

Let \mathcal{A} be a unital C^* -algebra. Given a faithful representation $\mathcal{A} \subset \mathcal{B}(\mathcal{L})$ in a Hilbert space \mathcal{L} , the set $G^+ \subset \mathcal{A}$ of positive invertible elements can be thought of as the set of inner products in \mathcal{L} , related to \mathcal{A} , which are equivalent to the original inner product. The set G^+ has a rich geometry: it is a homogeneous space of the invertible group G of \mathcal{A} , with an invariant Finsler metric. In the present talk I will describe the tangent bundle TG^+ of G^+ , as a homogenous Finsler space of a natural group of invertible matrices in $M_2(\mathcal{A})$, identifying TG^+ with the *Poincaré half space* \mathcal{H} of \mathcal{A} ,

$$\mathcal{H} = \{h \in \mathcal{A} : \text{Im}(h) \geq 0, \text{Im}(h) \text{ invertible}\}$$

I will show that $\mathcal{H} \simeq TG^+$ has properties similar to those of a space of non-positive constant curvature.

If time allows, I will define in \mathcal{H} *natural invariant non commutative Kähler structure* on \mathcal{H} and compute its moment map.