Álgebra Lineal 2 - Taller

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i) Sea $f \in \mathrm{Hom}_{\mathbb{Q}}(\mathbb{Q}^3, \mathbb{Q}^3)$ dada por

$$f(x,y,z) = (9x - 4z, 18x - y - 4z, \frac{15}{2}x - \frac{3}{2}y + z).$$

1. Si $\mathcal{C}$ la base canónica de $\mathbb{Q}^3$ entonces:

$$\Big[f\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 9 & 0 & -4 \\ 18 & -1 & -4 \\ 15/2 & -3/2 & 1 \end{bmatrix}$$

2. Tenemos $P_f(t) = P_1(t)P_2(t)P_3(t)$ donde $P_1(t) = (t-1)$, $P_2(t) = (t-3)$ y $P_3(t) = (t-5)$.
3. Sean $R_1(t) = P_2(t)P_3(t) = (t-3)(t-5)$, $R_2(t) = P_1(t)P_3(t) = (t-1)(t-5)$, $R_3(t) = P_1(t)P_2(t) = (t-1)(t-3)$. Tenemos

$$-\frac{1}{2}R_1(t) + \frac{1}{2}R_2(t) = (R_1(t), R_2(t))$$

y

$$-\frac{1+t}{8}(R_1(t), R_2(t)) + \frac{1}{8}R_3(t) = (R_1(t), R_2(t), R_3(t)) = 1$$

luego

$$\frac{1+t}{16}R_1(t) - \frac{1+t}{16}R_2(t) + \frac{1}{8}R_3(t) = 1$$

así si $\Pi_1(t) = \frac{1+t}{16}R_1(t) = \frac{(1+t)(t-3)(t-5)}{16}$, $\Pi_2(t) = -\frac{1+t}{16}R_2(t) = -\frac{(1+t)(t-1)(t-5)}{16}$ y $\Pi_3(t) = \frac{1}{8}R_3(t) = \frac{(t-1)(t-3)}{8}$, entonces, para $i = 1,2,3$, $p_i = \Pi_i(f)$ es la proyección sobre $V_i = \ker(P_i(f))$ de acuerdo a la descomposición

$$V = V_1 \oplus V_2 \oplus V_3.$$

4. Tenemos para $i = 1,2,3$

$$\Big[p_i\Big]_{\mathcal{C}}^{\mathcal{C}} = \Big[\Pi_i(f)\Big]_{\mathcal{C}}^{\mathcal{C}} = \Pi_i(\Big[f\Big]_{\mathcal{C}}^{\mathcal{C}})$$

y

$$\Big[f \circ p_i\Big]_{\mathcal{C}}^{\mathcal{C}} = \Big[f\Big]_{\mathcal{C}}^{\mathcal{C}}\Big[p_i\Big]_{\mathcal{C}}^{\mathcal{C}}.$$

Entonces

$$\Big[p_1\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -3/4 & 3/4 & -1 \\ -15/4 & 15/4 & -5 \\ -3/2 & 3/2 & -2 \end{bmatrix}$$

$$\Big[p_2\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -1/2 & -3/2 & 4 \\ -3/2 & -9/2 & 12 \\ -3/4 & -9/4 & 6 \end{bmatrix}$$

$$\Big[p_3\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 9/4 & 3/4 & -3 \\ 21/4 & 7/4 & -7 \\ 9/4 & 3/4 & -3 \end{bmatrix}$$

y

$$\Big[f \circ p_1\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -3/4 & 3/4 & -1 \\ -15/4 & 15/4 & -5 \\ -3/2 & 3/2 & -2 \end{bmatrix} = 1\Big[p_1\Big]_{\mathcal{C}}^{\mathcal{C}}$$

$$\Big[f \circ p_2\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -3/2 & -9/2 & 12 \\ -9/2 & -27/2 & 36 \\ -9/4 & -27/4 & 18 \end{bmatrix} = 3\Big[p_2\Big]_{\mathcal{C}}^{\mathcal{C}}$$

$$\left[f\circ p_3\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 45/4 & 15/4 & -15 \\ 105/4 & 35/4 & -35 \\ 45/4 & 15/4 & -15 \end{bmatrix} = 5\left[p_3\right]_{\mathcal{C}}^{\mathcal{C}}$$

5. Si $\mathcal{B} = \{(1,5,2),(2,6,3),(3,7,3)\}$ entonces

$$\left[f\right]_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

ii) Sea $f \in \mathrm{Hom}_{\mathbb{Q}}(\mathbb{Q}^3, \mathbb{Q}^3)$ dada por

$$f(x,y,z) = (2x - y + z, -2x + y + 2z, \frac{7}{3}x - \frac{5}{3}y + 2z).$$

1. Si $\mathcal{C}$ la base canónica de $\mathbb{Q}^3$ entonces:

$$\left[f\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 1 & 2 \\ 7/3 & -5/3 & 2 \end{bmatrix}$$

