A Brief Introduction to AdS/CFT Correspondence

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Outline of the Talk

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- Introduction

Outline of the Talk

- Introduction
- Motivation

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- Introduction
- Motivation
- Building the Correspondence

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- Introduction
- Motivation
- Building the Correspondence
- Remarks

Introduction

- From its foundations, String Theory was conceived to be a *theory of Strong Interactions*. The *confinement* can be introduce naturally using String methods. Mesons can be seen as a pair of quarks joined by a string.
- With the formal construction of the *gauge theory of strong interactions* in the 70's, String Theory and Strong Interaction fall apart.
- String Theory begin to be considered as the possible *Theory of Everything*.

Introduction

- G. 't Hooft's big idea: Large N expansion. Theories with large N parameter have a simpler diagrammatica
- In the 90's, two great events can be highlighted:
  - L. Susskind gives us the idea of *Holographic Principle* (HP) as a connection between Strings and Gauge Theories.
  - J. Maldacena in 1997 gives a demonstration of the HP and introduces the so-called *Gauge/Gravity Correspondence*
- Applications: Strongly Coupled Systems
  - Quark Gluon Plasma
  - Holographic Superconductors

F. Aprile, J. Russo, "Models of Holographic Superconductivity", arXiv:0912.0480v2 [hep-th].
D. Mateos, "String Theory and QCD" , arXiv:0709.1523v1 [hep-th].

Motivation: Large N Expansion

- The idea is to construct a link between a gauge theory and String theory. This link is the *large N expansion*.
- *'t Hooft's idea:* consider any pure gauge theory $U(N)$, take the limit $N \to \infty$ and then expand the physical quantities in $1/N$.
- we define the 't Hooft coupling

$$\lambda = g_{YM}^2 N \tag{1}$$

- Any amplitude, for example the vacuum – to – vacuum amplitude, can be written in terms of $1/N$ and the 't Hooft coupling as

$$\mathcal{A} = \sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda) \tag{2}$$

where (2) describes the diagrammatica of theory. From the large expansion, if we impose *fixed* λ, we can organize the diagrammatica by its topology into *planar* and *non-planar* diagrammatica.

G. 't Hooft, "A planar diagram theory for strong interactions". Nucl. Phys. B **72, 461 (1974)**

Motivation: Large Expansion for a $U(N)$ theory

Let's consider a gauge theory with symmetry group $U(N)$ with a Lagrangian given by

$$\mathcal{L} = \frac{1}{g_{YM}^2} \mathrm{Tr}\left[(\partial M)^2 + M^3 + M^4\right] \tag{3}$$

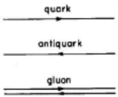
with $U(N)$ invariance. In general, from (3) we obtain the following scalings for the propagator and the vertices

$$D(x,y) \approx \frac{\lambda}{N} \tag{4}$$

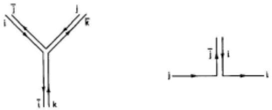
$$\text{Any Vertex} \approx \frac{N}{\lambda} \tag{5}$$

Motivation: Large N Expansion for a $U(N)$ Theory

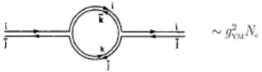
For simplicity, let's consider only vacuum diagrams, and let's introduce the double line notation to keep track of the matrix indices. So, the vertices, closed lines and the propagators are represented by to *fundamental*– *anti–fundamental* lines.



Double-line notation.



Vertices in double-line notation.



Gluon self-energy diagram in double-line notation.

Motivation: Large N Expansion for a $U(N)$ Theory

In a general way, for a diagram with V vertices, E propagators and F loops, the vacuum–to–vacuum amplitude can be written as

$$\mathcal{A} \approx N^{V-E+F} \lambda^{E-V} = N^{\chi} \lambda^{E-V} \tag{6}$$

with $\chi = E - V + F$ is the Euler Character of the diagram. If the diagram is *closed and orientable* we have

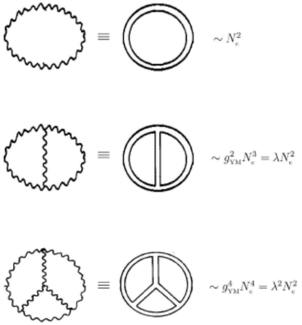
$$\chi = 2 - 2g \tag{7}$$

where we have introduces the *genus* of the 2–dimesional Feynman diagram.

Motivation: Large N expansion for a $U(N)$ Theory

In the 't Hooft limit, the diagrammatica splits into *planar* and *non–planar* diagrams:

Planar diagrams



Non–planar diagrams

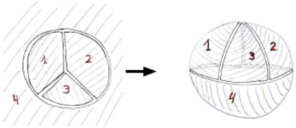
Conclusion

We thus see that non–planar diagrams are suppressed by higher powers of $1/N^2$ in the large–N expansion.

