We have done formal derivations for transition amplitudes, in terms of the T – matrix, to second orders in the coupling to the dot $V_{coupling}$ and consider a finite lead bandwidth, V. In fact, We put $V_{coupling} << V$. At $T = 0$ K, the devices filters singlet entangled pairs if $U \neq 0$. However, here the postselection is not used. Moreover, resonance structure for the singlet transition amplitude is studied as function of energy difference between the input lead and the dot single-particle state. In $U = 0$, there isn't tunneling and the two-fermion scattering matrix tend to the obtained by custom scattering matrix theory for noninteracting electrons. The first part of this work has been done in collaboration with Gladys León and Ernesto Medina D.

Many authors have studied the formulation interacting of the N-particle S-matrix

1. W. D. Oliver, F. Yamaguchi and Y. Yamamoto, Phys. Rev. Lett. **88**, 037901-1 (2002).
2. G. León, O. Rendón, H. M. Pastawski, V. Mujica and E. Medina, Europhys. Lett. 66, 624 (2004).
3. A. S. Alexandrov, A. M. Bratkovsky, and P. E. Kornilovitch, Phys. Rev. B **65**, 155209-1 (2002).
4. P. Metha, and N. Andrei, Phys. Rev. Lett. **96**, 216802 (2006).
5. M. C. Goorden and M. Büttiker, Phys. Rev. Lett. **99**, 146801 (2008).
6. A. V. Lebedev, G. B. Lesovik, and G. Blatter, Phys. Rev. Lett. **100**, 226805 (2008).
7. A. Dhar, D. Sen, D. Roy, Phys. Rev. Lett. **101**, 066805 (2008).
8. D. Roy, Phys. Rev. B **80**, 245304 (2009).
9. D. Roy, A. Soori, D. Sen, and A. Dhar, Phys. Rev. B **80**, 075302 (2009).
10. D. Roy, Phys. Rev. B **81**, 085330 (2010).

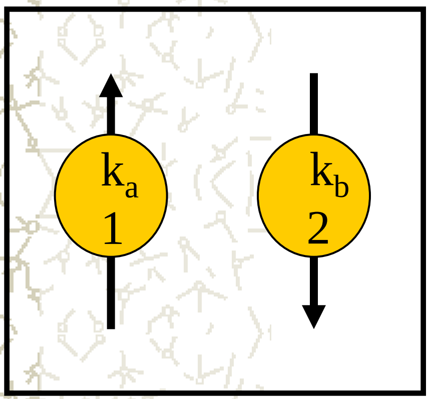
Outlook for…

- What's entanglement between two particles?
  Only qualitative aspect. (10.0min)
- Formal Theory of the T - Matrix.
  Only qualitative aspect. (13.0min)
- Application of T – Matrix to a system one dimensional with three leads. (10.0 min)



Qualitative properties of entangled pure state:

We have one system of two distinguishable particles with spin:



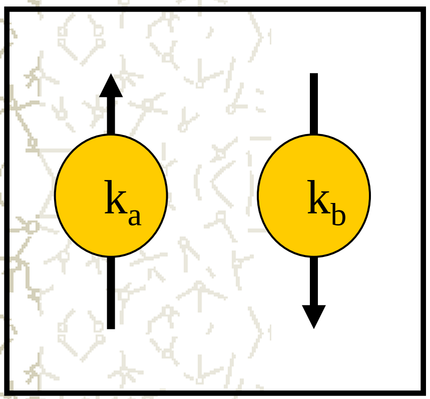
Graphic representation

$$|\Psi\rangle = \underbrace{|k_a \uparrow\rangle}_{\text{Particle 1}} \otimes \overbrace{|k_b \downarrow\rangle}^{\text{Particle 2}}$$

It isn't an entangled state.

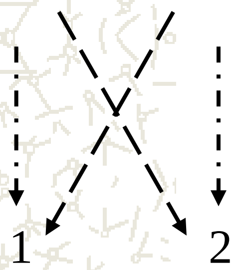
Now, We have two indistinguishable…

To see: J. Schielmann, D. Loss, and A. H. Mac Donald, Phys. Rev. B, **63**, 085311 (2001).



$$\left|k_a,\uparrow;k_b,\downarrow\right\rangle_{Slater} = \frac{1}{\sqrt{2}}\left\{\left|k_a,\uparrow;k_b,\downarrow\right\rangle - \left|k_b,\downarrow;k_a,\uparrow\right\rangle\right\}$$

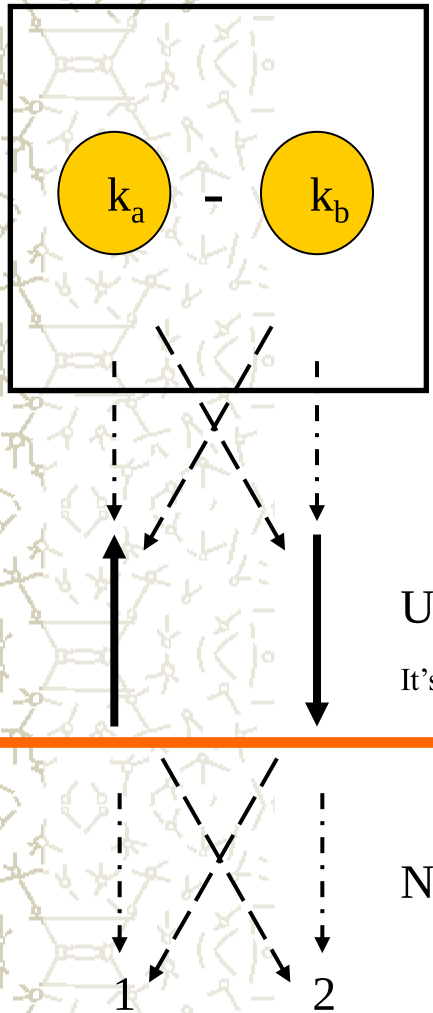
$$\left|k_a,\uparrow;k_b,\downarrow\right\rangle_{Slater} = \hat{a}^{+}_{k_a,\uparrow}\hat{a}^{+}_{k_b,\downarrow}\left|0\right\rangle$$



No useful entanglement

It isn't an entangled state.

