Algebra Universalis, 34 (1995) 314–321 0002-5240/95/020314-08$01.50 + 0.20/0

Finitely axiomatizable quasivarieties of graphs

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Abstract. Apart from four trivial quasivarieties of bipartite graphs, any finitely axiomatizable universal Horn class of graphs must contain graphs of arbitrarily large chromatic number. Hence, no finitely generated universal Horn class of graphs is finitely axiomatizable, except these four. On the other hand, there is a continuum of universal Horn classes of graphs.

W. Taylor [8] has shown that the class of n-chromatic graphs is not elementary for $n \geq 2$. His proof shows in fact that the quasivariety of n-colourable graphs is not finitely axiomatizable. It does not seem to have been noticed, however, that this holds for any finitely generated quasivariety of graphs, except in four trivial cases, as immediate consequence of an observation of Tarski on universally axiomatizable classes of structures [7] plus a theorem of Nešetřil and Pultr about classes of graphs defined by forbidden substructures [5]. We want to call attention to this fact and the more general one that, except in the four mentioned cases, *any finitely axiomatizable quasivariety of graphs must contain graphs of arbitrarily large chromatic number*. These results depend on a theorem of Erdös [3], theorem which may be utilized also to exhibit a continuum of universal Horn classes of graphs.

Graphs will be undirected and irreflexive. The non-finite axiomatizability phenomenon seems typical of undirected graphs since there are infinitely many elementary, finitely generated, quasivarieties of directed graphs. It fails also in the case of algebras, since by a result of Pigozzi [6] any finitely generated quasivariety of algebras with relative distributive congruences is finitely axiomatizable.

1. Forbidden substructures

Let σ be a finite relational vocabulary, St_σ the class of structures of type σ, and $\mathscr{F} \subseteq \mathrm{St}_\sigma$. Following [5], define:

$$\mathscr{F}\neg = \{\mathfrak{A} \in \mathrm{St}_\sigma : \text{no substructure of } \mathfrak{A} \text{ is isomorphic to an element of } \mathscr{F}\}.$$

Presented by J. Mycielski.
Received July 19, 1994; accepted in final form September 12, 1994.

LEMMA 1 (Tarski [7], Malcev [4] Chap. 33). *$\mathscr{C}$ is (finitely) axiomatizable by universal sentences if and only if $\mathscr{C} = \mathscr{F}\neg$ for some (finite) class $\mathscr{F}$ of structures.*

Proof. Given a universal sentence φ, let $\mathscr{F}_\varphi = \{\mathfrak{B} \in \mathrm{St}_\sigma \mid B \subseteq \{1, 2, \ldots, n\}, \mathfrak{B} \nvDash \varphi\}$. Then $\mathscr{C} = \mathscr{F}\neg$, if $\mathscr{F} = \bigcup_\varphi \mathscr{F}_\varphi$ where φ runs through some universal axiomatization of $\mathscr{C}$. As $\mathscr{F}_\varphi$ is finite then $\mathscr{F}$ will be finite whenever the axiomatization is finite. Reciprocally, if $\mathscr{C} = \mathscr{F}\neg$ where $\mathscr{F}$ consists of finite structures then $\mathscr{C}$ is axiomatized by the sentences:

$$\forall x_1 \cdots \forall x_{|B|} \neg D_{\mathfrak{B}}(x_1, \ldots, x_{|B|}), \qquad \mathfrak{B} \in \mathscr{F},$$

where $D_{\mathfrak{B}}$ is the conjunction of the atomic diagram of $\mathfrak{B}$. □

For example, if $\sigma = \{R\}$ where R is a binary relation symbol and $\mathscr{F}_0 = \{\langle \overset{\circlearrowleft}{\bullet} \rangle, \langle \bullet \to \bullet \rangle\}$ then $\mathscr{F}_0\neg$ is the class of (undirected) *graphs*, that is the *symmetric*, *irreflexive* relations. If

$$\mathscr{F}_1 = \mathscr{F}_0 \cup \{G : G \text{ is an undirected cycle of odd length}\}$$

then $\mathscr{F}_1\neg$ is the class of bipartite or 2-colourable graphs. For

$$\mathscr{F}_2 = \{\langle \overset{\circlearrowleft}{\bullet} \rangle, \langle \bullet \quad \bullet \rangle, \langle \bullet \rightleftarrows \bullet \rangle, \langle \bullet \nearrow \overset{\bullet}{} \searrow \bullet \leftarrow \rangle\},$$