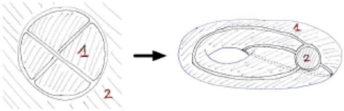
Motivation: Large N expansion for a $U(N)$ Theory

Let's introduce the concept of *Riemann Surface* to each diagram

Planar diagrams



Non–planar diagrams



The planar diagrams are related to *spheres*, and non–planar diagrams are related to *torus*. This surfaces are associated to the term N^{2-2g} in the expression for the amplitude, where we now define g as the genus of a compact, orientable and with no boundaries Riemann surface.

Motivation: Large N expansion for a $U(N)$ Theory

Conclusion

The expansion of any gauge theory diagrammatica takes the form

$$\mathcal{A} = \sum_{g=0}^{\infty} N^{2-2g} \sum_{n=0}^{\infty} c_{g,n} \lambda^n \tag{8}$$

where $c_{g,n}$ are constants.

Some remarks

- The first sum corresponds to the *loop expansion in Riemann Surfaces for a Closed String Theory* with coupling $g_s \sim 1/N^2$
- The second sum corresponds to the α*–expansion* in String Theory

This is the key for the correspondence! Any gauge theory can be written in terms of a String Theory! This is the so-called *Gauge/Gravity Correspondence*

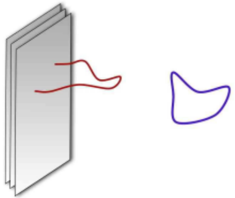
The AdS/CFT Correspondence

- The simplest example of a gauge/gravity duality: the equivalence between type IIB String Theory on $AdS_5 \times S^5$ and $\mathcal{N} = 4$ Super Yang Mills (SYM) Theory on 4–dimensional Minkowski space.

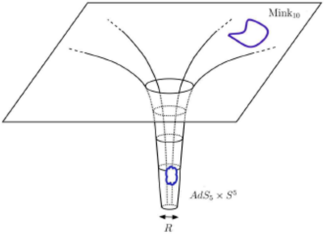
J. Maldacena, "The Large N limit for superconformal field theories and supergravity" . Adv. Theor, Math, Phys. **2, 231, 1998.**

The AdS/CFT Correspondence: Decoupling limit

Let's consider the "ground state" of a type IIB String Theory in presence of N D3–branes.



Since the D-branes carry mass and charge, they curved the spacetime



- far away from the branes the space is flat
- near to the branes we have the "throat" geometry of $AdS_5 \times S^5$

The AdS/CFT Correspondence: Decoupling Limit

Let's compare the gravitational radius R of the D3–branes with the string length (in string units).

$$\frac{R^4}{l_s} = 4\pi g_s N \tag{9}$$

This give us two descriptions

$$g_s N \ll 1 \tag{10}$$

- we have a description in terms zero–thickness objects in a flat space
- D3–branes are described as a defect in the spacetime, i.e., a boundary conditions for open string

$$g_s N \gg 1 \tag{11}$$

- The backreaction of the branes becomes important, i.e., it can not be neglected
- the size of the near–brane $AdS_5 \times S^5$ becomes large in string units, so the description in terms of an effective geometry for closed strings becomes simple

The AdS/CFT Correspondence: Decoupling Limit

We can now motivate the Correspondence in terms of excitations around the ground state in the two descriptions and taking the *decoupling limit*. For the First description we have:

- Excitations consist on open and closed strings, interacting with each other.
- At low energies, quantization of open strings give us a $\mathcal{N} = 4$SYM multiplet plus a tower of massive excitations propagating on the worldvolume of the branes: a $3+1$–dimensional flat space.
- Similary, quantization of the closed strings leads to a massless graviton supermultiplet plus a tower of massive modes, which vanish in this limit, becoming the closed strings into non-interacting objects (infrared freedom)

Conclusion

At low energies, open strings decouple from closed strings

$$S_{D3} \sim -T_{D3} \int d^4x \sqrt{-det\left(\eta^{\mu\nu} + \alpha'^2 F_{\mu\nu}\right)} \sim -\frac{1}{g_s} \int d^4x\, F_{\mu\nu}^2 \qquad (12)$$

The AdS/CFT Correspondence: Decoupling Limit

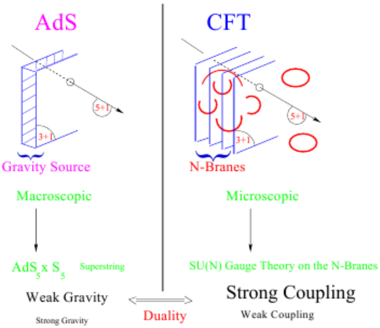
For the second case we have

- Low energy limit consist on focusing on the excitations that have arbitrary low energy with respect of to an observer in the asymptotically flat Minkowski region.
- we have two sets of degrees of freedom
  - those propagating in the Minkowski region
  - those propagating in the throat,
- At low energies only remain the massless 10–dimensional graviton supermultiplete modes in the Minkowski region, which decouple from the modes in the throat too, since the wave–length of the throat modes becomes larger than the size of the throat, R.
- In the throat region, the massive strings modes survives, i.e., the string is located sufficiently deep down in the throat.