Let us look at a singlet state:



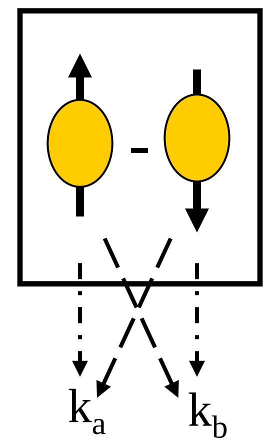
$$|s\rangle = \frac{1}{\sqrt{2}}\left\{ |k_a,\uparrow;k_b,\downarrow\rangle_{Slater} - |k_a,\downarrow;k_b,\uparrow\rangle_{Slater} \right\}$$

$$|s\rangle = \frac{1}{\sqrt{2}}\left\{ a^{+}_{k_a,\uparrow} a^{+}_{k_b,\downarrow} - a^{+}_{k_a,\downarrow} a^{+}_{k_b,\uparrow} \right\}|0\rangle$$

Useful entanglement

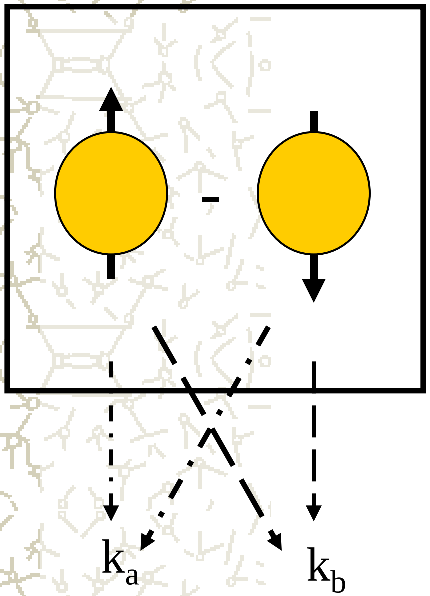
It's entangled with respect to spin degrees of freedom.

No useful entanglement



Another possible graphic representation, but if k_a and k_b are mesoscopic properties the entanglement is not useful, or is local entanglement.

Another singlet state…



$$|s_2\rangle = \frac{1}{\sqrt{2}}\left\{ |k_a,\uparrow;k_a,\downarrow\rangle_{Slater} + |k_b,\uparrow;k_b,\downarrow\rangle_{Slater} \right\}$$

$$|s_2\rangle = \frac{1}{\sqrt{2}}\left\{ \hat{a}^{+}_{k_a,\uparrow}\hat{a}^{+}_{k_a,\downarrow} + \hat{a}^{+}_{k_b,\uparrow}\hat{a}^{+}_{k_b,\downarrow} \right\}|0\rangle$$

State of double occupancies

It’s entangled with respect to orbital degrees of freedom,
but is local entanglement.

J. Schielmann, D. Loss, and A. H. Mac Donald, Phys. Rev. B, **63**, 085311 (2001).

Entanglement

- The proposals for entangling fermions involve physical phenomena that include:

  - Drawing entangled pairs from superconductors
  - Two electron collision with magnetic impurity
  - Two–particle scattering by a finite interacting region:
    Quantum dot Oliver, Yamaguchi, Yamamoto
    Stanford Group

Involve e-e interaction

  - Using Hall edge states and non-entangled particles (electron-hole) Beenakker's group

No e-e interaction

  - Non – interacting particles system are spin-entangled up to some finite distance. V. Vedral, University of Leeds, U.K.

To see: O. Rendón and E. Medina 2012 J.Phys.: Conf. Ser. **338** 012015

Formal theory of the T – Matrix: Only qualitative aspect.

- We have used the usual time independent scattering theory, where V(x) is taken “positive” and to be of finite support.
- The space of configuration of two particles in a system of one dimension is same that one particle in a two dimensional system.

⇓ then

The one-body probability amplitude for scattering from $k_{initial}$ to k_{final},

$$S^{(1)}_{k_{final},k_{initial}} = \delta\left(k_{final} - k_{initial}\right) - 2\pi\delta\left(\varepsilon_{k_{final}} - \varepsilon_{k_{initial}}\right) T^{(1)}_{k_{final},k_{initial}}$$

and two-body probability amplitude are always finite.

$$S^{(2)}_{q_{final},q_{initial}} = \delta^2\left(q_{final} - q_{initial}\right) - 2\pi\delta\left(E_{q_{final}} - E_{q_{initial}}\right) T^{(2)}_{q_{final},q_{initial}}$$

To see: W. Glöckle, The Quantum Mechanical Few – Body Problem, Springer –Verlag, Berlin (1983).

We assume that the Scattering Hamiltonian can be written as:

$$\hat{H} = \frac{P_1^2}{2m} + \frac{P_2^2}{2m} + V(x_1, x_2)$$

Where the two-body potential is:

$$V(x_1, x_2) = V(x_1) \otimes \mathbf{1}_2 + \mathbf{1}_1 \otimes V(x_2) + V(|x_2 - x_1|)$$

Then:

Interaction term among particles

- We computed the Transition Amplitude using the following relation.

$$\hat{T} = \hat{V}\hat{G}(\varepsilon + i\eta)\hat{G}_0^{-1}(\varepsilon + i\eta)$$

$$\hat{G}(\varepsilon + i\eta) = \frac{1}{\varepsilon + i\eta - \hat{H}}$$

- **G** is the Green function of the full Hamiltonian **H** and G_0 is the Green function of the unperturbed Hamiltonian H_0.

