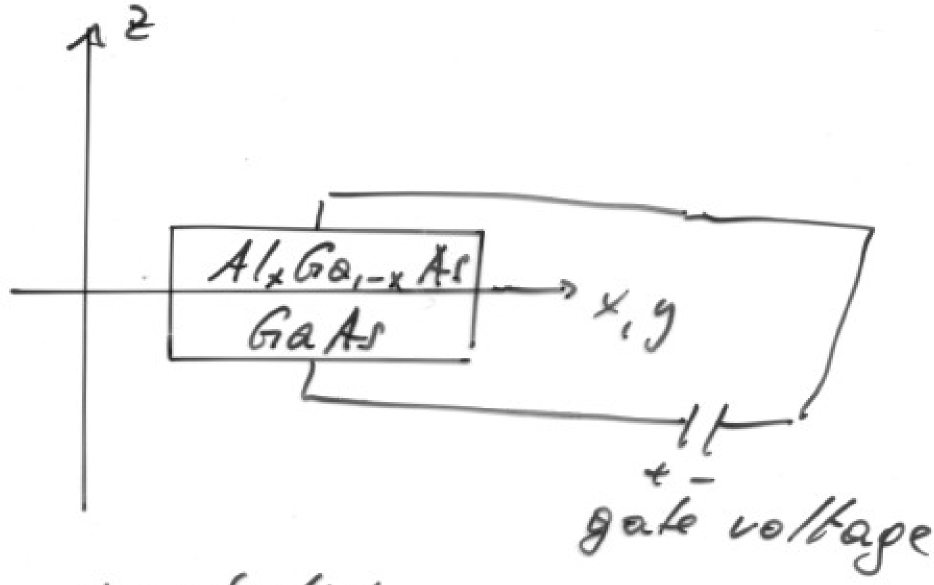
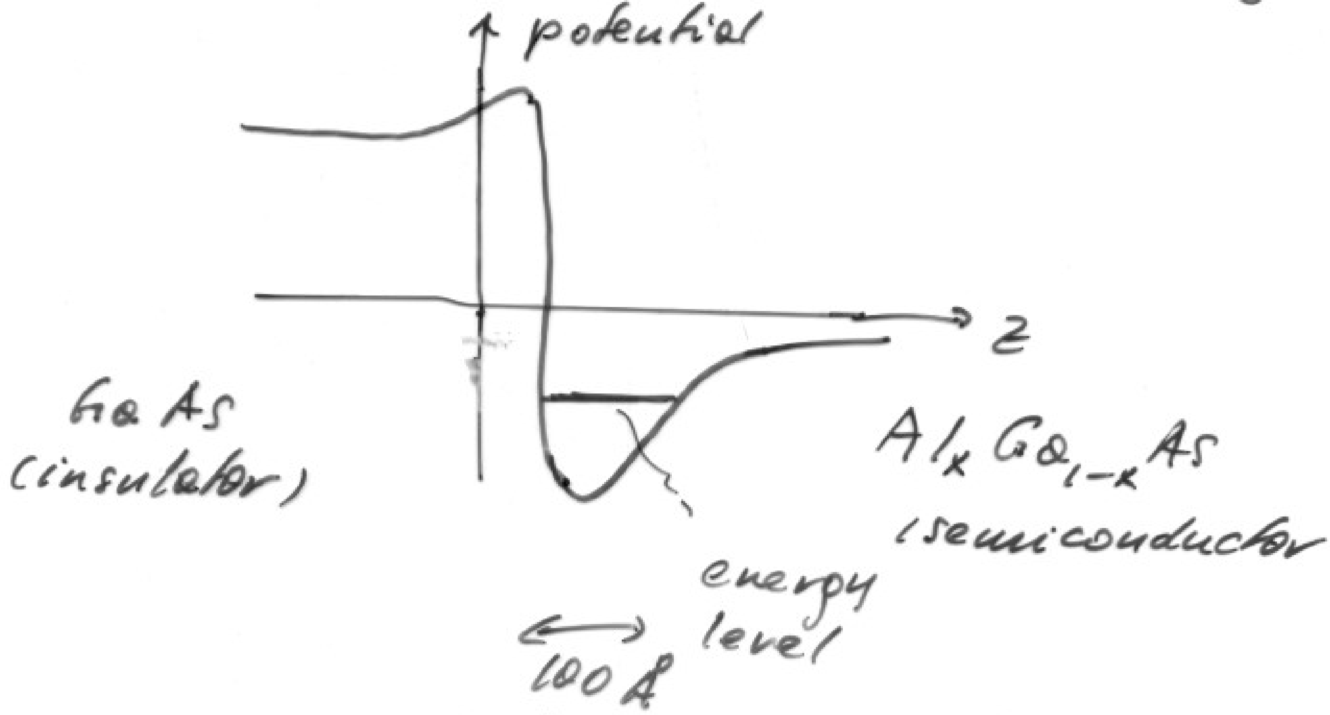
Plan