$\mathscr{F}_2\neg$ is the class of strict linear orders. If the structures in $\mathscr{F}$ are not all finite, $\mathscr{F}\neg$ is not necessarily first order axiomatizable, but the following are equivalent (compare with Adamek [1]):

(i) $\mathscr{C} = \mathscr{F}\neg$ where $\mathscr{F}$ consists of structures of power less than κ, $\omega \leq \kappa \leq \infty$.
(ii) $\mathscr{C}$ is axiomatizable by universal sentences of $L_{\kappa\kappa}$.

For example, if $\mathscr{F}_3 = \mathscr{F}_2 \cup \{\langle \cdots\cdot \to \bullet \to \bullet \to \bullet \to \bullet \rangle\}$ then $\mathscr{F}_3\neg$ is the class of well ordered sets, axiomatizable in $L_{\omega_1\omega_1}$.

2. Axiomatizable quasivarieties

Let a *quasivariety* be any non-empty class of structures of the same similarity type closed under isomorphisms, substructures and cartesian products (notice that we do not ask axiomatizability as in [4] nor closure under reduced products as in [2]). If $\mathbb{A}$ is a non-empty class of structures of type σ, then the *quasivariety* *generated by* $\mathbb{A}$, ISP($\mathbb{A}$), will be the closure of $\mathbb{A}$ under cartesian products, substructures, and isomorphic images. In general, ISP($\mathbb{A}$) does not need to be first order axiomatizable, unless it is closed under ultraproducts, or equivalently under direct limits. It is well known that in case it is axiomatizable, it will be axiomatized by *universal Horn sentences*; that is sentences of the form $\forall x_1 \cdots \forall x_2(\wedge \Phi \rightarrow \theta)$, where Φ is a finite set of atomic formulae and θ is an atomic or negated atomic formula. An axiomatizable quasivariety is called also a *universal Horn class*.

If $\mathbb{A}$ consists of finitely many finite structures of finite type, ISP($\mathbb{A}$) is said to be *finitely generated.* In such case ISP($\mathbb{A}$) is first order axiomatizable as it may be readily seen by showing that the class is closed under ultraproducts. Even better, in this case we may produce an explicit recursive universal Horn axiomatization as follows. By definition, $\mathfrak{A} \in$ ISP($\mathbb{A}$) if and only if $\mathfrak{A}$ is a substructure of a product of elements of $\mathbb{A}$. Equivalently, for any (n-ary) predicate symbol $P \in \sigma \cup \{=\}$ and elements $a_1, \ldots, a_n \in \mathfrak{A}$ such that $\mathfrak{A} \nvDash P[a_1, \ldots, a_n]$, there is an homomorphism $f: \mathfrak{A} \rightarrow \mathfrak{B} \in \mathbb{A}$ such that $\mathfrak{B} \nvDash P[f(a_1), \ldots, f(a_n)]$. When $\mathbb{A}$ and its elements are finite, it may be seen by a compactness argument that it is enough to have an homomorphism f with the above property for each finite substructure $\mathfrak{A} \mid \{a_1, \ldots, a_{n+k}\}$ of $\mathfrak{A}$ (finite partial substructure, if σ has function symbols). But this may be expressed by the following first order sentences $\Theta_{P,k}$:

$$\forall x_1 \cdots \forall x_n \forall x_{n+1} \cdots \forall x_{n+k}[\neg P(x_1, \ldots, x_n) \rightarrow \bigvee_{\mathfrak{B} \in \mathbb{A}} \varphi_{\mathfrak{B}}(x_1, \ldots, x_{n+k})],$$

where $k = 1, 2, 3, \ldots$, $P \in \sigma \cup \{=\}$, and $\varphi_{\mathfrak{B}}(x_1, \ldots, x_{n+k})$ is the formula:

