Tropical geometry

Exercises (third lecture)

Exercise 1 Fix a positive integer d. It is clear from their definitions that the numbers W_d and N_d are equal modulo 2. Prove that

$$W_d = N_d \mod 4.$$

Exercise 2 Show that for any generic collection of 8 points in $\mathbb{R}^2$, there exist exactly 9 or exactly 10 rational tropical cubics that pass through the points of the collection.

Exercise 3 Fix a positive integer d and an integer $0 \leq g \leq \frac{(d-1)(d-2)}{2}$.

(a) Prove that the Block-Göttsche polynomial $G_{d,\frac{(d-1)(d-2)}{2}}$ is equal to 1.

(b) Prove that the highest power of q which appears in the Block-Göttsche polynomial $G_{d,g}$ is $\frac{(d-1)(d-2)}{2} - g$, and that the coefficient of the corresponding monomial of $G_{d,g}$ is equal to

$$\binom{\frac{(d-1)(d-2)}{2}}{g}.$$