Algebraic number theory

Objectives

This course is design as an introduction to the classical algebraic number theory and it can be seen as a natural continuation of Abstract Algebra II. Its goal is to familiarize students with the unique factorization of ideals in algebraic number fields, which will lead us to the definition of the ideal class group. Furthermore, we will introduce Minkowskis geometry of numbers in order to prove Dirichlets unit theorem. As an example, we will take a closer look at the structure of the quadratic and cyclotomic fields. At the end of the course, some application of algebraic number theory to solving Diophantine equations will be discussed.

Contents

Linear algebra in number fields: norm, trace and discriminant; Rings of integers; Unique factorization of ideals; Norms of ideals; Ramification and degree; The ideal class group; The Kummer-Dedekind theorem; Quadratic and cyclotomic number fields; Geometry of numbers: Dirichlet's unit theorem; Units in quadratic fields.

The course will cover the following topics:

1. Introduction: number fields, rings of integers. (1 week)
2. Linear algebra in number fields: norm, trace, discriminant, integral basis. (2 weeks)
3. Ideal arithmetic: ideals, Dedekind rings, unique factorization of ideals, norms of ideals, ramification and degree, the ideal class group, Kummer-Dedekind theorem. (3 weeks)
4. Geometry of numbers: lattices and Minkowski's theorem, finiteness of the ideal class group, Dirichlet's unit theorem. (3 weeks)
5. Quadratic and cyclotomic fields (3 weeks)
6. The Rieman and Dedekind zeta functions (1 week)
7. Explicit computations and applications (2 weeks)

Literature

1. I. Stewart, D. Tall, *Algebraic Number Theory and Fermat's Last Theorem*, A K Peters/CRC Press, Third edition, 2001.

2. K. Ireland, M. Rosen, *A classical introduction to modern number theory*, Springer, Second edition, 1990.

3. Z. I. Borevich, I. R. Shafarevich, *Number theory*, Academic Press 1967.

4. M. R. Murthy, J. Esmonde, *Problems in algebraic number theory*, Springer, Second edition, 2010.

Evaluation

Partial exams: 25% (each)
Homework: 20%
Final exam: 30%

Professor

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