2. Tenemos $P_f(t) = P_1(t)^2 P_2(t)$ donde $P_1(t) = (t-1)$, $P_2(t) = (t-3)$.
3. Sean $R_1(t) = P_2(t) = (t-3)$, $R_2(t) = P_1(t)^2 = (t-1)^2$. Tenemos

$$-\frac{1+t}{4}R_1(t) + \frac{1}{4}R_2(t) = 1$$

así si $\Pi_1(t) = -\frac{1+t}{4}R_1(t) = -\frac{(1+t)(t-3)}{4}$ y $\Pi_2(t) = \frac{1}{4}R_2(t) = \frac{(t-1)^2}{4}$ entonces, para $i = 1,2$, $p_i = \Pi_i(f)$ es la proyección sobre $V_i = \ker(P_i(f))$ de acuerdo a la descomposición

$$V = V_1 \oplus V_2$$

4. Tenemos

$$\left[p_1\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -1/3 & 2/3 & 0 \\ -2/3 & 4/3 & 0 \\ -2 & 1 & 1 \end{bmatrix}$$

$$\left[p_2\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 4/3 & -2/3 & 0 \\ 2/3 & -1/3 & 0 \\ 2 & -1 & 0 \end{bmatrix}$$

y

$$\left[f\circ p_1\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -2 & 1 & 1 \\ -4 & 2 & 2 \\ -11/3 & 4/3 & 2 \end{bmatrix}$$

$$\left[f\circ p_2\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 4 & -2 & 0 \\ 2 & -1 & 0 \\ 6 & -3 & 0 \end{bmatrix} = 3\left[p_2\right]_{\mathcal{C}}^{\mathcal{C}}$$

5. Si $\mathcal{B} = \{(2,4,3),(0,0,1),(2,1,3)\}$ entonces

$$\left[f\right]_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} 3/2 & 1/2 & 0 \\ -1/2 & 1/2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

iii) Sea $f \in \mathrm{Hom}_{\mathbb{Q}}(\mathbb{Q}^4, \mathbb{Q}^4)$ dada por

$$f(x,y,z,w) = (x - y + w, -x - z + 2w, 2x - y - z - w, 2x - y)$$

1. Si $\mathcal{C}$ la base canónica de $\mathbb{Q}^4$ entonces:

$$\left[f\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 0 & -1 & 2 \\ 2 & -1 & -1 & -1 \\ 2 & -1 & 0 & 0 \end{bmatrix}$$

2. Tenemos $P_f(t) = P_1(t)P_2(t)P_3(t)$ donde $P_1(t) = (t+1)$, $P_2(t) = (t-1)$, $P_3(t) = (t^2-2)$.
3. Sean $R_1(t) = P_2(t)P3(t) = (t-1)(t^2-2)$, $R_2(t) = P_1(t)P_3(t) = (t+1)(t^2-2)$, $R_3 = (t-1)(t+1)$. Tenemos
$$\frac{1}{2}R_1(t) - \frac{1}{2}R_2(t) + R_3(t) = 1$$
así si $\Pi_1(t) = \frac{1}{2}R_1(t) = \frac{(t-1)(t^2-2)}{2}$, $\Pi_2(t) = -\frac{1}{2}R_2(t) = -\frac{(t+1)(t^2-2)}{2}$ y $\Pi_3(t) = R_3(t) = (t-1)(t+1)$ entonces, para $i = 1, 2, 3$, $p_i = \Pi_i(f)$ es la proyección sobre $V_i = \ker(P_i(f))$ de acuerdo a la descomposición
$$V = V_1 \oplus V_2 \oplus V_3.$$
4. Tenemos
$$\begin{bmatrix}p_1\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 2 & 0 & 0 & -2 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\begin{bmatrix}p_2\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -3 & 2 & -1 & 2 \\ -3 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ -3 & 2 & -1 & 2 \end{bmatrix}$$
$$\begin{bmatrix}p_3\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 3 & -2 & 1 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 0 & 1 & 1 \\ 3 & -2 & 1 & -1 \end{bmatrix}$$
y
$$\begin{bmatrix}f \circ p_1\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -1 & 0 & 0 & 1 \\ -2 & 0 & 0 & 2 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = -\begin{bmatrix}p_1\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}}$$
$$\begin{bmatrix}f \circ p_2\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -3 & 2 & -1 & 2 \\ -3 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ -3 & 2 & -1 & 2 \end{bmatrix} = \begin{bmatrix}p_2\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}}$$
$$\begin{bmatrix}f \circ p_3\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 5 & -3 & 1 & -2 \\ 4 & -2 & 0 & -2 \\ 3 & -1 & -1 & -2 \\ 5 & -3 & 1 & -2 \end{bmatrix}.$$
5. Si $\mathcal{B} = \{(1,2,1,0), (1,1,0,1), (1,0,-1,1), (1,1,1,1)\}$ entonces
$$\begin{bmatrix}f\end{bmatrix}_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 2 & 0 \end{bmatrix}.$$