If us look at the problem of two particles that hit an obstacle but without interaction among them, so We can factor $T^{(2)}$

$$T^{(2)} = T_1^{(1)} \otimes \mathbf{1}_2 + \mathbf{1}_1 \otimes T_2^{(1)} + T_1^{(1)} \otimes G_{02}^{(1)} + G_{01}^{(2)} \otimes T_2^{(1)}$$

In the case, distinguishable particle and conservative process, $E_{final}=E_{initial}$:

$$\left\langle k'_a,\uparrow;k'_b,\downarrow \middle| T^{(2)}(E) \middle| k_a,\uparrow;k_b,\downarrow \right\rangle = 2\pi\delta\left(k'_b - k_b\right)\left\langle k'_a,\uparrow \middle| T_1^{(1)}(\varepsilon_a) \middle| k_a,\uparrow \right\rangle + 2\pi\delta\left(k'_a - k_a\right)\left\langle k'_b,\uparrow \middle| T_2^{(1)}(\varepsilon_b) \middle| k_b,\uparrow \right\rangle$$

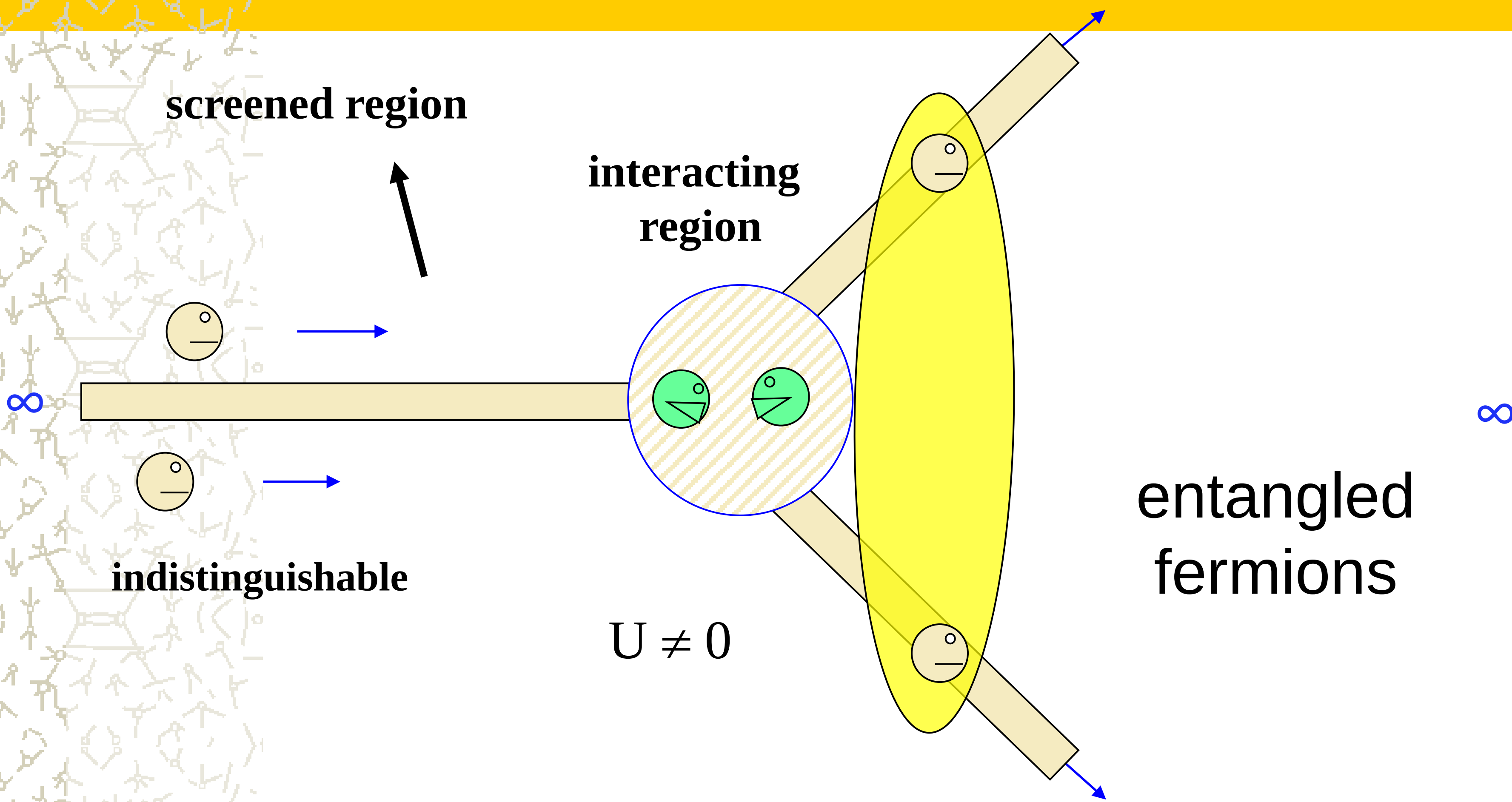
For the case of indistinguishable particle and conservative process, $E_{final}=E_{initial}$:

$$\left| k_a,\uparrow;k_b,\downarrow \right\rangle_{Slater} = \frac{1}{\sqrt{2}}\left\{ \left| k_a,\uparrow;k_b,\downarrow \right\rangle - \left| k_b,\downarrow;k_a,\uparrow \right\rangle \right\}$$

$$_{Slater}\left\langle k'_a,\uparrow;k'_b,\downarrow \middle| T^{(2)}(E) \middle| k_a,\uparrow;k_b,\downarrow \right\rangle_{Slater} = 2\pi\delta\left(k'_b - k_b\right)\,_{Slater}\left\langle k'_a,\uparrow \middle| T_1^{(1)}(\varepsilon_a) \middle| k_a,\uparrow \right\rangle_{Slater} + 2\pi\delta\left(k'_a - k_a\right)\,_{Slater}\left\langle k'_b,\uparrow \middle| T_1^{(1)}(\varepsilon_b) \middle| k_b,\uparrow \right\rangle_{Slater}$$

But this is the usual noninteracting electrons formulation of the scattering matrix in the situation of two-electron.

