Álgebra Lineal 2 - Taller 7

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1. Para cada uno de las siguientes parejas de transformaciones lineales f y funcionales lineales λ calcule $f^*(\lambda)$. Asuma que el cuerpo de base es $\mathbb{Q}$.

   i) $f(x,y) = (3x+y, y+x, 4x)$, $\lambda(x,y,z) = x+y+z$.
   ii) $f(x,y) = (3x+y, y+x, 4x)$, $\lambda(x,y,z) = x-2z$.
   iii) $f(x,y,z) = (x+y+z, x-z)$, $\lambda(x,y) = x-y$.
   iv) $f(x,y,z) = (x+y+z, x-z)$, $\lambda(x,y,z) = x+2y$.

2. Sea $V = \mathbb{R}^5$ con el producto interno

$$\langle (x_1,\ldots,x_5);(y_1,\ldots,y_5)\rangle = \sum_{i=1}^{5} x_i y_i$$

y

$$\begin{aligned} v_1 &= (0,1,1,1,1)\\ v_2 &= (1,0,1,1,1)\\ v_3 &= (1,1,0,1,1)\\ v_4 &= (1,1,1,0,1)\\ v_5 &= (1,1,1,1,0). \end{aligned}$$

Para

   i) $U = \langle v_1\rangle$
   ii) $U = \langle v_1, v_2\rangle$
   iii) $U = \langle v_1, v_2, v_3\rangle$
   iv) $U = \langle v_1, v_2, v_3, v_4\rangle$

   (a) Encuentre una base ortonormal de U y $U^\perp$.
   (b) La representación matricial de $p_U^\perp$ y $p_{U^\perp}^\perp$ en la base canónica.

<u>Solución</u>:

1. i) $f^*(\lambda)(x,y) = 8x+2y$.
   ii) $f^*(\lambda)(x,y) = -5x+y$.
   iii) $f^*(\lambda)(x,y,z) = y+2z$.
   iv) $f^*(\lambda)(x,y,z) = 3x+y-z$.

2. $\{v_1,v_2,v_3,v_4,v_5\}$ es una base de $\mathbb{R}^5$, si usamos Gram-Schmidt obtenemos $\{u_1,u_2,u_3,u_4,u_5\}$ donde

$$u_1 = \frac{1}{2}(0,1,1,1,1) \tag{1}$$

$$u_2 = \frac{\sqrt{7}}{14}(4,-3,1,1,1) \tag{2}$$

$$u_3 = \frac{\sqrt{70}}{70}(4,4,-6,1,1) \tag{3}$$

$$u_4 = \frac{\sqrt{130}}{130}(4,4,4,-9,1) \tag{4}$$

$$u_5 = \frac{\sqrt{13}}{13}(1,1,1,1,-3) \tag{5}$$

(a)
i) $U = \langle u_1 \rangle$ y $U^\perp = \langle u_2, u_3, u_4, u_5 \rangle$
ii) $U = \langle u_1, u_2 \rangle$ y $U^\perp = \langle u_3, u_4, u_5 \rangle$
iii) $U = \langle u_1, u_2, u_3 \rangle$ y $U^\perp = \langle u_4, u_5 \rangle$
iv) $U = \langle u_1, u_2, u_3, u_4 \rangle$ y $U^\perp = \langle u_5 \rangle$

(b)
i)
$$\big[p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{4}\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \big[p_{U^\perp}^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \big[\mathrm{id} - p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 & 0 & 0 \\ 0 & 3 & -1 & -1 & -1 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & -1 & 3 \end{bmatrix}$$

ii)
$$\big[p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{7}\begin{bmatrix} 4 & -3 & 1 & 1 & 1 \\ -3 & 4 & 1 & 1 & 1 \\ 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 2 & 2 & 2 \\ 1 & 1 & 2 & 2 & 2 \end{bmatrix} \quad \big[p_{U^\perp}^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \big[\mathrm{id} - p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{7}\begin{bmatrix} 3 & 3 & -1 & -1 & -1 \\ 3 & 3 & -1 & -1 & -1 \\ -1 & -1 & 5 & -2 & -2 \\ -1 & -1 & -2 & 5 & -2 \\ -1 & -1 & -2 & -2 & 5 \end{bmatrix}$$

iii)
$$\big[p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{10}\begin{bmatrix} 8 & -2 & -2 & 2 & 2 \\ -2 & 8 & -2 & 2 & 2 \\ -2 & -2 & 8 & 2 & 2 \\ 2 & 2 & 2 & 3 & 3 \\ 2 & 2 & 2 & 3 & 3 \end{bmatrix} \quad \big[p_{U^\perp}^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \big[\mathrm{id} - p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{10}\begin{bmatrix} 2 & 2 & 2 & -2 & -2 \\ 2 & 2 & 2 & -2 & -2 \\ 2 & 2 & 2 & -2 & -2 \\ -2 & -2 & -2 & 7 & -3 \\ -2 & -2 & -2 & -3 & 7 \end{bmatrix}$$

iv)
$$\big[p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{13}\begin{bmatrix} 12 & -1 & -1 & -1 & 3 \\ -1 & 12 & -1 & -1 & 3 \\ -1 & -1 & 12 & -1 & 3 \\ -1 & -1 & -1 & 12 & 3 \\ 3 & 3 & 3 & 3 & 4 \end{bmatrix} \quad \big[p_{U^\perp}^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \big[\mathrm{id} - p_U^\perp\big]_{\mathcal{C}}^{\mathcal{C}} = \frac{1}{13}\begin{bmatrix} 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 1 & -3 \\ 1 & 1 & 1 & 1 & -3 \\ -3 & -3 & -3 & -3 & 9 \end{bmatrix}$$