Basics of Convex and Discrete Geometry

Instructor: Susanna Dann, office H 402
Time: MW 16:00 – 17:50
Location: C-107
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Descripción: This course will serve as an introduction to convexity. We start with the basics of convex sets, with theorems of Helly, Radon and Caratheodory and their applications. We continue with convex polytopes and prove three jewels of convexity: theorems of Steiniz, Cauchy and Minkowski. Lastly, we start to develop the beautiful Brunn-Minkowsky theory that combines ideas of algebra, geometry and analysis.

Prerequisits: MATE 1105 Linear Algebra, MATE 1207 Calculo Vectorial, MATE 1102 Matematica Estructural.

Objectives: This course will provide students with basic notions of convexity. We will discuss some classical and beautiful results in convexity and their connections and applications to different areas of mathematics. We will point out open problems related to the material we cover. Along the way, students will sharpen und deepen their knowledge of linear algebra and calculus.

Content:

Part One: Separation theorem, Caratheodory`s theorem, Radon`s theorem, Helly`s theorem, Blaschke selection principle and their applications (e.g. Given a finite set of points in Euclidean n-dimensional space with cardinality greater or equal to n+1. What is the minimal radius of the ball containing this set?)

Part Two: Polytopes: vertices, faces and facets; Euler`s theorem; Main theorem for polytopes: equivalence of definitions as a convex hull of finitely many points or intersection of finitely many half-spaces; duality. Theorems of Steiniz, Cauchy and Minkowski.

Part Three: Brunn-Minkowski theory: Minkowski`s theorem (volume of a linear combination of convex bodies is a homogeneous polynom in the coefficients), Brunn-Minkowski inequality, Brunn`s theorem.

If time allows: Steiner symmetrization and isoperimetric inequality.

Evaluation:

Homework:

There will be bi-weekly homework assignments, 6 total. Each homework is worth 15points of the final grade.

Collaboration Policy: you can and are encouraged to collaborate on the homework assignments. However, you are required to turn in individual homework and to write the names of your collaborators.

Midterm Exams: There will be two in class exams (March 13, May 8), 90 min each. Each exam is worth 10 points. The lowest grade is dropped. A single double-sided page of cheat-sheet is allowed.

Bibliografía:

- A. Barvinok, "A Course in Convexity".
- T. Bonnesen and W. Fenchel "Theory of Convex Bodies".
- Ludwig Danzer, Branko Gruenbaum, and Victor Klee "Helly`s theorem and its relatives".
- R. Gardner "THE BRUNN-MINKOWSKI INEQUALITY" http://faculty.wwu.edu/gardner/gorizia12.pdf
- Branko Gruenbaum "Convex Polytopes".
- J. Matousek "Lectures on Discrete Geometry".
- Roger Webster, "Convexity".
- Guenter Ziegler "Lectures on Polytopes".