Interacting Quantum dot



Oliver et al, PRL 88 (2002) 7901

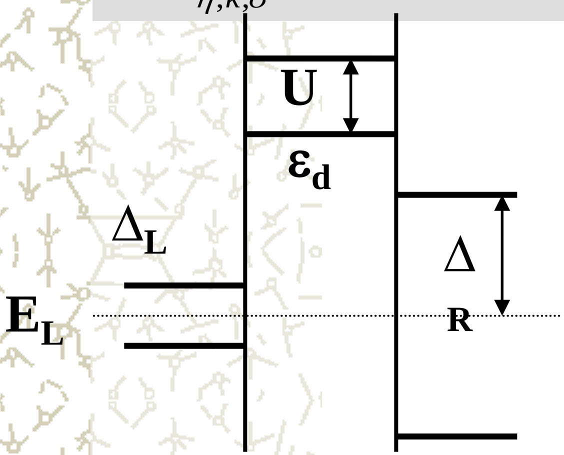
- The system is described by a tight-binding Hamiltonian with an on-site Coulomb energy term U with no single-electron excitations:

Leads **Dot** **e-e interaction within dot**

$$\hat{H} = \hat{H}_0 + \hat{V}$$

$$\hat{H}_0 = \sum_{\eta,k,\sigma} \varepsilon_{\eta,k} \hat{a}^{\dagger}_{\eta,k,\sigma} \hat{a}_{\eta,k,\sigma} + \sum_{\sigma} \varepsilon_d \hat{c}^{\dagger}_{\sigma} \hat{c}_{\sigma} + U \hat{n}_{\uparrow} \hat{n}_{\downarrow}$$

$$\hat{V} = \sum_{\eta,k,\sigma} (V_\eta \hat{a}^{\dagger}_{\eta,k,\sigma} \hat{c}_{\sigma} + h.c.)$$



$$2E_L = (\varepsilon_{L,k} + \varepsilon_{L,k'}) = (\varepsilon_{R_1,k_1} + \varepsilon_{R_2,k_2})$$

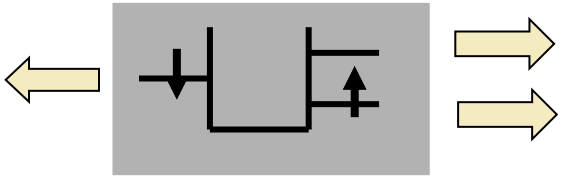
$$\varepsilon_{L,k}, \varepsilon_{L,k'} = E_L \pm \Delta_L$$

$$\varepsilon_{R_1,k_1}, \varepsilon_{R_2,k_2} = E_L \pm \Delta_R$$

- Two electron co-tunneling can occur when $E_i = E_f$ and the two electrons end up on different outgoing leads R_1 and R_2.

Coupling to the leads

Escape to reservoir



Escape to reservoir

- The semi-infinite leads have a semi-circular density of state and a bandwith of $4V_{leads}$.
- They dress the dot with a complex self-energy that shift and broadens the internal dot levels.

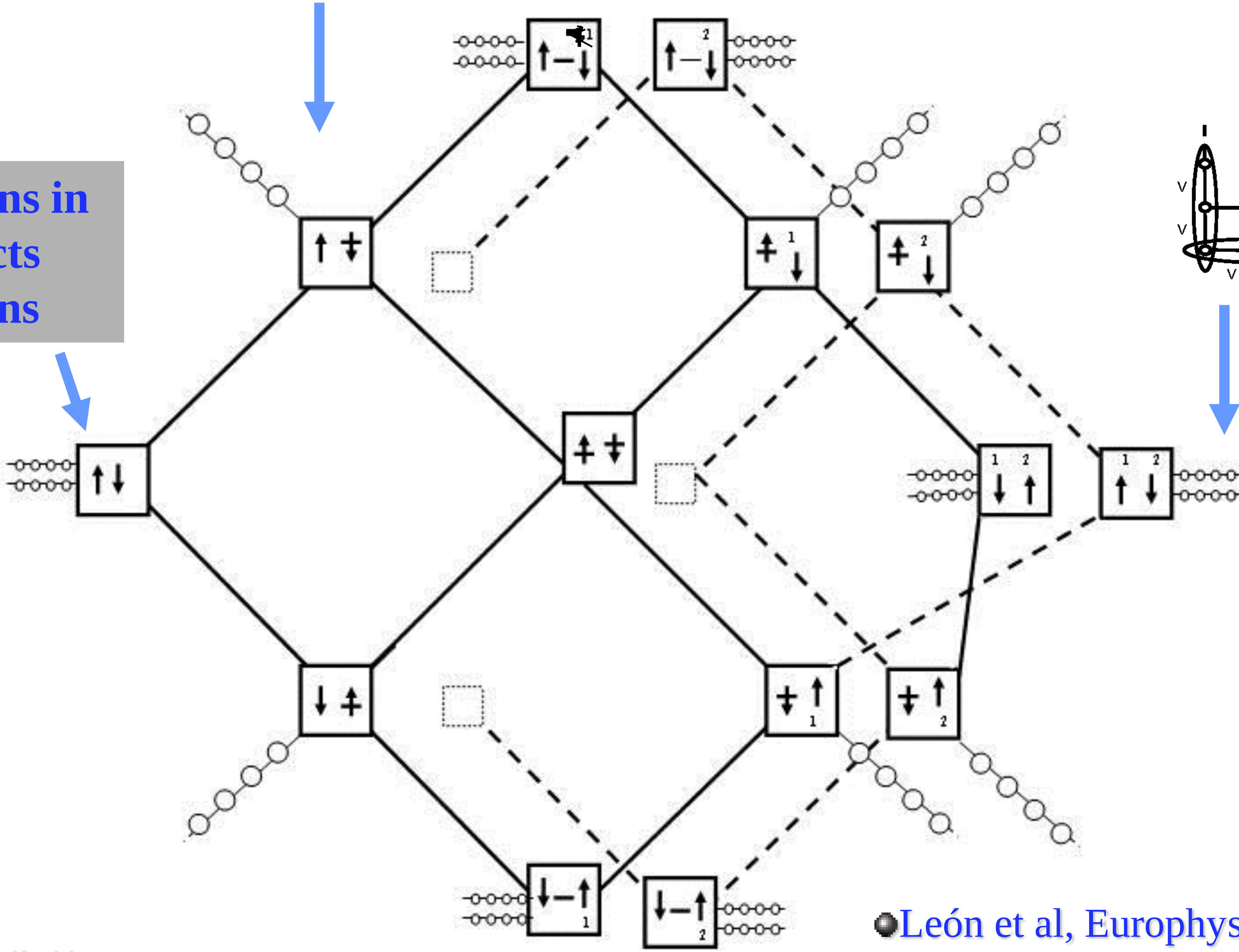
$$\Sigma(\varepsilon) = \varepsilon / 2 + i\sqrt{V_{leads}^2 - (\varepsilon/2)^2}.$$