1. The classical Hall effect
2. The quantum Hall effect (QHE)
3. Quantum mechanics: bare essentials
4. Landau levels
5. Stability of the Hall conductance (role of disorder)
6. The topological approach to the QHE
7. The QHE as a pump
8. Edge currents
9. Mathematical reformulations of 6-8
10. Equivalences

The quantum Hall effect: experimental setup

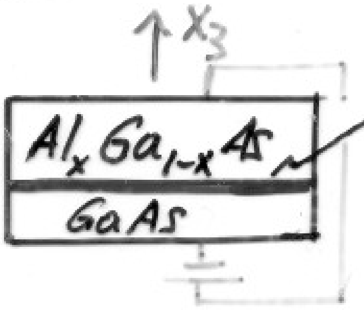




The experiment (von Klitzing, 1980)

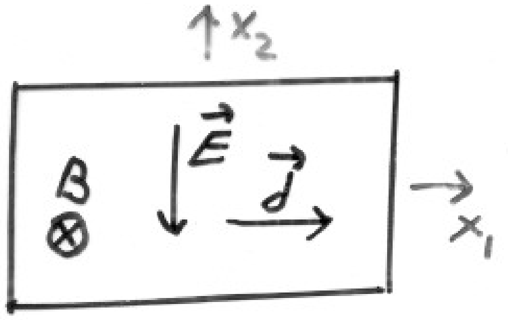
a typical sample

from the side:



electron gas
- confined to the interface (dim = 2)
- of density n (or Fermi energy μ) tunable through gate voltage

from above:

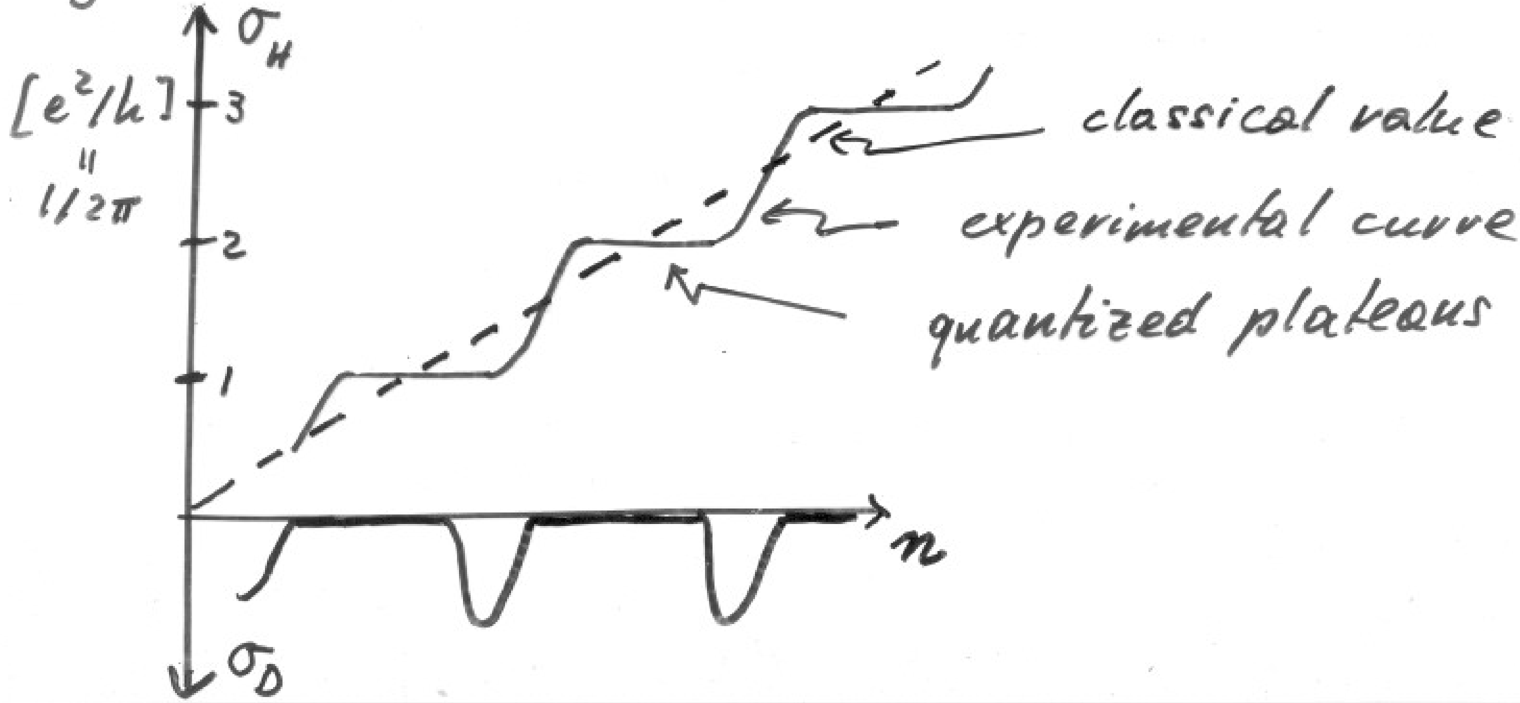


Hall-Ohm law

$$\vec{j} = \sigma \vec{E} \quad , \quad \sigma = \begin{pmatrix} \sigma_D & -\sigma_H \\ \sigma_H & \sigma_D \end{pmatrix}$$