$$\bigvee_{\substack{f:\{x_1,\ldots,x_{n+k}\}\rightarrow B \\ \mathfrak{B} \nvDash P(f(x_1),\ldots,f(x_n))}} \left\{ \bigwedge_{\substack{Q \in \sigma \\ x_{i_1},\ldots,x_{i_r} \in \{x_1,\ldots,x_{n+k}\} \\ \mathfrak{B} \nvDash Q(f(x_{i_1}),\ldots,f(x_{i_r}))}} \neg Q(x_{i_1}, \ldots, x_{i_r}) \right\}$$

saying that there is an homomorphism f from the substructure induced by $\{x_1, \ldots, x_{n+k}\}$ to $\mathfrak{B}$ such that $\mathfrak{B} \nvDash P(f(x_1), \ldots, f(x_n))$. If σ has function symbols, they must be treated as relations Q in the above formulae. Since σ, $\mathbb{A}$ and $\mathfrak{B} \in \mathbb{A}$ are all finite then the sentences $\Theta_{P,k}$ are first order. They are not universal Horn but have the form $\forall \bar{x}\ (\vee_i \wedge_j \sigma_{ij})$ where the σ_{ij} are all negated atomic, except for one which is atomic. By distributing $\vee$ over $\wedge$ and then $\forall$ over $\wedge$, they become conjunctions of universal Horn sentences.

A *graph* will be a non-empty set with an irreflexive, symmetric binary relation, that is, a model (V, E) of the universal Horn sentences

$$G1: \forall x\ (x = x \rightarrow \neg Rxx), \qquad G2: \forall x \forall y\ (Rxy \rightarrow Ryx).$$

The class $\mathscr{C}_n$ of n-colourable graphs, $n \geq 2$, is a finitely generated quasivariety and so a universal Horn class. In fact, it is generated by a single graph, as shown by Nešetřil and Pultr [5], and independently by Wheeler [9]. We give a proof here for the case of bipartite graphs.

LEMMA 2. $\mathscr{C}_2 = \mathrm{ISP}(L)$, where $L = \langle \bullet - \bullet - \bullet - \bullet \rangle$.

Proof. Given a bipartite graph $G = (V, E)$ and vertices $a \neq b$, $a \not\mathrel{E} b$ in G, define a homomorphism $h = h_{ab} : G \to L = \langle 4 - 1 - 2 - 3 \rangle$ with $h(a) \neq h(b)$, $h(a) \not\mathrel{E} h(b)$ as follows. Fix a $\{1, 2\}$-colouring $c : G \to L$. Now, if a and b have the same colour, say 1, define $h(b) = 3$, $h(x) = c(x)$ for $x \neq b$; if a and b have distinct colours, say 1 and 2, define $h(a) = 3$, $h(b) = 4$, and $h(x) = c(x)$ for $x \neq a, b$. The family $\{c, h_{ab}\}_{ab}$ induces a substructure embedding $f(x) = \langle c(x), h_{ab}(x) \rangle_{ab}$ of G in a power of L. □

It follows that any non-bipartite graph generates all bipartite graphs:

COROLLARY 1. *If* $G \notin \mathscr{C}_2$ *then* $\mathscr{C}_2 \subseteq \mathrm{ISP}(G)$.

Proof. From the hypothesis, G must have a full subgraph which is a k-cycle $T = \langle 1 - 2 - \cdots - k - 1 \rangle$ with odd k. If $k \geq 5$ then L is subgraph of T. If $k = 3$ then L is a subgraph of $T \times T$ which contains as full subgraph the 6-cycle: $\langle 11 - 22 - 13 - 21 - 12 - 23 - 11 \rangle$. In any case $L \in \mathrm{ISP}(G)$. □

3. Finitely axiomatizable quasivarieties of graphs

Consider the following proper sub-quasivarieties of $\mathscr{C}_2$ generated by a single graph each:

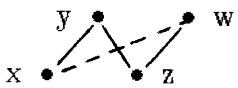
$\mathscr{D}_1 = \mathrm{ISP}(\langle \bullet \rangle) =$ One vertex graphs
$\mathscr{D}_2 = \mathrm{ISP}(\langle \bullet\ \bullet \rangle) =$ Discrete graphs
$\mathscr{D}_3 = \mathrm{ISP}(\langle \bullet - \bullet \rangle) =$ Disjoint sums of $\langle \bullet - \bullet \rangle$ and $\langle \bullet \rangle$
$\mathscr{D}_4 = \mathrm{ISP}(\langle \bullet - \bullet - \bullet \rangle) =$ Disjoint sums of complete bipartite graphs and $\langle \bullet \rangle$.