iv) Sea $f \in \mathrm{Hom}_{\mathbb{R}}(\mathbb{R}^4, \mathbb{R}^4)$ dada por
$$f(x,y,z,w) = (x - y + w, -x - z + 2w, 2x - y - z - w, 2x - y).$$

1. Si $\mathcal{C}$ la base canónica de $\mathbb{R}^4$ entonces:
$$\begin{bmatrix}f\end{bmatrix}_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 1 & -1 & 0 & 1 \\ -1 & 0 & -1 & 2 \\ 2 & -1 & -1 & -1 \\ 2 & -1 & 0 & 0 \end{bmatrix}$$
2. Tenemos $P_f(t) = P_1(t)P_2(t)P_3(t)P_4(t)$ donde $P_1(t) = (t+1)$, $P_2(t) = (t-1)$, $P_3(t) = (t-\sqrt{2})$, $P_4(t) = (t+\sqrt{2})$.

3. Sean $R_1(t) = P_2(t)P3(t)P_4(t) = (t-1)(t^2-2)$, $R_2(t) = P_1(t)P_3(t)P_4(t) = (t+1)(t^2-2)$, $R_3 = (t-1)(t+1)(t-\sqrt{2})$ y $R_4 = (t-1)(t+1)(t+\sqrt{2})$. Tenemos

$$-\frac{\sqrt{2}}{8}(t^2+3+2\sqrt{2}t)(t-\sqrt{2})R_1(t)+\frac{\sqrt{2}}{8}(t^2+3+2\sqrt{2}t)(t-\sqrt{2})R_2(t)-\frac{\sqrt{2}}{4}(t^2+3+2\sqrt{2}t)R_3(t)+\frac{\sqrt{2}}{4}R_4(t)=1$$

así si $\Pi_1(t) = -\frac{\sqrt{2}}{8}(t^2+3+2\sqrt{2}t)(t-\sqrt{2})R_1(t)$, $\Pi_2(t) = \frac{\sqrt{2}}{8}(t^2+3+2\sqrt{2}t)(t-\sqrt{2})R_2(t)$, $\Pi_3(t) = -\frac{\sqrt{2}}{4}(t^2+3+2\sqrt{2}t)R_3(t)$ y $P_4(t) = \frac{\sqrt{2}}{4}R_4(t)$ entonces, para $i=1,2,3,4$, $p_i = \Pi_i(f)$ es la proyección sobre $V_i = \ker(P_i(f))$ de acuerdo a la descomposición

$$V = V_1 \oplus V_2 \oplus V_3 \oplus V_4.$$

4. Tenemos

$$\left[p_1\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 2 & 0 & 0 & -2 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\left[p_2\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -3 & 2 & -1 & 2 \\ -3 & 2 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ -3 & 2 & -1 & 2 \end{bmatrix}$$

$$\left[p_3\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 3/2-5\sqrt{2}/4 & -1+3\sqrt{2}/4 & 1/2-\sqrt{2}/4 & -1/2+\sqrt{2}/2 \\ 1/2-\sqrt{2} & -1/2+\sqrt{2}/2 & 1/2 & \sqrt{2}/2 \\ -1/2-3\sqrt{2}/4 & \sqrt{2}/4 & 1/2+\sqrt{2}/4 & 1/2+\sqrt{2}/2 \\ 3/2-5\sqrt{2}/4 & -1+3\sqrt{2}/4 & 1/2-\sqrt{2}/4 & -1/2+\sqrt{2}/2 \end{bmatrix}$$

$$\left[p_4\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 3/2+5\sqrt{2}/4 & -1-3\sqrt{2}/4 & 1/2+\sqrt{2}/4 & -1/2-\sqrt{2}/2 \\ 1/2+\sqrt{2} & -1/2-\sqrt{2}/2 & 1/2 & -\sqrt{2}/2 \\ -1/2+3\sqrt{2}/4 & -\sqrt{2}/4 & 1/2-\sqrt{2}/4 & 1/2-\sqrt{2}/2 \\ 3/2+5\sqrt{2}/4 & -1-3\sqrt{2}/4 & 1/2+\sqrt{2}/4 & -1/2-\sqrt{2}/2 \end{bmatrix}$$

