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Weyl-Dirac Equation in Condensed Matter Physics: Graphene

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Contents

Dirac Equation (1928)

$$\boxed{\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x) = 0}$$

Covariant form, Invariance under a Lorentz transformation*

$$\psi'(x) = U\psi(x)$$

$$\left[\gamma^{\mu}, \gamma^{\upsilon}\right]_{+} = 2\mathrm{g}^{\mu\upsilon}$$

$\psi(x)$ **4-component spinor.**
Decoupled when m=0.

Dirac Lagrangian and Transformation

$$L(x) = \overline{\psi}(x)\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(x)$$

Mass: Flux of the grav. Field through a surface enclosing the particle

if $$\psi'(x') = S(\Lambda)\psi(x)$$

DE retains its Lorentz-invariant form when:

$$S(\Lambda) = e^{-(i/2)\alpha_{\mu\upsilon}\Sigma^{\mu\upsilon}} \qquad \Sigma^{\mu\upsilon} = -\frac{1}{4i}\left[\gamma^{\mu}, \gamma^{\upsilon}\right]$$

$\Sigma^{\mu\upsilon}$ **Spin term (Gen. Of the Lorentz group)**

↓

Finite mass particle

Massless Weyl Equation

- Setting m=0 in DE → Weyl Eq.
- 4-comp. Parity → 2-comp. Maximal parity violation (x2 eq).
- Invariant under the global (chirality) transformation:

$$\psi \rightarrow e^{i\alpha\gamma_5}\psi$$

$$\psi_{\pm}(x) = \frac{1}{2}\left(1 \pm \gamma_5\right)\psi(x) \quad \Longrightarrow \quad \left(\partial_t + \sigma \cdot \nabla\right)\psi_{\pm}(x) = 0$$

Two uncoupled Weyl equations (reduced representation). In momentum space => Chirality.

Weyl's formulation(1)

- Weyl gauge theory: Invariance and rescaling.
- Conformal vs. Weyl gauge symmetry.

$$g'_{\mu\upsilon} = S(x)\eta_{\mu\upsilon}$$

- Weyl-Cartan geometry: The length of a vector has no absolute geometric meaning.
- Conf. Transformation in Riemann space: The *mapping* is conformal if

$$g_{\mu\upsilon}(x')dx'^{\mu}\, dx'^{\upsilon} = s(x')g_{\mu\upsilon}(x)dx^{\mu}dx^{\upsilon}$$

Weyl's formulation(2)

- Group W_r in V_4 (Riemann space) is defined by:

Weyl (conformal) rescalings of the metric

$$g_{\mu\upsilon}(x) \rightarrow g^{r}{}_{\mu\upsilon}(x) = s(x) g_{\mu\upsilon} \equiv e^{2\lambda(x)} g_{\mu\upsilon}$$

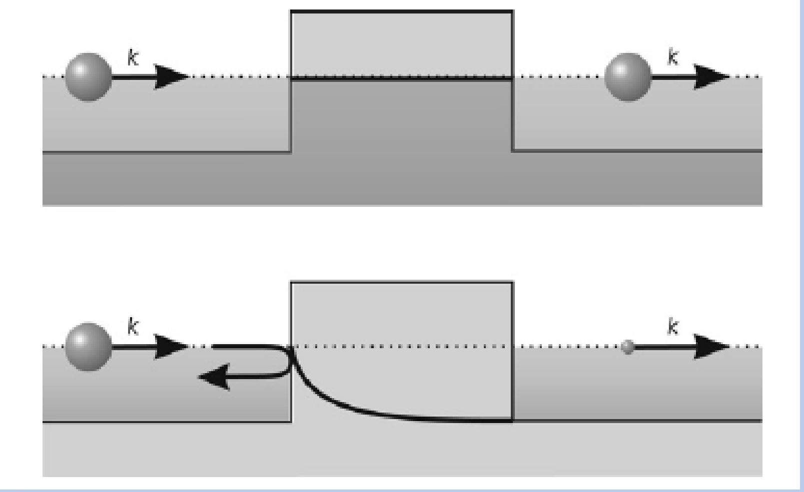
and transf. All other dynamical variables

$$\phi \rightarrow \phi^{r} = [s(x)]^{\omega/2} \phi \equiv e^{\omega\lambda} \phi$$

- ω:= Weyl dimension of the field.