The right hand descriptions of $\mathscr{D}_1$ and $\mathscr{D}_2$ are obvious. The description of $\mathscr{D}_3$ follows because $\langle \bullet - \bullet \rangle^I$ consists of $2^{|I|}$ copies of $\langle \bullet - \bullet \rangle$. The description of $\mathscr{D}_4$ is less immediate: suppose $G = (V, E)$ is bipartite with complete bipartite components, then $a \neq b$, $a \not\mathrel{E} b$ imply that a and b may be given the same colour and so we do not need the vertex 4 in the construction of the embedding in Lemma 2. Conversely, given a chain $a \mathrel{E} b \mathrel{E} c \mathrel{E} d$ in $S = \langle 1 - 2 - 3 \rangle$ then $|d - a| = \pm 1 \pm 1 \pm 1 \equiv$

1 (mod 2) and so $a \mathrm{E} d$. That is, the universal Horn sentence

$$\forall x \forall y \forall z \forall w \ (Rxy \wedge Ryz \wedge Rzw \rightarrow Rxw)$$

holds in S. Therefore it holds in any substructure of a power of S. But this sentence says that the graph must have complete bipartite components because it prohibits the configuration:



with the broken edge missing. Moreover, it prohibits triangles because it yields $(Rxy \wedge Ryz \wedge Rzx \rightarrow Rxx)$ and graphs are irreflexive. Also, given an $n+3$-cycle it implies that there is an $n+1$-cycle; therefore, it forbids all odd cycles forcing the graph to be bipartite.

From the above discussion, each $\mathscr{D}_i$ is axiomatized by the finite set of sentences $\{G_1, G_2, \varphi_i\}$ where:

$$\varphi_1 : \forall x \forall y \ (x = x \rightarrow x = y)$$

$$\varphi_2 : \forall x \forall y \ (x = x \rightarrow \neg xRy)$$

$$\varphi_3 : \forall x \forall y \forall z \ (Rxy \wedge Rxz \rightarrow y = z)$$

$$\varphi_4 : \forall x \forall y \forall z \forall w \ (Rxy \wedge Ryz \wedge Rzw \rightarrow Rxw).$$

By Lemma 1 and the following Proposition, $\mathscr{D}_i$, $i = 1, 2, 3, 4$, are the only finitely axiomatizable, finitely generated, quasivarieties of graphs.

PROPOSITION (Nešetřil and Pultr [5], Th. 3.2). *$\mathscr{D}_1$, $\mathscr{D}_2$, $\mathscr{D}_3$ and $\mathscr{D}_4$ are the only finitely generated quasivarieties of graphs of the form $\mathscr{F}\neg$ for a finite family $\mathscr{F}$ of finite graphs.*

The proof of this proposition by Nešetřil and Pultr, based on a theorem of Erdös [3], may be modified to yield a stronger result which does not depend on finite generation. First notice the following fact of independent interest:

LEMMA 3. *$\mathscr{D}_1$, $\mathscr{D}_2$, $\mathscr{D}_3$ and $\mathscr{D}_4$ are the only proper sub-quasivarieties of $\mathscr{C}_2$.*

Proof. If $\mathscr{C}$ is a proper sub-quasivariety of $\mathscr{C}_2$, then $L = \langle \bullet - \bullet - \bullet - \bullet \rangle \notin \mathscr{C}$ by Lemma 2. Let F be a minimal full subgraph of L not belonging to $\mathscr{C}$. Cases:

$F = \langle \bullet \rangle$, then $\mathscr{C} = \varnothing$.

