

*Escuela de Física-Matemática 2011*

# Teorías cuánticas conformes y sus aplicaciones

*Departamento de Matemáticas*

*Departamento de Física*

*Universidad de los Andes*

*16 Mayo – 20 Mayo 2011*

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Lunes, 16 de Mayo (Salón: R-209)

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8:00 – 9:00 *Registration*

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9:00 – 9:15 *Opening*

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9:15 – 10:15 Germán Sierra: *General aspects of CFT*

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10:15 – 11:15 Giuseppe Mussardo: *CFT and statistical physics*

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11:15 – 11:45 Break

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11:45 – 12:45 Daniel Cabra: *CFT and condensed matter*

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12:45 – 14:00 Lunch Break

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14:00 – 14:40 Jean Carlos Cortissoz: *The Probabilistic Method and Navier-Stokes*

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14:40 – 15:20 Juan Camilo Orduz: *Prequantization and Dirac structures*

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15:20 – 15:50 Break

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15:50 – 16:30 Alexander Cardona: *Geometric structures on infinite dimensional spaces induced by trace defects*

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16:30 – 17:10 Mikhail Malakhaltsev: *Hopf bundle and quantum mechanics*

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Martes, 17 de Mayo (Salón: R-109)

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9:00 – 10:00 Germán Sierra: *General aspects of CFT*

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10:00 – 11:00 Giuseppe Mussardo: *CFT and statistical physics*

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11:00 – 11:30 Break

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11:30 – 12:30 Daniel Cabra: *CFT and condensed matter*

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12:30 – 14:00 Lunch Break

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14:00 – 14:40 Nicolás Avilán: *Cuantización por grupo canónico*

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14:40 – 15:20 Juan Manuel Guerra: *Weyl-Dirac Equation In Condensed Matter Physics:  
Graphene*

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15:20 – 15:50 Break

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15:50 – 16:30 Sofyan Iblisdir: *Boundary quantum critical phenomena with entanglement renormalization*

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16:30 – 17:10 Miguel Ángel Martín Contreras: *A Brief Introduction to the AdS/CFT Correspondence*

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Miercoles, 18 de Mayo (Salón: R-209)

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9:00 – 10:00 Germán Sierra: *General aspects of CFT*

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10:00 – 11:00 Giuseppe Mussardo: *CFT and statistical physics*

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11:00 – 11:30 Break

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11:30 – 12:30 Daniel Cabra: *CFT and condensed matter*

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Jueves, 19 de Mayo (Salón: R-109)

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9:00 – 10:00 Germán Sierra: *General aspects of CFT*

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10:00 – 11:00 Giuseppe Mussardo: *CFT and statistical physics*

---

11:00 – 11:30 Break

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11:30 – 12:30 Daniel Cabra: *CFT and condensed matter*

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12:30 – 14:00 Lunch Break

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14:00 – 14:40 Andrés Ángel: *Topología de Cuerdas*

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14:40 – 15:20 Andrés Vargas: *Conformal Killing vectors and twistor spinors in Riemannian geometry*

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15:20 – 16:00 Monika Winklmeier: *On the spectrum of the Klein-Gordon equation*

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16:00 – 17:00 Break

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17:00 – Movie: *Ludwig Boltzmann: the genius of disorder* (by G. Mussardo)

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Viernes, 10 de Mayo (Salón: R-209)

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9:00 – 10:00 Germán Sierra: *General aspects of CFT*

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10:00 – 11:00 Giuseppe Mussardo: *CFT and statistical physics*

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11:00 – 11:30 Break

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11:30 – 12:30 Daniel Cabra: *CFT and condensed matter*

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12:30 – 14:00 Lunch Break

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14:00 – 14:40 Alonso Botero: *Entanglement Invariants, Asymptotic Rates and the Quantum Method of Types*

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14:40 – 15:20 Mohammed El Aïdi: *The lower bound of the spectrum of the Schrödinger operator defined in an open set of a geodesically complete Riemannian Manifold*

