Perturbations of periodic Sturm–Liouville operators

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The work by G.W. Hill in 1886 has led to the 'Hill's equation' for the linear second-order ordinary differential equation with periodic coefficients,

$$\frac{1}{r_0}\left(-\frac{\mathrm{d}}{\mathrm{d}x}p_0\frac{\mathrm{d}}{\mathrm{d}x}+q_0\right)y=\lambda y.$$

The above time-independent Schrödinger equation in one spatial dimension with a periodic potential is used within the description of certain effects of atomic nuclei in a crystal. Here the spectral parameter λ has a physical interpretation as the total energy of an electron, and the band structure of the essential spectrum to regions of admissible and forbidden energies. Moreover, impurities (i.e. perturbations) can lead to additional discrete energy levels in the forbidden regions (i.e. eigenvalues in the gap of the essential spectrum).

Here we investigate the change of the spectrum under L^1-assumptions on the differences of the coefficients. We describe the essential spectrum and the absolutely continuous spectrum of the perturbed operator. If a finite first moment condition holds for the differences of the coefficients, then at most finitely many eigenvalues appear in the spectral gaps. This observation extends a seminal result by the Ukrainian mathematician Rofe-Beketov from the 1960ies.

This is based on joint works with J. Behrndt (Graz), P. Schmitz (Ilmenau), and G. Teschl (Vienna).