Title: On Elementary Equivalence of Real Semigroups of Preordered Rings

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Summary: Due to the nature of the functor that associates a Real Semigroup (RS) to a preordered ring (p-ring), it has proven to be a hard model-theoretic question to established if elementary equivalence is preserved by this functor, in particular, because it preserves arbitrary directed colimits and only **finite** products, making unavailable some of the standard model-theoretic methods, such as the Kiesler-Shelah ultrapower Theorem.

Furthermore:

(1) The basic properties of the RS associated to p-rings, such as equality of terms, is not, in general, first-order describable;

(2) One counter example that the functor from p-rings to RSs does preserve arbitrary products (or even countable products) is the ring $\mathbb{C}([0, 1])$, preordered by squares: it is a Pythagorean, Archimedean f-ring with bounded inversion and, in fact, real closed. This example and item (1) above indicated that a new approach was necessary.

However, interesting information can be obtained from $L_{\omega_1,\omega}$-elementary equivalence of p-rings. In this setting, we establish that if two p-rings are $L_{\omega_1,\omega}$-elementarily equivalent, then their associated RSs are elementarily equivalent and that this also holds true for important constructions in the category of p-rings. To this end, we introduce two lifting properties that are crucial for these results.

We also present and study a especial class of p-rings, those of bounded exponent, showing, in particular that if two von Neumann regular p-rings are elementarily equivalent, the same is true of their associated RSs.

Moreover, we apply this method to Boolean powers of p-rings, proving that if two p-rings are $L_{\omega_1,\omega}$-elementarily equivalent, the RSs associated to their Boolean powers are (first-order) elementarily equivalent.

All pertinent definitions and concepts will be succinctly explained in the conference.

This is joint work with Hugo Mariano and will appear in the Logic Journal of the IGPL.