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15:20 – 15:50 Break

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15:50 – 16:30 Leonardo Cano: *Geometria espectral de variedades con esquina y operadores de Schrödinger de varios cuerpos*

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# *Morning Lectures*

**Daniel Cabra** (Universidad Nacional de la Plata, Argentina)

## *CFT and condensed matter*

1. One dimensional strongly correlated systems: Luttinger liquid
2. Bosonization and low energy behaviour. CFT description
3. Hubbard and Heisenberg models and the Bethe Ansatz
4. Applications of the low energy CFT description to spin chains and ladders
5. Thermodynamics and transport properties. Comparison with experiments

**Giuseppe Mussardo** (SISSA and INFN, Trieste, Italia)

## *CFT and statistical physics*

1. Critical Phenomena in Statistical Physics and the challenge they pose. Examples.
2. Renormalization Group analysis: fixed point, deformations and scaling region.
3. Continuum limit, quantum field theory approach. Examples: Ising model as Majorana fermion, Tricritical Ising Model,  $O(N)$  model.
4. Integrable models in StatPhys, scattering theory and bootstrap program.
5. Breaking of integrability and analysis of the classes of universality.

**Germán Sierra** (CSIC, Madrid, España)

## *General aspects of CFT*

1. Conformal group. Energy momentum tensor. Virasoro algebra.
2. Conformal blocks and null vectors.
3. Vertex operators. Coulomb gas. Minimal models and WZW models.
4. Fusion and braiding matrices. Modular invariance.
5. Entanglement in CFT.

# *Short Communications*

***Andrés Angel***

(Universidad de los Andes, Colombia)

## **Topología de cuerdas**

La topología de cuerdas es el estudio de estructuras algebraicas asociadas al espacio de lazos libres en una variedad diferenciable de dimensión finita. En esta charla presentaré distintas encarnaciones de estas estructuras algebraicas y posibles generalizaciones.

***Nicolás Avilán***

(Universidad de los Andes, Colombia)

## **Cuantización por grupo canónico**

Se presenta el método de cuantización canónica, el cual parte del estudio de un grupo que actúa apropiadamente sobre el espacio de configuración del sistema clásico (el grupo canónico) y del estudio de sus representaciones unitarias irreducibles, con el fin de obtener operadores cuánticos y un espacio de funciones de onda. Se examinan las restricciones del método y se ilustran algunos ejemplos en los cuales ha sido aplicado de manera exitosa. Finalmente se examinan sistemas clásicos en los que se espera realizar futuras aplicaciones.

***Alonso Botero***

(Universidad de los Andes, Colombia)

## **Entanglement Invariants, Asymptotic Rates and the Quantum Method of Types**

The so-called Quantum Method of Types, a quantum version of the classical information-theoretic method of counting typical sequences, provides an elegant connection between certain asymptotic rates in Quantum Information and the invariants that serve to characterize entanglement classes (LU or SLOCC). We shall briefly discuss this connection and some partial results obtained thus far.

***Leonardo Cano***

(Universidad de los Andes, Colombia)

## **Geometría espectral de variedades con esquina y operadores**

de Schrödinger de varios cuerpos

***Alexander Cardona***

(Universidad de los Andes, Colombia)

### **Geometric structures on infinite dimensional spaces induced by trace defects**

$\zeta$ -function regularized traces can be used as a tool, in infinite-dimensional geometry, to define analogs of usual geometric structures (e.g. symplectic or Dirac structures) in the infinite-dimensional setting, and to build topological invariants. In this talk we will introduce the analytic setting needed in order to define the  $\zeta$ -function regularization of traces and such tools will be used to define geometric structures on infinite-dimensional Lie algebras and infinite-dimensional manifolds.

***Jean Carlos Cortissoz***

(Universidad de los Andes, Colombia)

### **The Probabilistic Method and Navier-Stokes**

In this talk we introduce a probabilistic approach to show the existence of initial data with arbitrarily large  $L^2(\mathbf{R}^3)$ ,  $\dot{H}^{\frac{1}{2}}(\mathbf{R}^3)$  and  $\mathcal{PM}^2$ -norms for which a Generalized Navier-Stokes system generate a global regular solution. The main feature of the approach we present is that it is not deterministic: we show that from a certain family of possible large initial data most of them give raise to global regular solutions to a given Generalized Navier-Stokes system.