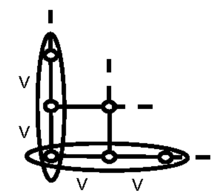
Inclusion of self-energies

Semi-infinite chain with self-energyv

$$\Sigma = \Delta \mp i\Gamma$$

2 electrons in contacts
2 chains

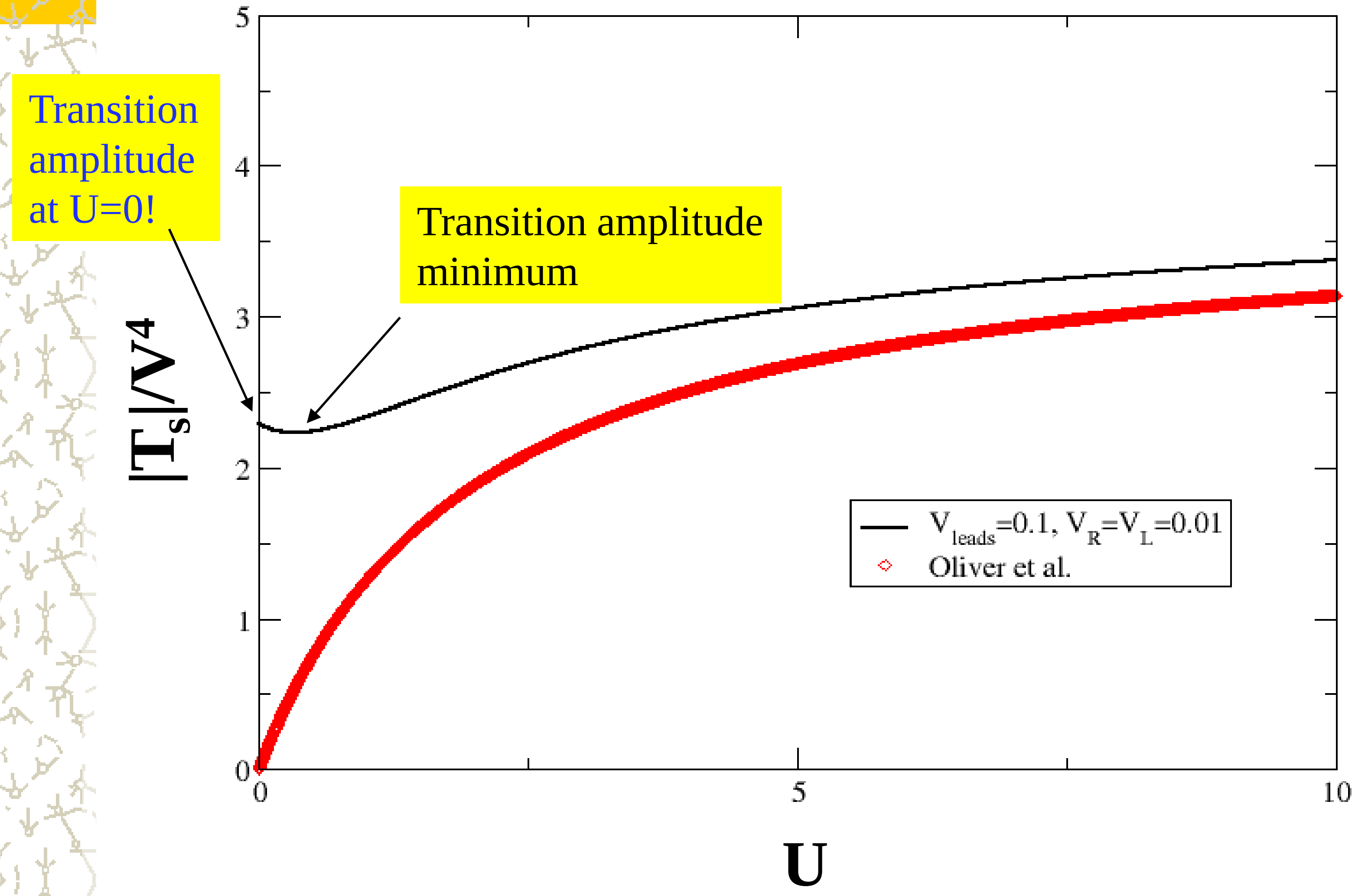




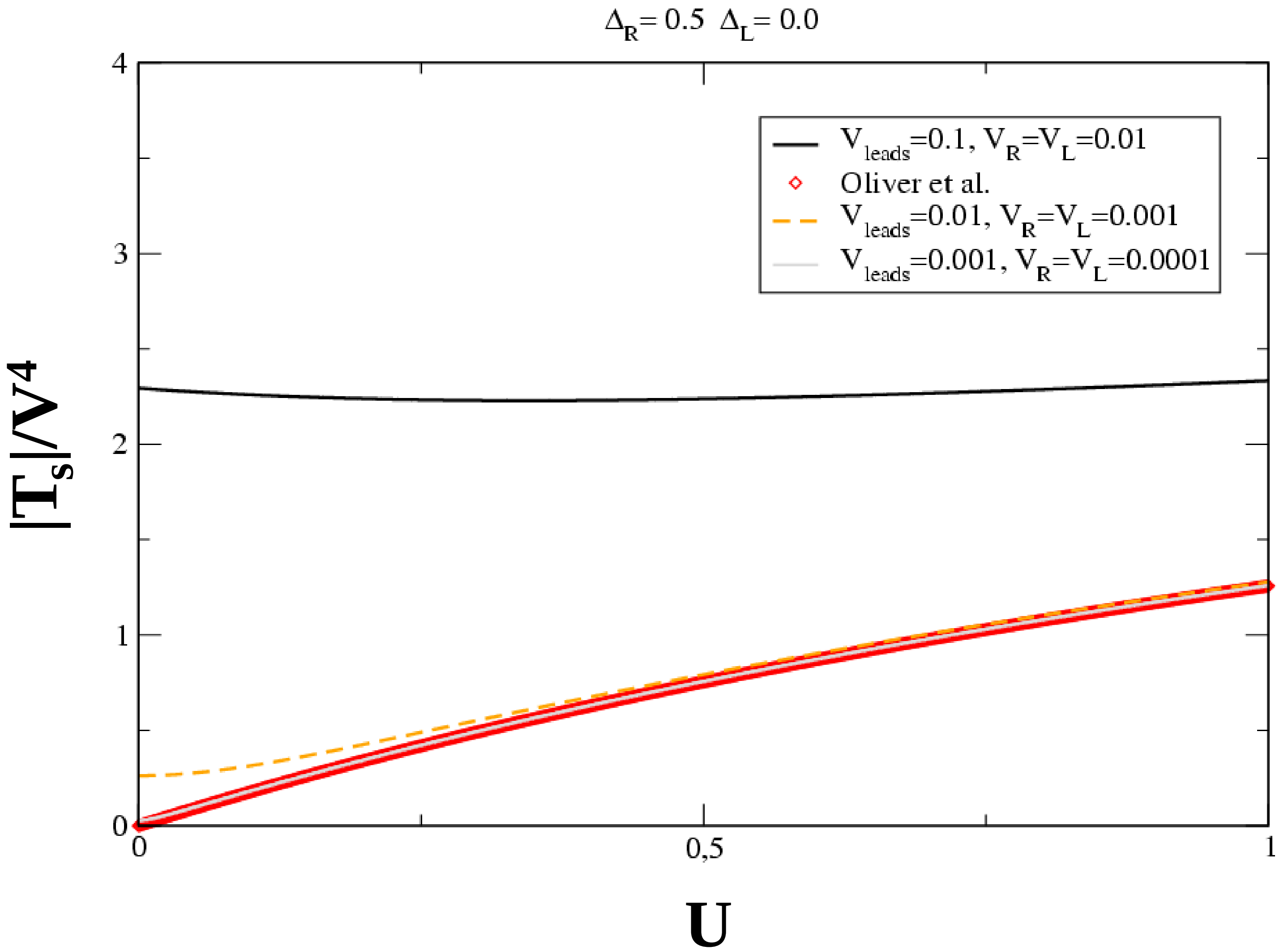
León et al, Europhysics Lett.