$F = \langle \bullet \ \ \bullet \rangle$, then $\mathscr{C} \subseteq \mathscr{D}_1$.
$F = \langle \bullet - \bullet \rangle$, then $\mathscr{C} \subseteq \mathscr{D}_2$.
$F = \langle \bullet - \bullet \ \ \bullet \rangle$, then $F \in \mathrm{ISP}(\langle \bullet - \bullet \rangle)$; hence, $\langle \bullet - \bullet \rangle \notin \mathscr{C}$ and F is not minimal.
$F = \langle \bullet - \bullet - \bullet \rangle$, then $\mathscr{C} \subseteq \mathscr{D}_3$, because the situation 1 E 2 E 3 in a graph of $\mathscr{C}$ implies 1 E 3 or $1 = 3$, but 1 E 3 yields a triangle which can not be in $\mathscr{C}$ and so the degree of any vertex is at most 1.
$F = \langle \bullet - \bullet - \bullet - \bullet \rangle$, then $\mathscr{C} \subseteq \mathscr{D}_4$, since for bipartite graphs prohibiting the subgraph F amounts to imposing the condition φ_4 introduced above.

In each case, the inclusion $\mathscr{C} \subseteq \mathscr{D}_i$ is in fact an equality because the generator of $\mathscr{D}_i$ is a proper subgraph of F; hence, by minimality of F it is not forbidden and so it belongs to $\mathscr{C}$. □

THEOREM 1. *A finitely axiomatizable quasivariety of graphs, distinct from* $\mathscr{D}_1$, $\mathscr{D}_2$, $\mathscr{D}_3$ *and* $\mathscr{D}_4$, *has graphs of arbitrarily large chromatic number.*

Proof. By Lemma 1, a finitely axiomatizable quasivariety $\mathscr{C}$ is defined by finitely many finite forbidden subgraphs $F_1, \ldots, F_n$. If any F_i has cycles, let $k =$ maximum of the lengths of cycles in the F_i's. By the mentioned theorem of Erdös [3] there are graphs of arbitrarily large chromatic number without cycles of length less than $k + 1$; those graphs necessarily belong to $\mathscr{C}$. If some F_j does not have cycles then it is bipartite; hence, $\mathscr{C}$ does not contain $\mathscr{C}_2$, and by the Corollary to Lemma 2 it must be a proper subquasivariety of $\mathscr{C}_2$. Now apply Lemma 3. □

COROLLARY 2. *For any n, no subquasivariety of* $\mathscr{C}_n$ *is finitely axiomatizable, except* $\mathscr{D}_1$, $\mathscr{D}_2$, $\mathscr{D}_3$ *and* $\mathscr{D}_4$.

COROLLARY 3. *No finitely generated quasivariety of graphs is finitely axiomatizable, except* $\mathscr{D}_1$, $\mathscr{D}_2$, $\mathscr{D}_3$ *and* $\mathscr{D}_4$.

As we have mentioned in the introduction, the last Corollary does not hold for quasivarieties of directed graphs nor for quasivarieties of algebras. Consider, for example, the directed cycle D_n of length n. Any power D_n^I consists exactly of $n^{|I|-1}$ copies of D_n; hence, the graphs in $\mathrm{ISP}(D_n)$ are disjoint sums of copies of D_n and chains of length less than n. It follows that $\mathrm{ISP}(D_n)$ is axiomatized by the following finite set of universal Horn sentences:

$$\forall x \forall y \forall z \ (Rxy \wedge Rxz \rightarrow y = z)$$

$$\forall x \forall y \forall z \ (Rxy \wedge Rzy \rightarrow x = z)$$

$$\forall x_1 \forall x_2 \cdots \forall x_n \ (Rx_1x_2 \wedge Rx_2x_3 \wedge \cdots \wedge Rx_{n-1}x_n \rightarrow Rx_nx_1)$$

$$\forall x_1 \cdots \forall x_k \ (Rx_1x_2 \wedge \cdots \wedge Rx_{k-1}x_k \rightarrow \neg\, Rx_kx_1), \qquad \text{for any } k < n.$$