σ_H : Hall conductance

σ_D : ohmic (dissipative) conductance



The bare essentials of quantum mechanics

A quantum mechanical system is characterized by

- a Hilbert space $\mathcal{H}$ (with inner product $(\cdot,\cdot)$)
- some self-adjoint operators A
- a distinguished self-adjoint operator H (the Hamiltonian), possibly time-dependent

meaning:

- $\psi \in \mathcal{H}$ with $(\psi,\psi) = 1$ is the state of the system at a given time (with ψ and $e^{i\alpha}\psi$, $(\alpha \in \mathbb{R})$ representing the same state)
- A are observables:

  $$(\psi, A\psi)$$

  is the expectation value of the measurement of A in the state ψ
- Energy H generates evolution ($H = H(t)$ if system subject to external forces) through

  $$i\hbar \frac{\partial\psi}{\partial t} = H(t)\,\psi(t) \ ;$$

equivalently: propagator $U(t,t') : \psi(t') \mapsto \psi(t)$

$$i\hbar \frac{\partial}{\partial t} U(t,t') = H(t)\,U(t,t') \quad , \quad U(t,t) = 1$$

Example: Single-particle systems

- $\mathcal{H} = L^2(\mathbb{R}^d)$, d = dimension of physical space

states ψ "wave functions"

- Observables x_i position $(i = 1, .. d)$

$$p_i = -i\hbar \frac{\partial}{\partial x_i} \quad \text{momentum}$$

and, for $d = 3$, $\vec{L} = \vec{x} \wedge \vec{p}$ angular momentum

... ..

- 1) $H = \frac{\vec{p}^{\,2}}{2m} + V(\vec{x}) = -\frac{\hbar^2 \Delta}{2m} + V(\vec{x})$

particle in a potential V (e.g. harmonic oscillator, hydrogen atom)

2) $A_0(\vec{x},t)$, $\vec{A}(\vec{x},t)$ electromagnetic potentials

$\vec{E} = -\vec{\nabla} A_0 - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}$ electric field

$\vec{B} = \text{rot}\, \vec{A}$ $(d=3)$, $B = \frac{\partial A_2}{\partial x_1} - \frac{\partial A_1}{\partial x_2}$ $(d=2)$

magnetic field

$\frac{1}{m}(\vec{p} - \frac{e}{c}\vec{A}(\vec{x},t))$: velocity

$$H(t) = \frac{1}{2m}(\vec{p} - \frac{e}{c}\vec{A}(\vec{x},t))^2 + A_0(\vec{x},t)$$

<u>Example</u> : Systems of many non-interacting fermionic particles

may be described in terms of correspon-ding single-particle system $\mathcal{H}$.

- Pauli principle : Each single-particle state can be occupied at most once

  - Projection P onto the occupied state $\varphi_i \in \mathcal{H}$ defines a many-particle state

- Expectation of A in $P = \sum_i \varphi_i (\varphi_i, \cdot)$

$$tr(AP) = \sum_i (\varphi_i, A\varphi_i)$$

- Evolution from t' to t

$$P \longmapsto U(t,t') P\, U(t,t')^*$$

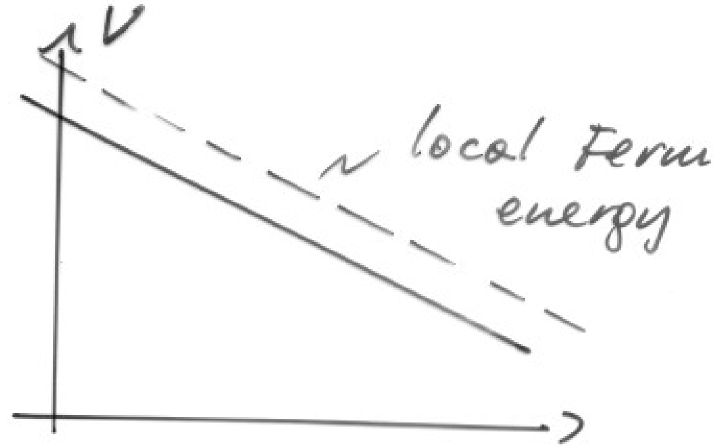
- Many-body ground state

  P is the projection onto the eigen-states of H with lowest eigenvalues (energies) up to some energy μ (Fermi energy)

μ $\sigma(H)$: spectrum of H

– pure sample

cross section



– disordered sample