Exact results, Degenerate input electrons

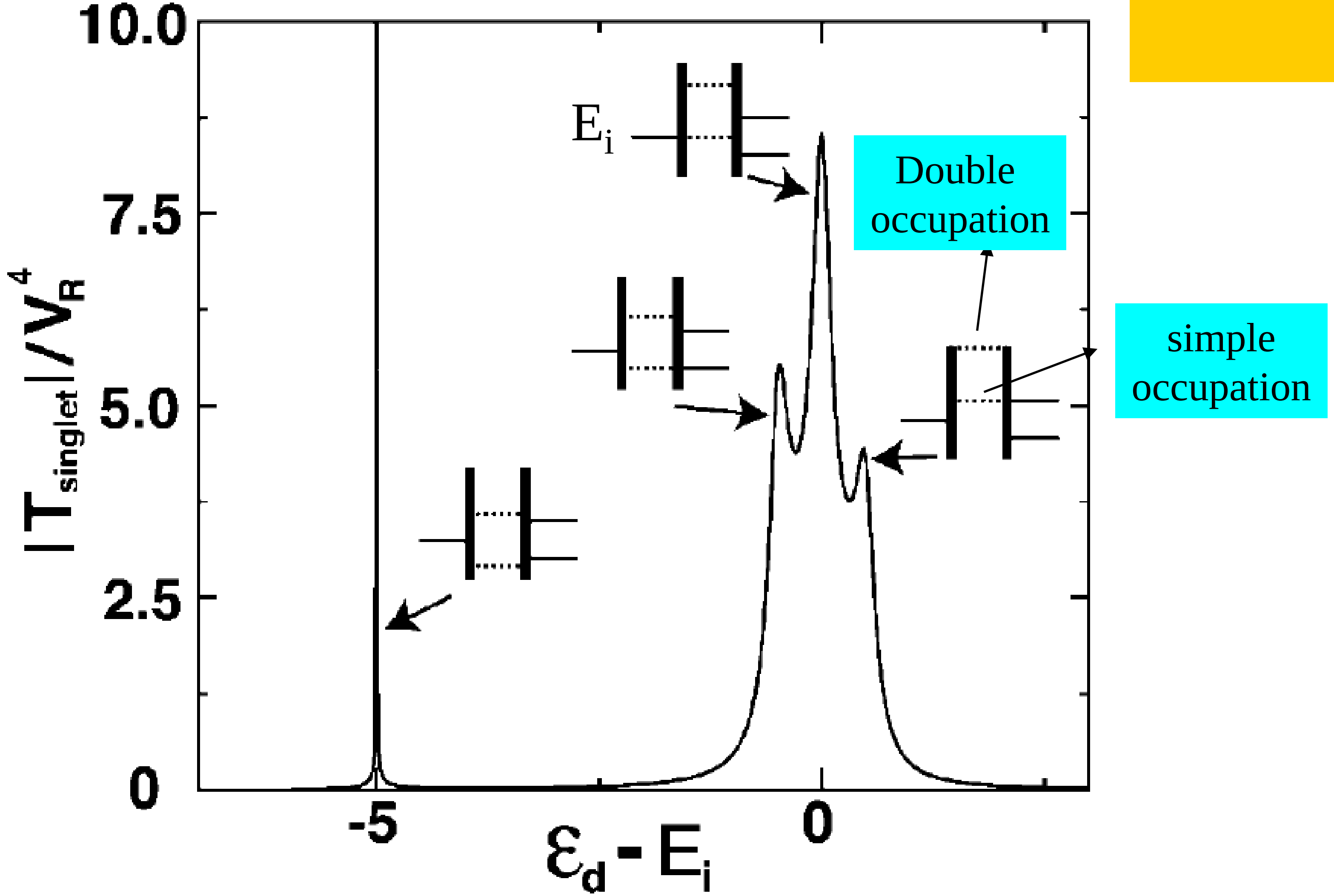
$$\Delta_R = 0.5 \quad \Delta_L = 0$$

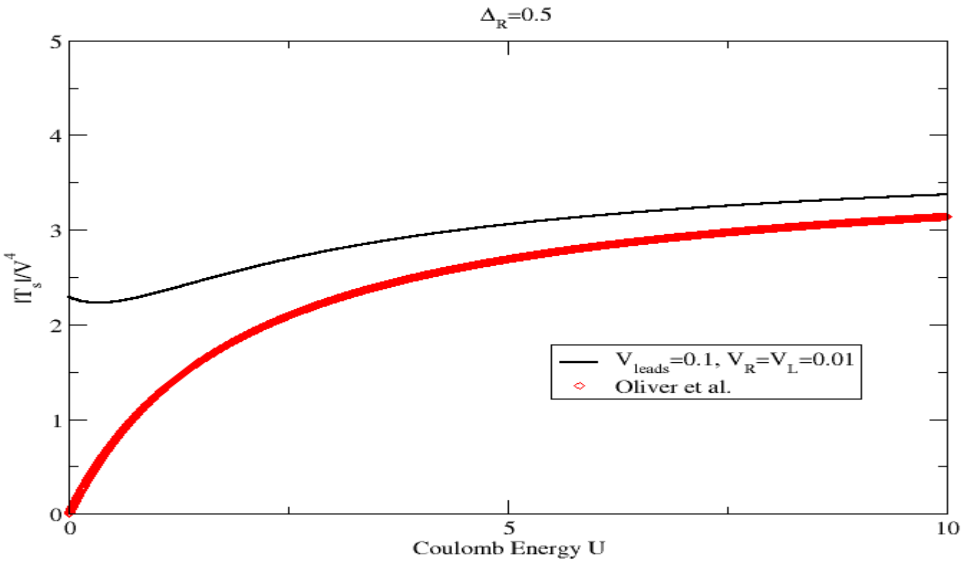


Limit of zero broadening due to leads

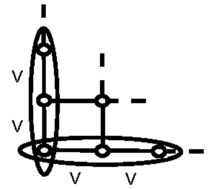


Gate voltage effects



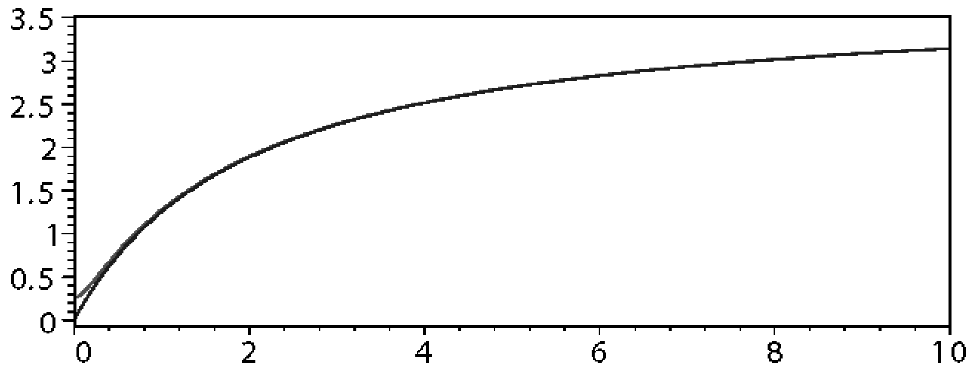


Δ_R=0.5
5
4
3
2
1
0
$|T|/V_s^4$
0
5
10
Coulomb Energy U
V_{leads}=0.1, V_R=V_L=0.01
Oliver et al.