Conclusion

At low energies, we have interacting closed strings in $AdS_5 \times S^5$ plus free gravity in flat 10–dimensional spacetime. Type IIB String Theory reduces to type IIB Supergravity

The AdS/CFT Correspondence: The Conjecture



Conjecture (The AdS/CFT Correspondence)

4–dimensional $\mathcal{N} = 4$ $SU(N)$ SYM and type IIB Sring Theory on $AdS_5 \times S^5$ are two different descriptions of the same underlying physics, and we say that the two theories are dual to each other

The AdS/CFT Correspondence: Matching Parameters

Let's examine the parameters that enter the definition of each theory, and the relation (map) between them.

- Gauge Theory: Is specified by the 't Hooft coupling $\lambda = g_{YM} N^2$ and the parameter N.
- String Theory: Is specified by the coupling constant g_s and the size of the AdS_5 and S^5 spaces, which are *maximally symmetric* , i.e., the spaces are totally specified by a single scale: the curvature radius R.
- Both spaces in the string solutions are sourced by D3–branes have the same radii, so this imply

$$\frac{R^2}{\alpha'} \sim \sqrt{g_s N} \sim \sqrt{\lambda} \tag{13}$$

- String Coupling is related to the Gauge Theory parameters through

$$g_s \sim g_{YM}^2 \sim \frac{\lambda}{N} \tag{14}$$

which means that, for a fixed-size $AdS_5 \times S^5$ geometry (λ fixed), the string loop expansion corresponds to the $1/N$ expansion in the Gauge Theory.

The AdS/CFT Correspondence: Matching Parameters

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Conclusion

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- Equivalently, If we take the radius in Planck units

$$\frac{R^4}{l_p^4} \sim \frac{R^4}{\sqrt{G}} \sim N \tag{16}$$

Conclusion

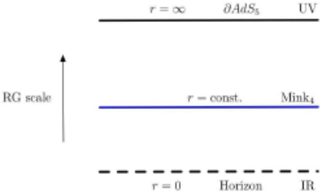
Quantum corrections on the string side are suppressed by powers of $1/N$, so classical limit of the string side corresponds to the planar limit of the Gauge Theory

The AdS/CFT Correspondence: Matching Symmetries

Why AdS_5

$$dS^2 = \frac{r^2}{R^2}\left(-dt^2 + \sum_{i=1}^{3} dx_i\, dx^i\right) + \frac{R^2}{r^2} dr^2 \tag{17}$$

- x^{μ} coordinates lies along the wordlvolume of the D3–branes, so they are connected with the Gauge Theory coordinates.
- The coordinate r (and those over S^5), span in direction transverse to the branes.



Conclusion

AdS_5 is foliated by r- constant slices, each of which is isometric to 4–dimensional Minkowski space time. If $r \to \infty$, we are in the conformal boundary, where the Gauge Theory lives

The AdS/CFT Correspondence: Matching Symmetries

What about the Gauge Theory?

- $\mathcal{N} = 4$ SYM is a conformal theory, i.e., is invariant under dilatations given by

$$D: \; x^\mu \to \Lambda x^\mu \tag{18}$$

  As we would expect, (18) is also a symmetry of (17). Indeed, if we take the rescaling $r \to r/\Lambda$ we can show it.

- this means that short distance physics in the Gauge Theory is related to physics near the *AdS* boundary, where we have long distance physics
- r can be identified as a Renormalization Group (RG) scale of the gauge theory, which in this case has a trivial flow (since D is a isometry of AdS_5)

Conclusion

Symmetries are the same on both sides of the duality

How can we map observables in the two theories?

- Since $g_s \sim g_{YM}^2$, we can conjecture that
$$Z\left[\phi\right]_{CFT} = Z\left[\mathbf{\Phi}|_{\partial \mathbf{AdS}}\right] \tag{19}$$
which means that Any operator in the gauge side is sourced by a string field on the boundary of AdS
- Given the correspondence of the gauge symmetries on the string side, we expect that the field dual to a conserved current j^μ to be the gauge field A_μ. So, the coupling
$$\int d^4x\, A_\mu\left(x\right) J^\mu\left(x\right) \tag{20}$$
must be gauge invariant under transformations $\delta A_\mu = \partial_\mu f$.
- A particular set are the translationally invariant currents with energy momentum tensor $T_{\mu\nu}$ which couple to a 2–spin field, namely, the graviton
$$\int d^4x\, T_{\mu\nu}\left(x\right) g^{\mu\nu}\left(x\right) \tag{21}$$

Remarks

- The AdS/CFT Correspondence described here is not proven.
- The Correspondence is a deep statement about the equivalence of two completely different theories.
- Any String vacuum must have a dual Gauge Theory. The real problem is to find it. (For example, the QCD string dual is unknown)
- AdS/CFT Correspondence is our most concrete implementation of the Holographic Principle: A quantum gravity living in the bullk of a given d+1–spacetime is equivalent to a gauge theory residing on its boundary.