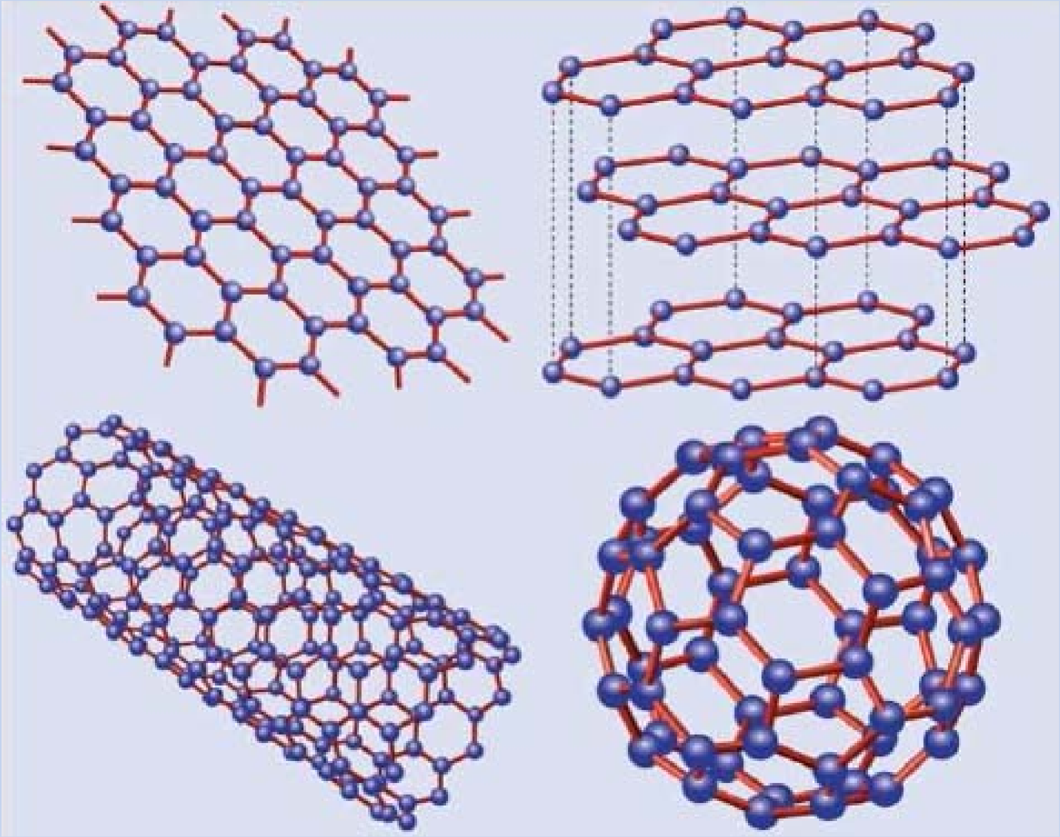
<u>Klein Paradox</u> (Chiral Tunneling)

- *"…If the potential is on the order of the electron mass, the barrier is nearly transparent."*



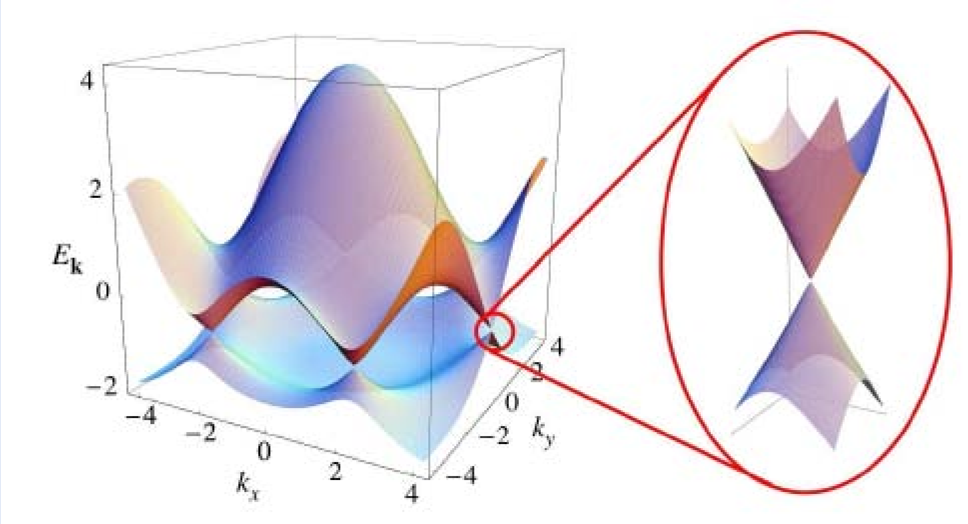
- RQM can be consistently formulated only in terms of fields rather than individual particles.

Graphene and Nanotubes



Graphene and Nanotubes (2)

- Electronic structure of graphene: Dirac points.



$$\hat{H}_0 = -i\hbar v_f \sigma \nabla \qquad v_f \approx 10^6 m/s$$

Graphene and Nanotubes (3)

- New bridge between condensed matter physics and QED.
- Dirac-like electrons => honecomb structure.
- Quasiparticles in graphene $E = \hbar k v_f$
- Pseudospin degree of freedom (Sublattice)
- e- and e+ (holes) are interconnected.
- Coherent transport in graphene (impurities don’t scatter).

Research

- Electric transport through C-nanotubes and graphene nanoribbons double quantum dots coupled by a superconductor.

    Non-equilibrium Green's functions.

References

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Tks!