y

$$\left[f\circ p_1\right]_{\mathcal{C}}^{\mathcal{C}} = -\left[p_1\right]_{\mathcal{C}}^{\mathcal{C}}$$

$$\left[f\circ p_2\right]_{\mathcal{C}}^{\mathcal{C}} = \left[p_2\right]_{\mathcal{C}}^{\mathcal{C}}$$

$$\left[f\circ p_3\right]_{\mathcal{C}}^{\mathcal{C}} = \sqrt{2}\left[p_3\right]_{\mathcal{C}}^{\mathcal{C}}$$

$$\left[f\circ p_4\right]_{\mathcal{C}}^{\mathcal{C}} = -\sqrt{2}\left[p_3\right]_{\mathcal{C}}^{\mathcal{C}}$$

5. Si $\mathcal{B} = \{(1,2,1,0),(1,1,0,1),(1,0,-1,1)+\sqrt{2}(1,1,1,1),(1,0,-1,1)-\sqrt{2}(1,1,1,1)\}$ entonces

$$\left[f\right]_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & -\sqrt{2} \end{bmatrix}.$$

v) Sea $f \in \mathrm{Hom}_{\mathbb{Q}}(\mathbb{Q}^4,\mathbb{Q}^4)$ dada por

$$f(x,y,z,w) = (2x-2y+2z-2w, -x+2y+2z, 5x-6y+z-5w, -x+3z-w)$$

1. Si $\mathcal{C}$ la base canónica de $\mathbb{Q}^4$ entonces:

$$\left[f\right]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 2 & -2 & 2 & -2 \\ -1 & 2 & 2 & 0 \\ 5 & -6 & 1 & -5 \\ -1 & 0 & 3 & -1 \end{bmatrix}$$

2. Tenemos $P_f(t) = P_1(t)P_2(t)$ donde $P_1(t) = t^2-2*t+2$, $P_2(t) = t^2-2*t+10$.

3. Sean $R_1(t) = P_2(t)$, $R_2(t) = P_1(t)$. Tenemos

$$\frac{1}{8}R_1(t) - \frac{1}{8}R_2(t) = 1$$

así si $\Pi_1(t) = \frac{1}{8}R_1(t)$, $\Pi_2(t) = -\frac{1}{8}R_2(t)$ entonces, para $i = 1, 2$, $p_i = \Pi_i(f)$ es la proyección sobre $V_i = \ker(P_i(f))$ de acuerdo a la descomposición

$$V = V_1 \oplus V_2.$$

4. Tenemos

$$\big[p_1\big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 3 & -2 & -1 & -1 \\ 1 & 0 & 0 & -1 \\ 2 & -2 & -1 & 0 \\ 2 & -2 & -1 & 0 \end{bmatrix}$$

$$\big[p_2\big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -2 & 2 & 1 & 1 \\ -1 & 1 & 0 & 1 \\ -2 & 2 & 2 & 0 \\ -2 & 2 & 1 & 1 \end{bmatrix}$$

y

$$\big[f \circ p_1\big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 4 & -4 & -2 & 0 \\ 3 & -2 & -1 & -1 \\ 1 & -2 & -1 & 1 \\ 1 & -2 & -1 & 1 \end{bmatrix}$$

$$\big[f \circ p_2\big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -2 & 2 & 4 & -2 \\ -4 & 4 & 3 & 1 \\ 4 & -4 & 2 & -6 \\ -2 & 2 & 4 & -2 \end{bmatrix}$$

5. Si $\mathcal{B} = \{(1,0,1,1), (1,1,0,0), (1,0,2,1), (1,1,0,1)\}$ entonces

$$\big[f\big]_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 3 & 1 \end{bmatrix}.$$

vi) Sea $f \in \operatorname{Hom}_{\mathbb{C}}(\mathbb{C}^4, \mathbb{C}^4)$ dada por

$$f(x, y, z, w) = (2x - 2y + 2z - 2w, -x + 2y + 2z, 5x - 6y + z - 5w, -x + 3z - w).$$

1. Si $\mathcal{C}$ la base canónica de $\mathbb{C}^4$ entonces:

$$\big[f\big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 2 & -2 & 2 & -2 \\ -1 & 2 & 2 & 0 \\ 5 & -6 & 1 & -5 \\ -1 & 0 & 3 & -1 \end{bmatrix}$$

2. Tenemos $P_f(t) = P_1(t)P_2(t)P_3(t)P_4(t)$ donde $P_1(t) = t - (1+i)$, $P_2(t) = t - (1-i)$, $P_3(t) = t - (1+3i)$, $P_4(t) = t - (1-3i)$.