***Mohammed El Aïdi***

(Universidad Nacional de Colombia, Bogotá)

### **The lower bound of the spectrum of the Schrödinger operator defined in an open set of a geodesically complete Riemannian Manifold**

The purpose of my talk is devoted to show the existence with accuracy the lower bound of the spectrum corresponding to the Schrödinger operator defined in an open set of a geodesically complete Riemannian manifold, the condition boundary is the Dirichlet. The assumptions on the potential are the average of its positive part is semi-bounded from below, the average operates in a negligible set and regarding the negative part of the potential is bounded in each geodesically cube, i.e. normal cubes.

*Juan Manuel Guerra Castro*  
(Universidad Nacional de Colombia, Bogotá)

## Weyl-Dirac Equation In Condensed Matter Physics: Graphene

The Dirac equation provides a description of a spin-1/2 particle, such as electrons and neutrinos, consistent with quantum mechanical and relativistic principles [1]. In 1928, Hermann Weyl introduced a generalization of Riemannian geometry in an attempt to formulate a unified field theory. He proposed a two-component equation to describe massless spin-1/2 particles, which violates parity invariance. For this reason, the motion of the neutrino is described by the Weyl-Dirac (WD) equation [2]. The structure of Weyl's geometry is characterized by the symmetric tensor density  $g_{\mu\nu}$  and the *pseudovector*  $\phi_\mu$  (where it is reasonable to assume that the  $g_{\mu\nu}$  represent the gravity field and the  $\phi_\mu$  are the components of the vector potential). In contrast to Poincaré group, the spinorial representation of the Lorentz group can be reduced if a couple of projection operators are defined. The *massless* condition makes reducible the representation of the Poincaré group since the mass term in the Lagrangian density is not invariant under *two* separate Lorentz transformations [3]. Dirac equation (as well as WD equation) predicts the so called Klein's paradox (the dispersion of a Dirac electron is nearly transparent when the barrier is of the order of the electron mass). When Dirac equation is written in a *curved*-Weyl, some consequences arise such as the *minimal* coupling of the electron and the electromagnetic field and the Klein's paradox holds [4].

When the 2D honeycomb lattice of graphene is studied, its band structure is given by an effective *mass description* (or *k.p* approximation) describing the states in the vicinity of the first Brillouin zone points (Dirac points). The effective Hamiltonian leads to Weyl-Dirac naturally from Schrödinger equation (Tight-Binding approximation). Therefore, the charge carriers of Graphene (as well as C-Nanotubes) behave like WD electrons near the Dirac points, so *coherent transport* is possible in graphene, due to the lack of back-scattering of impurities.

The predictions of the WD particles have been observed from outstanding experiments in graphene as well as relativistic superconductivity. The purpose of the present lecture is to review how the predictions of the relativistic quantum mechanics in curved spaces can be observed (and measured) in solid state systems.

### References

- [1] J.J. Sakurai. Advanced Quantum Mechanics. 1967.
- [2] W. Greiner. Relativistic Quantum Mechanics. 3rd Ed. Springer, 1987.
- [3] M. Kaku, Quantum Field Theory (A modern Introduction). Oxford Un. Press. 1993.
- [4] T. Ando, J. Phys. Soc. Jpn. 74, 777 (2005)
- [5] H. Weyl, Zeits. f. Physik 56, 330 (1929)

- [6] M. Novello. Dirac's Equation In a Weyl Space. Cen. Bra. Pes. Fis. LXIV, N4. 1969
- [7] A. Castro, F. Guinea, N. Peres, K. Novoselov, and A. Geim, Rev. Mod. Phys. 81, 109 (2009).