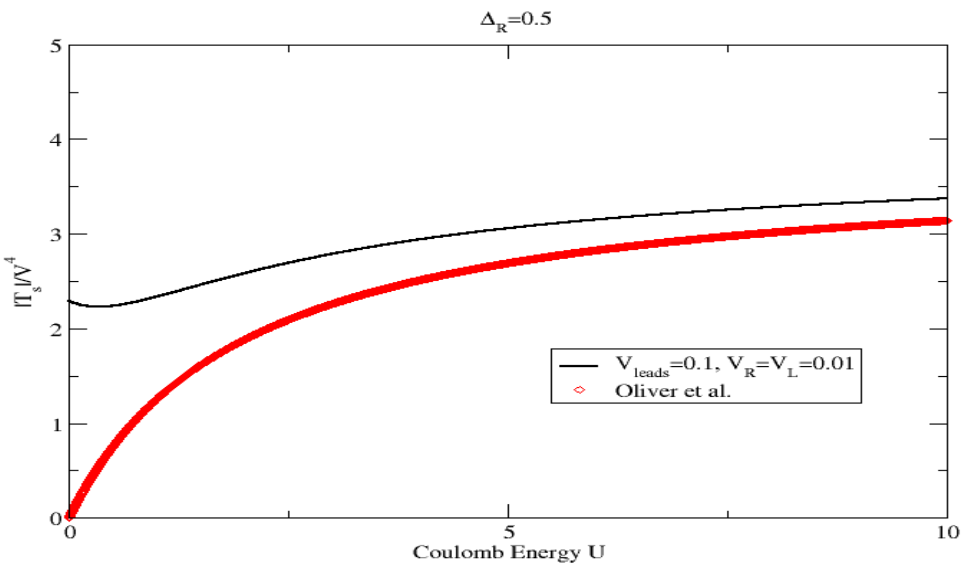


v
v
v
v

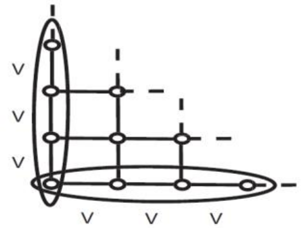
N=2



3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10

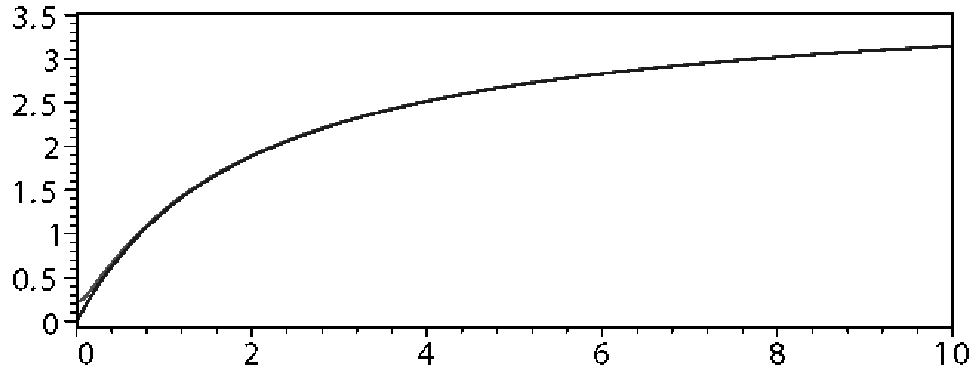


Δ_R=0.5
V_leads=0.1, V_R=V_L=0.01
Oliver et al.
Coulomb Energy U
0
5
10
1
2
3
4

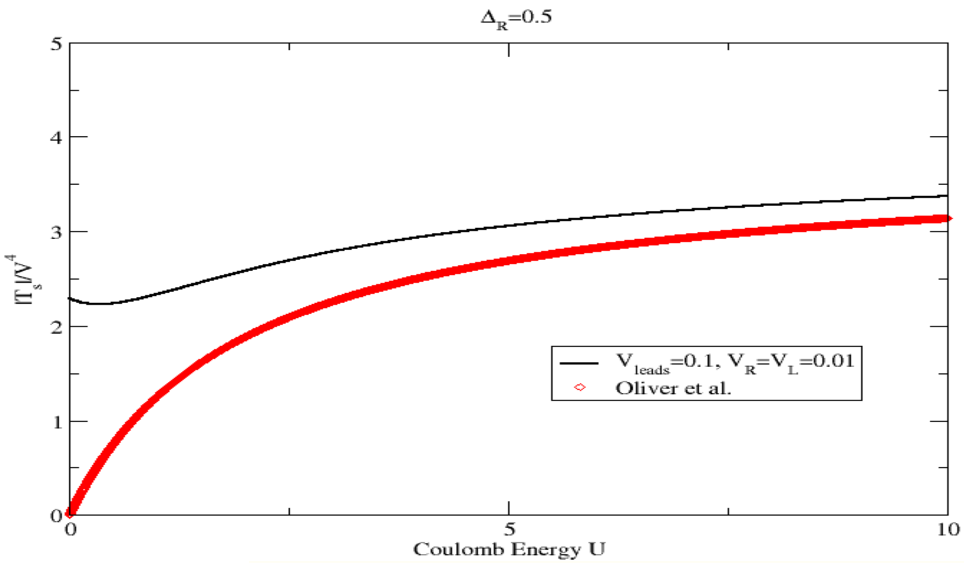


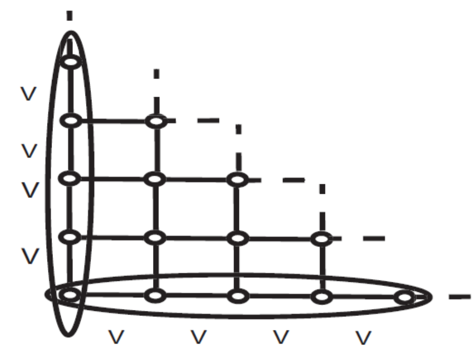
V
V
V
V
V
V

N=3

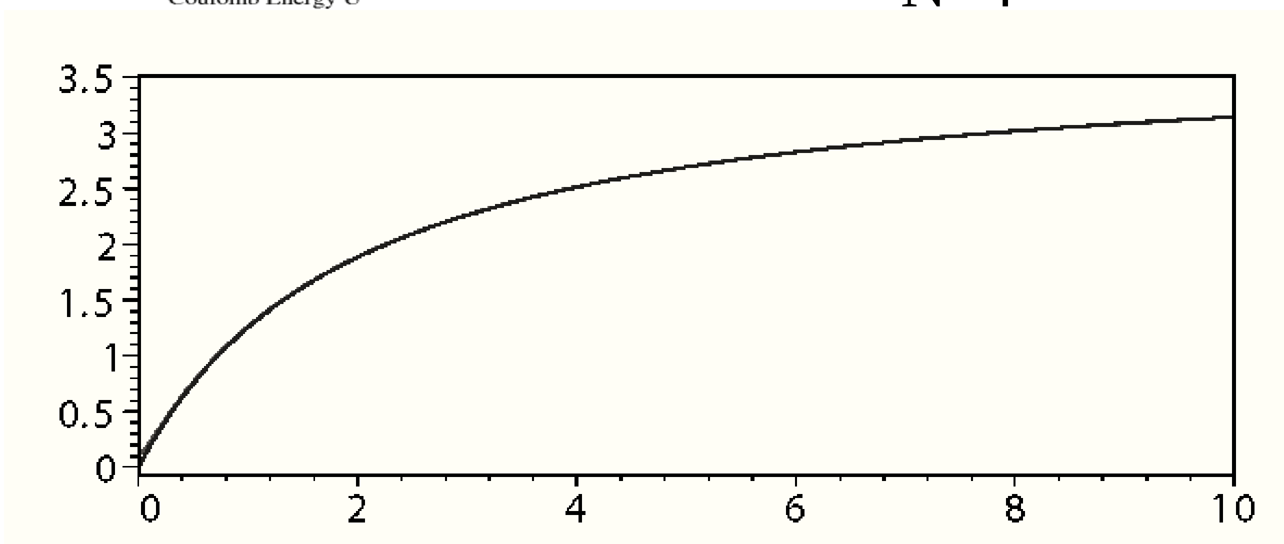


3.5
3
2.5
2
1.5
1
0.5
0
2
4
6
8
10


ΔR=0.5
|T|/Vs4
Vleads=0.1, VR=VL=0.01
Oliver et al.
Coulomb Energy U
0
1
2
3
4
5
0
5
10


V
V
V
V
V
V
V
V

N=4


3.5
3
2.5
2
1.5
1
0.5
0
0
2
4
6
8
10

Conclusion.

For $V_{coupling} << V$.

•At T = 0 K, the devices filters singlet entangled pairs if $U \neq 0$.

•The resonance structure for the singlet transition amplitude is studied as function of energy difference between the input lead and the dot single-particle state.

•In U = 0, there isn't tunneling and the two-fermion scattering matrix tend to the obtained by standard scattering matrix theory for noninteracting electrons