3. Sean $R_1(t) = P_2(t)P_3(t)P_4(t)$, $R_2(t) = P_1(t)P_3(t)P_4(t)$, $R_3(t) = P_1(t)P_2(t)P_4(t)$, $R_4(t) = P_1(t)P_2(t)P_3(t)$. Tenemos

$$\frac{1}{768}(t^2-(2-6i)t-(16-6i))(t-(1-3i))R_1(t)-\frac{1}{768}(t^2-(2-6i)t-(16-6i))(t-(1-3i))R_2(t)-\frac{i}{384}(t^2-(2-6i)t-(16-6i))(t-(1-$$

así si $\Pi_1(t) = \frac{1}{768}(t^2 - (2-6i)t - (16-6i))(t-(1-3i))R_1(t)$, $\Pi_2(t) = -\frac{1}{768}(t^2 - (2-6i)t - (16-6i))(t - (1-3i))R_2(t)$, $\Pi_3(t) = -\frac{i}{384}(t^2 - (2-6i)t - (16-6i))(t - (1-3i))R_3(t)$, $\Pi_4(t) = -\frac{i}{48}R_4(t)$ entonces, para $i = 1, 2, 3, 4$, $p_i = \Pi_i(f)$ es la proyección sobre $V_i = \ker(P_i(f))$ de acuerdo a la descomposición

$$V = V_1 \oplus V_2 \oplus V_3 \oplus V_4.$$

4. Tenemos

$$\Big[p_1\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 3/2 - i/2 & -1+i & -1/2+i/2 & -1/2-i/2 \\ 1/2 - i & i & 1/2i & -1/2 \\ 1+1/2i & -1 & -1/2 & -1/2i \\ 1+1/2i & -1 & -1/2 & -1/2i \end{bmatrix}$$

$$\Big[p_2\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} 3/2 + i/2 & -1-i & -1/2-i/2 & -1/2+i/2 \\ 1/2 + i & -i & -1/2i & -1/2 \\ 1-1/2i & -1 & -1/2 & 1/2i \\ 1-1/2i & -1 & -1/2 & 1/2i \end{bmatrix}$$

$$\Big[p_3\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -1 & 1 & 1/2-i/2 & 1/2+i/2 \\ -1/2+i/2 & 1/2-i/2 & -i/2 & 1/2 \\ -1-i & 1+i & 1 & i \\ -1 & 1 & 1/2-i/2 & 1/2+i/2 \end{bmatrix}$$

$$\Big[p_4\Big]_{\mathcal{C}}^{\mathcal{C}} = \begin{bmatrix} -1 & 1 & 1/2+i/2 & 1/2-i/2 \\ -1/2-i/2 & 1/2+i/2 & i/2 & 1/2 \\ -1+i & 1-i & 1 & -i \\ -1 & 1 & 1/2+i/2 & 1/2-i/2 \end{bmatrix}$$

y

$$\Big[f \circ p_1\Big]_{\mathcal{C}}^{\mathcal{C}} = (1+i)\Big[p_1\Big]_{\mathcal{C}}^{\mathcal{C}}$$

$$\Big[f \circ p_2\Big]_{\mathcal{C}}^{\mathcal{C}} = (1-i)\Big[p_2\Big]_{\mathcal{C}}^{\mathcal{C}}$$

$$\Big[f \circ p_3\Big]_{\mathcal{C}}^{\mathcal{C}} = (1+3i)\Big[p_3\Big]_{\mathcal{C}}^{\mathcal{C}}$$

$$\Big[f \circ p_4\Big]_{\mathcal{C}}^{\mathcal{C}} = (1-3i)\Big[p_4\Big]_{\mathcal{C}}^{\mathcal{C}}$$

5. Si $\mathcal{B} = \{(1,0,1,1)-i(1,1,0,0), (1,0,1,1)+i(1,1,0,0), (1,0,2,1)-i(1,1,0,1), (1,0,2,1)+i(1,1,0,1)\}$ entonces

$$\Big[f\Big]_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} 1+i & 0 & 0 & 0 \\ 0 & 1-i & 0 & 0 \\ 0 & 0 & 1+3i & 0 \\ 0 & 0 & 0 & 1-3i \end{bmatrix}.$$