On the other hand, $\mathrm{ISP}(D_n) \neq \mathrm{ISP}(D_m)$ for $n \neq m$. Similarly, if $A_n = (\{1, 2, \ldots, n\}, S)$ with $S(x) = x + 1 \pmod n$, then the associated quasivarieties of algebras $\mathrm{ISP}(A_n)$ are all distinct and axiomatized each by:

$$\forall x \forall y \ (f(x) = f(y) \rightarrow x = y).$$

$$\forall x \ (f^n(x) = x)$$

$$\forall x \neg (f^k(x) = x), \qquad \text{for } 1 \leq k < n.$$

4. A continuum of universal Horn classes of graphs

In contrast to Lemma 3, $\mathscr{C}_n$ has infinitely many axiomatizable subquasivarieties for $n \geq 3$. If U_n denotes the undirected cycle of length n then we have a strict descending chain of universal Horn subclasses of $\mathscr{C}_3$:

$$\mathscr{C}_3 > \mathrm{ISP}(U_5) > \mathrm{ISP}(U_7) > \mathrm{ISP}(U_9) > \cdots.$$

Utilizing Erdös graphs, we may exhibit continuously many universal Horn classes of graphs, although we do not know if this may be obtained inside some $\mathscr{C}_n$, $n \geq 3$.

A family of graphs $\mathscr{F}$ will be called *homomorphism independent* if for any distinct $A, B \in \mathscr{F}$ there is no homomorphism $h : A \rightarrow B$. For any finite structure B, let ϕ_B be the universal closure of $\neg \wedge D^+(B)$, where $D^+(B)$ is the positive atomic diagram of B. Evidently, ϕ_B is (equivalent to) a universal Horn sentence.

LEMMA 4. (a) $A \models \phi_B$ *if and only if there is no homomorphism* $h : B \rightarrow A$.
(b) $\phi_{B_1}, \ldots, \phi_{B_n} \models \phi_B$ *if and only if* $\exists h : B_1 \rightarrow B$ *or* $\cdots$ *or* $\exists h : B_n \rightarrow B$.

Proof. (a) It is well known that some expansion of A satisfies the positive diagram of B if and only if there is an homomorphism from B into A. As B is finite, the existence of such an expansion is expressible by the existential closure of $\wedge D^+(B)$; negating it we get the statement.

(b) Assume $\phi_{B_1}, \ldots, \phi_{B_n} \models \phi_B$. If there is no homomorphism from any B_i into B, then $B \models \phi_{B_i}$ for all i, and so $B \models \phi_B$ which is absurd. Reciprocally, if $\exists h : B_i \rightarrow B$ for some i, and $A \models \phi_{B_1}, \ldots, \phi_{B_n}$ then there is no $f : B_i \rightarrow A$. A fortiori, there is no $g : B \rightarrow A$; that is $A \models \phi_B$. □

COROLLARY 4. *The family of sentences* $\{\phi_B : B \in \mathscr{F}\}$ *is logically independent if and only if* $\mathscr{F}$ *is an homomorphism independent family of structures.*

Let $ch(G)$ denote the chromatic number of G, and $og(G)$ the smallest length of odd cycles in a non bipartite graph G. Given an homomorphism $f: A \to B$, then $ch(A) \leq ch(B)$; moreover, the homomorphic image of an odd cycle of length n in A contains an odd cycle of length $m \leq n$; hence, $og(B) \leq og(A)$. Utilizing Erdös' theorem [3] we may construct a sequence of graphs $E_1, E_2, E_3, \ldots$ such that $ch(E_n) < ch(E_{n+1})$, $og(E_n) < og(E_{n+1})$. This family is homomorphism independent by the last observations.

THEOREM 2. *There are continuously many universal Horn classes of graphs.*

Proof. Any subset of the logically independent family $\{\phi_{E_i} : i \in \omega\}$ axiomatizes a different quasivariety. □

QUESTION. Does $\mathscr{C}_n$ contain an homomorphism independent infinite family of finite graphs, for $n \geq 3$? In such case $\mathscr{C}_n$ would contain a continuum of universal Horn subclasses.

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