***Sofyan Iblisdir***

(Facultat de Física, Universitat de Barcelona, España)

### **Boundary quantum critical phenomena with entanglement renormalization**

***Mikhail Malakhaltsev***

(Universidad de los Andes, Colombia)

### **Hopf bundle and quantum mechanics**

Heinz Hopf discovered the bundle, which nowadays is referred to as “Hopf bundle”, in 1931, in order to calculate the homotopy group  $\pi_3(S^2)$ . The Hopf bundle is a wonderful example of a principal bundle with rich geometry which appears in various geometrical situations. The aim of the talk is to demonstrate that Berry's phases can be described using the Hopf bundle of a Hilbert space.

***Miguel Ángel Martín Contreras***

(Universidad de los Andes, Colombia)

### **A Brief Introduction to the AdS/CFT Correspondence**

String Theory is widely known by its fantastic predictions in Cosmology and Particle Physics, but far away from this topics, String Theory has a strong feature: its mathematical structure. On the picture of String Theory, when we consider a collection of p-branes, we find a space-time geometry (a Gravity Theory in  $d+1$  dimensions) that can be related to Conformal Theories (that live in  $d$  dimensions) with the particularity that the couplings of both theories are inverse-related when we impose the correct conditions (the Large  $N_c$  expansion). The main objective of this talk is to introduce the basic ideas in order to obtain the Correspondence (in the extremal case) between the Supergravity Type IIB (SUGRA IIB) in the Bulk of  $AdS_5$  with the  $N = 4$  Super Yang-Mills theory ( $N = 4$  SYN) living on the boundary of same space.

***Juan Camilo Orduz***  
(Universidad de los Andes, Colombia)

### **Prequantization and Dirac structures**

First of all we review the integrality condition for the prequantization of symplectic and Poisson manifolds. Then we introduce Dirac structures, which generalize Poisson and symplectic manifolds, and discuss their prequantization integrality condition introduced by Weinstein and Zambon. Finally we will discuss the case of twisted-Dirac structures.

***Andrés Vargas***  
(Pontificia Universidad Javeriana)

### **Conformal Killing vectors and twistor spinors in Riemannian geometry**

Vector fields whose flow preserve the conformal class of a metric are well studied objects in conformal geometry. We will present some well-known properties of these vector fields on Riemannian manifolds and additionally their “spinorial” counterpart (also known as twistor spinors) will be constructed on Riemannian spin manifolds. For this, the Dirac and twistor operators need to be defined. Important result will be briefly mentioned and if time permits some easy explicit examples will be given.

***Monika Winklmeier***  
(Universidad de los Andes, Colombia)

### **On the spectrum of the Klein-Gordon equation**

The Klein-Gordon equation describes a quantum mechanical spin-0 particle. It can be associated with a block operator matrix and with a operator valued function. We will describe the spectrum of the operator associated with the Klein-Gordon equation.

	Lunes	Martes	Miercoles	Jueves	Viernes
8:00 – 9:00	Registration				
9:00 – 10:00	G. Sierra	G. Sierra	G. Sierra	G. Sierra	G. Sierra
10:00 – 11:00	G. Mussardo	G. Mussardo	G. Mussardo	G. Mussardo	G. Mussardo
11:00 – 11:30	<i>Break</i>	<i>Break</i>	<i>Break</i>	<i>Break</i>	<i>Break</i>
11:30 – 12:30	D. Cabra	D. Cabra	D. Cabra	D. Cabra	D. Cabra
12:30 – 14:00	<i>Break</i>	<i>Break</i>		<i>Break</i>	<i>Break</i>
14:00 – 14:40	J.C. Cortissoz	N. Avilán		A. Ángel	A. Botero
14:40 – 15:20	J.C. Orduz	J.M. Guerra		A. Vargas	M. El Aïdi
15:20 – 15:50	<i>Break</i>	<i>Break</i>		M. Winklmeier	<i>Break</i>
15:50 – 16:30	A. Cardona	S. Iblisdir			L. Cano
16:30 – 17:10	M. Malakhaltsev	M. Martín			